Aim: Students will be able to explore the quadratic functions and understand the role of $\boldsymbol{a}$ in $\boldsymbol{y}=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c}$.
DoNow: Graph the functions on graphing paper and compare their graphs:

1. $y=x^{2}$

| $x$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |

2. $y=2 x^{2}$

| $x$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |

3. $y=-x^{2}$

| $x$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |

4. $y=-2 x^{2}$

| $x$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $y$ |  |  |  |  |  |

## 1. Parabolas \& Axis of Symmetry

a. The path of a projectile is called a parabola, and the line in the middle of the parabola is the axis of symmetry.
b. Exercise: Draw the axis of symmetry of the following parabolas.




c. Each parabola that you have seen is the graph of a quadratic function.

## 2. Quadratic Function \& Standard Form

a. For $\boldsymbol{a} \neq \boldsymbol{0}$, the function $\boldsymbol{y}=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c}$ is a quadratic function. When a quadratic function is written in the form $\boldsymbol{y}=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}$ $+c$, it is in standard form.
b. Exercise: Name the value of $a, b$, and $c$ for each quadratic function.
i. $y=x^{2}+3 x+5 \quad a=$ $\qquad$ , $b=$ $\qquad$ , $c=$ $\qquad$ iv. $y=5 x^{2}-12$
$a=$ $\qquad$ , $b=$ $\qquad$ ,$c=$ $\qquad$
ii. $y=-2 x^{2}-5$
$a=$ $\qquad$ , $b=$ $\qquad$ , $c=$ $\qquad$ v. $y=-3 x^{2}$
$a=$ $\qquad$ , $b=$ $\qquad$ ,$c=$
iii. $y=2 x^{2}+x+7 \quad a=$ $\qquad$ , $b=$ $\qquad$ $c=$ $\qquad$ vi. $y=4 x^{2}+2 x$
$a=$ $\qquad$ $b=$ $\qquad$ $c=$ $\qquad$
c. Exercise: Write each quadratic function in standard form.
i. $y=3-4 x^{2}+2 x$ standard form $\qquad$
ii. $y=7 x+3+5 x^{2}$ standard form $\qquad$
3. The role of " $a$ " - Vertex (the highest or lowest point of a parabola)
a. When a parabola opens upward, the y-coordinate of the vertex is the minimum value of the function.
b. When a parabola opens downward, the y-coordinate of the vertex is the maximum value of the function.
c. How $\boldsymbol{a}$ affect the parabola?

| $a>0$, | opens upward |
| :--- | :--- |
| $a<0$, | opens downward |

a is positive, smiley face
a is negative, sad face

d. Exercise: Tell whether each parabola opens upward or downward and whether the y-coordinate of the vertex is a maximum or minimum.
i. $y=0.2 x^{2}$ opens $\qquad$ $y$-coordinate of the vertex is a $\qquad$
ii. $y=-0.5 x^{2}$
opens $\qquad$ $y$-coordinate of the vertex is a $\qquad$
4. The role of " $a$ " -- Width of a parabola
a. The value of $\boldsymbol{a}$ also affects the width of a parabola.

| The larger the $\|a\|$, the narrower the graph <br> The smaller the $\|a\|$, the wider the graph |
| :--- | :--- |

b. Example: The quadratic functions: $y=-4 x^{2}, y=x^{2} / 4$, and $y=x^{2}$, where $y=-4 x^{2}$ is the narrowest.

c. Exercise: Roughly draw and order a group of quadratic functions $y=2 x^{2}, y=x^{2} / 2, y=x^{2}$.

d. Order each group of quadratic functions from widest to narrowest graph.

1) (A) $y=4 x^{2}$, (B) $y=-2 x^{2}$, (C) $y=\frac{1}{3} x^{2}$
2) (A) $y=-2 x^{2}$, (B) $y=-x^{2}$, (C) $y=\frac{1}{5} x^{2}$
e. Match each of the following function with corresponding graph.
3) $y=3 x^{2}$
4) $y=-3 x^{2}$
5) $y=\frac{1}{3} x^{2}$

6) $y=-\frac{1}{3} x^{2}$
$\operatorname{Aim}$ : Students will be able to explore the quadratic functions and understand the role of $\boldsymbol{c}$ and $\boldsymbol{b}$ in $\boldsymbol{y}=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c}$.
DoNow: Graph the functions on graphing paper and compare their graphs:
1. Changing $c$ : Changing the value of $\boldsymbol{c}$ in the function $\boldsymbol{y}=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{c}$ changes the vertical position of the parabola. $\boldsymbol{c}$ is the $\boldsymbol{y}$ intercept for the graph.

| If $c>0$, | the graph shifts up. |
| :--- | :--- |
| If $c<0$, | the graph shifts down. |

2. Example: Graphing and comparing the quadratic functions $y=-x^{2}, y=-x^{2}+3, y=-x^{2}-1$, and find the maximum value for each equation.
a. $y=-x^{2}+3$, shifts the parabola $y=-x^{2}$ up 3 units.
b. $y=-x^{2}-1$, shifts the parabola $y=-x^{2}$ down 1 unit.
3. Exercise: If graph (a) is $\mathrm{y}=0.5 \mathrm{x}^{2}$, can you graph
a. $y=0.5 x^{2}$
b. $y=0.5 x^{2}+2$
c. $y=0.5 x^{2}-2$
d. $y=-0.5 x^{2}+2$
e. $y=-0.5 x^{2}-2$
(a)

(b)



. Graphing Calculator: Using graphing calculator to graph quadratic functions.
a. $y=x^{2}-2 x$
b. $y=x^{2}-3 x+2$
c. $y=x^{2}-9$


4. Real-life data range: No negative part for $x$ or $y$ values or for both values.
5. Describe whether each quadratic function has a maximum or minimum.

1) $y=2 x^{2}+7$maximumminimum
value $=$ $\qquad$
2) $y=x^{2}-3$maximumminimum
value $=$ $\qquad$
3) $y=-x^{2}-4$maximumminimum
4) $y=-5 x^{2}+12$maximum
minimum
value $=$ $\qquad$
value $=$ $\qquad$
7. Match each of the following function with corresponding graph.
1) $y=x^{2}-1$
2) $y=x^{2}+2$
3) $y=-x^{2}+1$
4) $y=-x^{2}-2$
5) $y=3 x^{2}+2$
6) $y=-\frac{1}{2} x^{2}-2$

8. If graph (1) is $y=x^{2}$, can you graph
2) $y=x^{2}-3$
3) $y=x^{2}+3$
4) $y=-2 x^{2}+3$
5) $y=-2 x^{2}-3$
(1)

(2)

(3)



9. Changing $\boldsymbol{b}$ : So far, we only deal with $\boldsymbol{b}=\boldsymbol{0}$. When $\boldsymbol{b} \neq \boldsymbol{0}$, the parabola shifts right or left. The axis $\boldsymbol{o f}$ symmetry is no longer the $y$-axis.
1) The graph of $\boldsymbol{y}=\boldsymbol{a} \boldsymbol{x}^{2}+\boldsymbol{b} \boldsymbol{x}+\boldsymbol{c}$, where $\boldsymbol{a} \neq \boldsymbol{0}$, has the line

$$
x=\frac{-b}{2 a} \text { as its axis of symmetry }
$$

2) The $\boldsymbol{x}$-coordinate of the vertex is $\frac{-b}{2 a}$ and the

## $y$-intercept of the graph is $c$

10. Example: Graph the quadratic functions $y=5-4 x-x^{2}$.
1) Rewrite the function in standard form: $y=-x^{2}-4 x+5$
2) Find the $\boldsymbol{y}$-intercept $\boldsymbol{c}: 5$.
3) Find the equation of the axis of symmetry: $\boldsymbol{x}=-\boldsymbol{b} /(2 \boldsymbol{a})=-(-4) /(2 *(-1))=-2$, i.e., $\boldsymbol{x}=-2$
4) Find the coordinates of the vertex: $\boldsymbol{x}$-coordinate of the vertex is -2 . Substitute -2 for $\boldsymbol{x}$, and we get $\boldsymbol{y}=-(-2)^{2}-4(-2)$ $+5=9$. So, the vertex is at $(-2,9)$.
5) Make the table of value and graph the function.
11. Exercise: Find the equation of the axis of symmetry for the following quadratic functions.
1) $y=x^{2}-4 x+4 \quad$ axis of symmetry: $\qquad$
2) $y=x^{2}+6 x+5 \quad$ axis of symmetry: $\qquad$
12. Exercise: Find the coordinates of the vertex and the $\boldsymbol{y}$-intercepts for the following quadratic functions.
1) $y=x^{2}-2 x+1 \quad$ vertex: $\qquad$ $y$-intercept: $\qquad$
2) $y=x^{2}+4 x+4 \quad$ vertex: $\qquad$ $y$-intercept: $\qquad$
13. Exercise: Graph $y=x^{2}-6 x+9$. Find the equation of the axis of symmetry, the coordinates of the vertex, and the $y$ intercept.
axis of symmetry: $\qquad$
vertex: $\qquad$
$y$-intercept: $\qquad$

Find the equation of the axis of symmetry, coordinates of the vertex, and y-intercept and graph the following quadratic functions.
a. $y=x^{2}-4 \quad$ axis of symmetry: $\qquad$ vertex: $\qquad$ $y$-intercept: $\qquad$


b. $y=x^{2}-5 x+6$ axis of symmetry: $\qquad$ vertex: $\qquad$ $y$-intercept: $\qquad$


c. $y=-x^{2}+3 x-2$ axis of symmetry: $\qquad$ vertex: $\qquad$ $y$-intercept: $\qquad$



