Chapter Four

VECTOR QUANTITIES AND THEIR APPLICATIONS

KEY IDEAS

Vectors are quantities that have both magnitude and direction. Forces and displacements are examples of vector quantities. A vector is best represented by an arrow: it points in the vector's direction, and its length is proportional to the vector's magnitude.

Vector quantities can be combined with one another. The process is known as vector addition, and the sum is equal to the combined result of the interaction of the individual vectors. The vector sum is also known as the resultant vector. Vectors can be added geometrically by using a measured scale or mathematically by using the laws of algebra and trigonometry.

The inverse process of vector addition is known as resolution. In resolution, a single vector is separated into a number of components. Resolution usually simplifies the solution of vector-related problems such as two-dimensional motion, static equilibrium, and motion on an inclined plane.

KEY OBJECTIVES

At the conclusion of this chapter you will be able to:

- Define the terms *scalar* and *vector* and list scalar and vector quantities.
- Represent a vector quantity by an arrow drawn to scale.
- Relate the direction of a vector to compass directions.
- Define the term *resultant vector.*
- Add vector quantities (1) graphically and (2) algebraically.
- Relate vector subtraction to vector addition.
- Define the term *vector resolution*, and resolve a vector into its *x*- and *y*-components. (Treatment is limited to two-dimensional analysis.)
- Add vector quantities by adding their *x*- and *y*-components.
- Define the term *static equilibrium.*
- Solve static equilibrium problems.
- Identify and calculate the parallel and perpendicular components of an object's weight when the object is on an inclined plane.
- Solve problems involving motion on an inclined plane.
- Solve problems involving the motion of an object in two dimensions.

✱ 4.1 INTRODUCTION

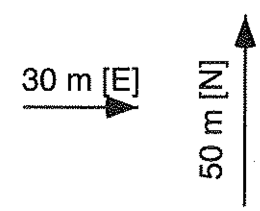
If we want to measure the mass of an object, it makes no difference whether we face in any particular direction while taking the measurement. Quantities such as mass, time, and temperature are called **scalar** quantities because they can be described solely in terms of their magnitudes (sizes).

If we are traveling in a car at 60 miles per hour, however, it makes a great deal of difference which way the car is facing: A car traveling east from Los Angeles might end up in New York, but if the car faced west it could end up in the Pacific! Quantities such as velocity, acceleration, and force are called vector quantities because they must be described in terms of their magnitudes *and* directions.

✱ 4.2 DISPLACEMENT, AND REPRESENTATION OF VECTOR QUANTITIES

In our study of motion in Chapter 2, we defined the simplest vector, called *displacement*, as a directed change in the position of an object. In other words, displacement is a distance (magnitude) in a given direction. For example, the quantity 30 meters [east] represents a displacement. (In this book, we will indicate the direction in brackets following the magnitude of the vector.)

We can easily represent a vector quantity by using an arrow. The length of the arrow represents the magnitude of the vector, and the direction of the vector points from the tail of the arrow toward its tip. The diagrams illustrate two displacement vectors.



It should be obvious that the arrows shown above are not really 30 meters and 50 meters long. They are drawn *to scale*, as a map maker does when creating a map. In this instance, 1 centimeter of arrow length represents 20 meters of distance, and the arrows are 1.5 and 2.5 centimeters long, respectively.

✱ indicates that material is part of the New York State core curriculum.

PROBLEM
If 0.20 meter represents a displacement of 30. kilometers, what displacement is represented by a vector 0.80 meter long?

SOLUTION

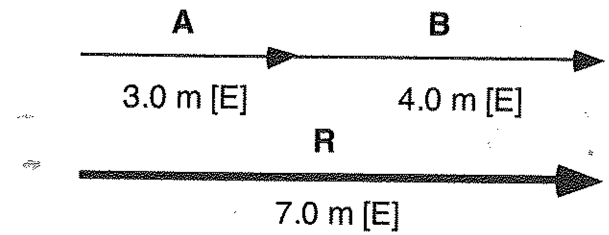
On occasion, it may be necessary to measure the angle that the vector makes with the *y*-axis (vertical). The conversion is an easy matter since the angles are complementary to one another. (In the diagram above, the angles would be 60° east of north, 20° west of north, and 45° west of south.)

4.3 VECTOR ADDITION

An airplane traveling at 300 meters per second [east] enters the jet stream, whose velocity is 100 meters per second [north]. What is the velocity of the airplane when measured by a person on the ground?

This problem, as well as a host of related problems, can be solved by a process called *vector addition*. When applied to vector quantities, the term *addition* means calculating the net effect of two or more vectors acting on the same object. It does not matter whether the vector quantities are velocities, displacements, forces, or others. The techniques of vector addition are the same in each case.

Suppose a person walks 3.0 meters [east] and then walks 4.0 meters [east]. What is the net displacement of the person? This is a simple problem in vector addition.



The addition is performed by placing the two vectors (drawn to scale, 1 cm = 1 m) in a line, head to tail, as shown in the diagram. The sum of the vectors (indicated by the bold arrow) is called the **resultant vector (R)** and is determined by drawing an arrow beginning at the tail of the first vector and extending to the head of the second vector. The vector equation for this sum is:

A	+	**B**	=	**R**
3.0 m [E]		4.0 m [E]		7.0 m [E]

Note that symbols representing vector quantities are set in boldface type (**A**). Another way of representing a vector quantity is to place an arrow above the symbol ($\vec{A}$). In this book we will use boldface type for vectors.

When vectors are oriented in opposite directions, their magnitudes are *subtracted*, as shown in the following problem.

PROBLEM
A bird flies north 3.0 kilometers and then south 4.0 kilometers. What is the resultant displacement of the bird?

A	+	B	=	R
3.0 m [N]		4.0 km [S]		1.0 km [S]

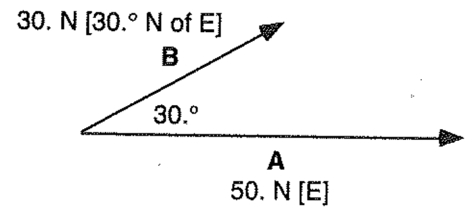
Now we will consider how vectors at right angles are added.

PROBLEM

A plane flies with a velocity of 300. meters per second [east] and enters the jet stream, whose velocity is 100. meters per second [north]. What is the resultant velocity of the plane?

PROBLEM*

As shown in the diagram, an object is subjected to two concurrent forces: **A** = 50. newtons [east] and **B** = 30. newtons [30.° north of east]. What is the resultant force on the object?



Then we have:

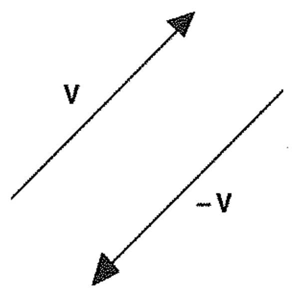
* Solving by this method is

✱ 4.4 VECTOR SUBTRACTION

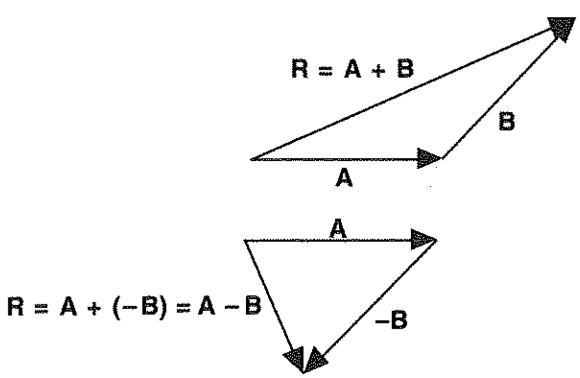
Subtraction is really a special case of addition. In mathematics, the expression $a - b$ is equivalent to the expression $a + (-b)$, where $-b$ is called the negative of b. Similarly, we subtract two vectors by adding one of them to the negative of the other:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Since vectors have both magnitude and direction, the negative of a vector **V** is equal to **V** in magnitude but opposite in direction, as illustrated below.



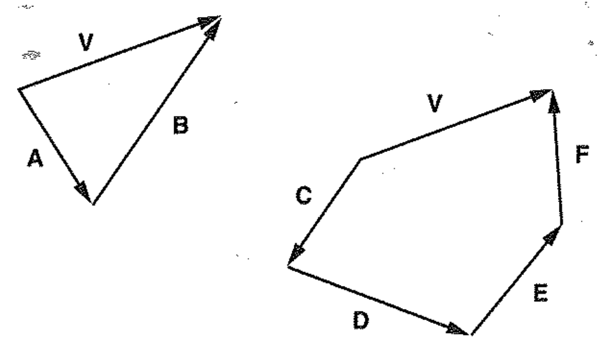
The following diagram compares vector addition and subtraction:



✱ 4.5 RESOLUTION OF VECTORS

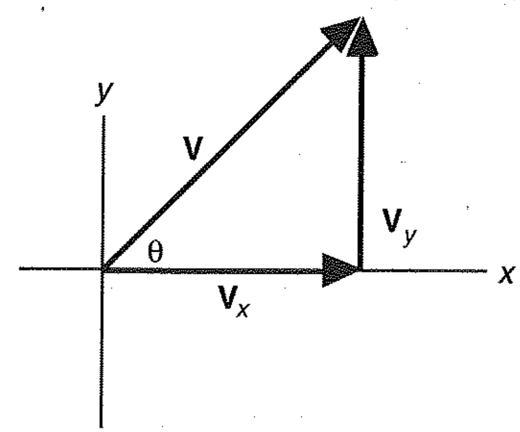
If an automobile is traveling at 20 meters per second at 45° north of east, we know that the vehicle is traveling north and east at the same time. But *how fast* is it going in each direction? To solve this problem we use a technique known as **vector resolution.**

Resolving a vector means breaking it down into a number of components that will add to produce the original vector, as shown in the diagram.



Here **A** and **B** are components of **V**, as are **C**, **D**, **E**, and **F**. As the diagram implies, a vector can be resolved into any number of components in virtually any direction. However, it is usually most helpful to resolve a vector into just *two* components that lie along the x- and the y-axis (i.e., the components are *perpendicular* to each other).

This resolution is illustrated in the following diagram:



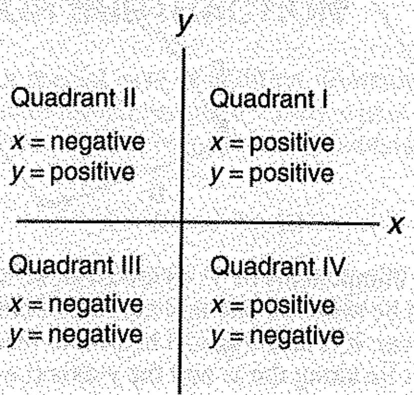
$\mathbf{V}_x$ and $\mathbf{V}_y$ are called the x- and y-components of **V**, respectively, and θ is the angle that **V** makes with the x-axis. Since the vector and its components form a right triangle, it follows that:

PHYSICS CONCEPTS

$$\frac{V_x}{V} = \cos\theta \Rightarrow V_x = V\cos\theta$$

$$\frac{V_y}{V} = \sin\theta \Rightarrow V_y = V\sin\theta$$

- and y-components of a vector have algebraic signs that depend on the
ant in which the original vector lies, as shown in the diagram below:



LEM
e x- and y-components of the force **F** = 30. newtons [30.° north of east].

TION
the for … positive:

LEM
e is flying at 500 meters per second at 37° south of east. How fast is
g east, and how fast is it flying south?

4.6 USING RESOLUTION TO ADD VECTORS

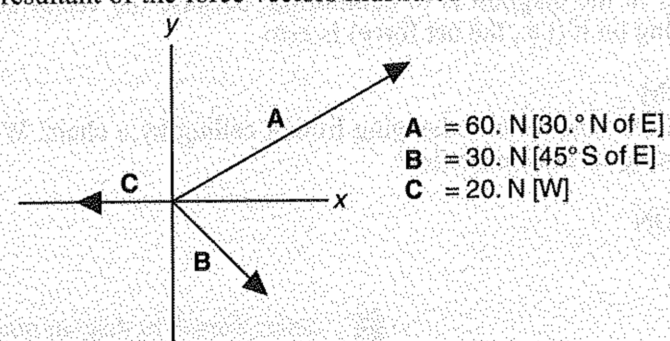
To solve a vector problem graphically (i.e., using a ruler and protractor) is usually fairly easy. However, we cannot achieve the precision of a mathematical solution. Unfortunately, mathematical solutions using the laws of cosines and sines are tedious and difficult.

We now develop a technique for vector addition that is relatively simple and yet gives us precision. The rules for this method are as follows:

1. Resolve each of the vectors to be added into its x- and y-components. Remember to include the proper sign (positive or negative), depending on the quadrant of each vector.
2. Be aware that, if a vector lies on the x-axis, its y-component is zero; if a vector lies on the y-axis, its x-component is zero.
3. Add the x-components together to produce the x-component of the resultant vector ($\mathbf{R}_x$).
4. Add the y-components together to produce the y-component of the resultant vector ($\mathbf{R}_y$).
5. Calculate the magnitude of the resultant vector by means of the Pythagorean theorem: $R = \sqrt{\mathbf{R}_x{}^2 + \mathbf{R}_y{}^2}$
6. Find the angle that the resultant vector makes with the x-axis from this relationship: $\tan\theta = \frac{\mathbf{R}_y}{\mathbf{R}_x}$.

Let's add three vectors using this method.

PROBLEM
Find the resultant of the force vectors illustrated below:



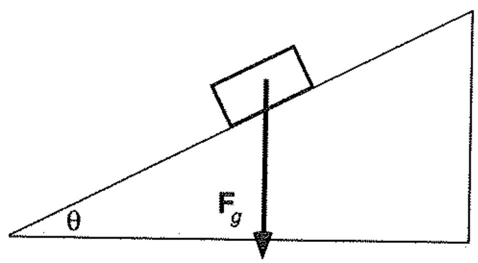
SOLUTION
We begin by calc … ector:

Substituting in the

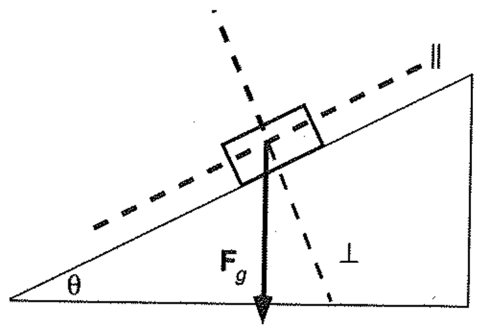
10°)

4.8 INCLINED PLANES

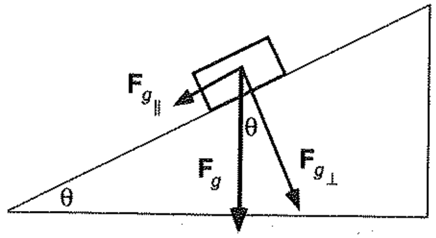
Let us now consider an object whose weight is $\mathbf{F}_g$, sliding down a frictionless inclined plane that makes an angle of θ with the ground. The diagram illustrates the situation.



If we examined the object closely, we would find that it has an acceleration directly parallel to the surface of the plane. To analyze the forces acting on the object, we choose a set of axes that are parallel and perpendicular to the surface of the plane as shown in the diagram below.



It is most convenient to resolve the weight of the object into components that are parallel and perpendicular to the inclined plane.



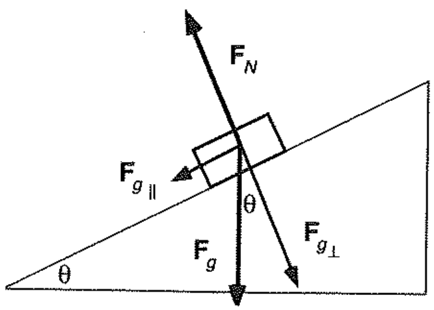
Simple trigonometry yields these relationships

$$F_{g\parallel} = F_g \sin\theta \quad \text{and} \quad F_{g\perp} = F_g \cos\theta$$

Hint: The above equations as they appear are not printed on the Reference Table and should therefore be memorized.

Since the object's acceleration is parallel to the plane, force $\mathbf{F}_{g\parallel}$ is unbalanced. The object does not accelerate in the perpendicular direction, therefore, $\mathbf{F}_{g\perp}$ is balanced by the normal force $\mathbf{F}_N$, which also acts perpendicularly to the surface of the plane and is equal to $\mathbf{F}_g \cos\theta$.

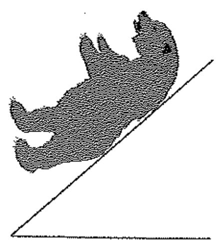
The diagram below illustrates all of the forces acting on the object as it travels down the plane.



PROBLEM

A 5000-newton polar bear slides down an ice slide without friction. If the slide makes an angle of 37° with the ground, calculate the accelerating force and the normal force on the polar bear.

SOLUTION



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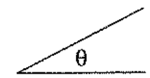


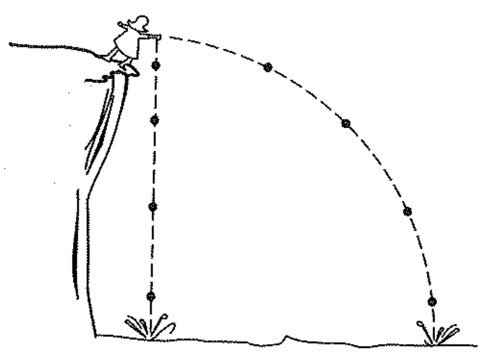
PROBLEM

A wooden crate whose weight ($\mathbf{F}_g$) is 1200 newtons slides down a metal ramp at constant speed as it is unloaded from an airplane. The ramp makes an angle of 30° with the horizontal. Calculate the force of friction ($\mathbf{F}_f$), the normal force ($\mathbf{F}_N$), and the coefficient of kinetic friction (μ_k) between the crate and the ramp.

SOLUTION

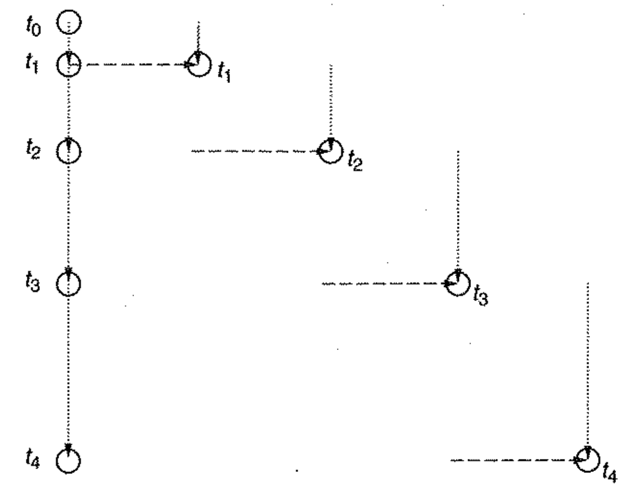
4.9 MOTION IN A PLANE (TWO-DIMENSIONAL MOTION)

Every student of physics is familiar with the following problem: An object is thrown horizontally away from a tall cliff at the same instant that an identical object is dropped vertically from the same height down the cliff. If air resistance is ignored, which object will reach the ground first? The diagram illustrates the problem.



As unreasonable as it may seem, both objects hit the ground at exactly the same instant. Why does this happen? The explanation is based on the fact that the force due to gravity is the only factor that governs the downward motion of both objects. Since both objects fall at the same rate, they reach the ground at the same time. The difference is that the object thrown outward travels horizontally as well as vertically.

The following diagram represents a time lapse picture of this situation.



Notice that at any instant the vertical displacement of each object is the same. Also notice that in each time interval the horizontal displacement traveled by the object thrown outward is constant.

In the y-direction, both objects can be considered as falling freely from rest (with an acceleration of -9.8 meter per second per second) and are governed by the equations of motion developed in Chapter 2:

$$\mathbf{d}_y = \mathbf{v}_{iy}\, t + \frac{1}{2}\mathbf{a}_y\,(t)^2$$

$$\mathbf{v}_{fy}{}^2 = \mathbf{v}_{iy}{}^2 + 2\mathbf{a}_y\,\mathbf{d}_y$$

$$\mathbf{a}_y = \frac{\Delta \mathbf{v}_y}{t} = \frac{\mathbf{v}_{fy} - \mathbf{v}_{iy}}{t}$$

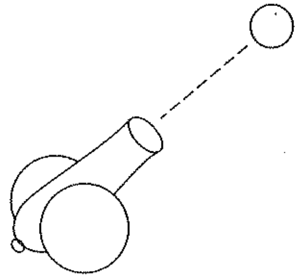
In the x-direction, the thrown object travels at constant speed. Since $\mathbf{v}_x$ does not change, it is equal to the average speed, and we can use a variation of the equation $\mathbf{v} = \mathbf{d}/t$ to describe its motion:

$$\mathbf{v}_x = \frac{\mathbf{d}_x}{t}$$

It is important to note that the x- and y-motions of the two objects are *completely independent* of one another while occurring in the same amount of time.

PROBLEM

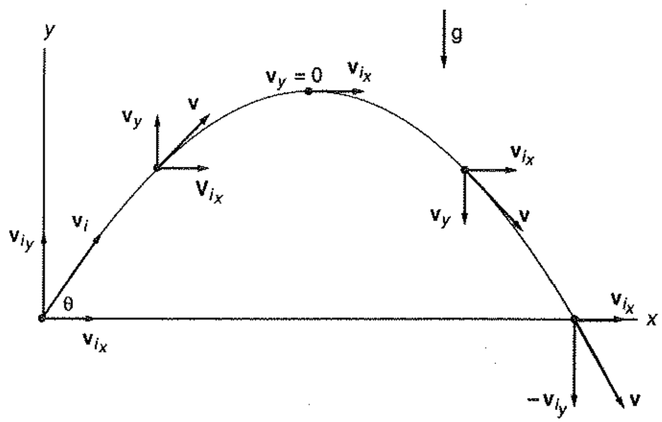
An object is thrown outward from a cliff with a horizontal velocity of 20. meters per second. With air resistance ignored, the object takes 15 seconds to reach the bottom of the cliff. Calculate (a) the height of the cliff and (b) the horizontal distance traveled.



The shape of the path (known as the *trajectory*) is parabolic. If the initial velocity of the object is $\mathbf{v}_i$ and the angle is θ, we must resolve the velocity into its x- and y-components before we can solve any problem undergoing this type of motion:

$$\mathbf{v}_{i_x} = \mathbf{v}_i \cos\theta$$
$$\mathbf{v}_{i_y} = \mathbf{v}_i \sin\theta$$

In the x-direction, the object travels at constant speed. In the y-direction, we treat the object as if it were tossed directly upward and then returned to the ground: Its speed decreases as it rises and increases as it returns to the ground.



The motion equations developed in Chapter 2 can be used to solve situations involving this type of motion.

It is important to note that the up portion of the motion graph is symmetrical to the down half; therefore, $t_{up} = t_{down}$. In addition, the velocity in the vertical direction at any point on the up side is equal in magnitude and opposite in direction to the symmetrical point on the down side of the graph. This information can be used to quickly calculate the total time of the object's flight using the equation $\mathbf{a} = \mathbf{v}_y/t$. The initial velocity can be divided by 9.81 m/s^2 to solve for the time it takes for the up portion. The total time is calculated simply by multiplying the time obtained by 2.