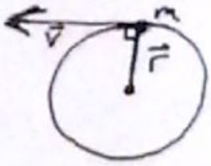
Angular Momentum Notes

Point mass

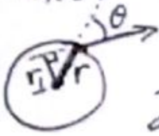
$\vec{p} = m \cdot \vec{v}$ $\quad \frac{kg \cdot m}{s}$ $\quad \vec{\omega} = \frac{\vec{v}}{r}$

$\vec{L} = I \vec{\omega}$ $\quad \frac{kg \cdot m^2}{s}$



$\vec{L} = I\vec{\omega} = mr^2\left(\frac{\vec{v}}{r}\right) = (m\vec{v})r = p \cdot r$

$\boxed{\vec{L} = \vec{p} \cdot r}$ $\quad L = \mathcal{L}$ $\quad \boxed{L = m\vec{v}r}$



NOT ON AP EXAM

$\mathcal{L} = \vec{r}\vec{p}\sin\theta$

$\theta = \angle(\vec{r}, \vec{p})$

$\mathcal{L} = \vec{r}_\perp \vec{p}$ $\quad \boxed{\mathcal{L} = \vec{r}_\perp m\vec{v}}$

Conservation of Angular Momentum

$\tau = \frac{\Delta \mathcal{L}}{\Delta t}$ $\quad \Delta\mathcal{L} = \mathcal{L}_f - \mathcal{L}_0 = \tau \Delta t$

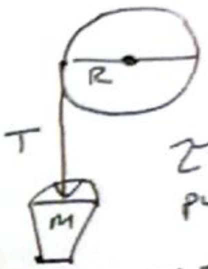
If $\Sigma \tau = 0$, then $\mathcal{L}_f = \mathcal{L}_0$

Work

Work $= \tau \cdot \Delta\theta$ just like work $= F \cdot \Delta x$ $\quad S = R \cdot \theta$

Work $= KE_f - KE_0 = \tau \cdot \Delta\theta = \frac{1}{2} I \omega^2$

$\tau \cdot \Delta\theta = KE_f = \frac{1}{2} I \omega_F$



$\tau = I\alpha$

$\tau = R \cdot T$ pulley

$\omega_F = \sqrt{\frac{2\tau \cdot \Delta\theta}{I}} = \sqrt{\frac{2RT\Delta\theta}{I}}$

$\alpha I = RT$

$\alpha = \frac{RT}{I}$

$\Delta\theta = \frac{1}{2}\left(\frac{RT}{I}\right)t^2 = \frac{1}{2}(\alpha)t^2$

$\omega_f = \sqrt{\frac{2RT\,(\frac{1}{2})(RT/I)\,t^2}{I}} = \left(\frac{RT}{I}\right)t$

$\Delta L = \tau \cdot \Delta t$ just like $\Delta p = F \cdot \Delta t$ Impulse

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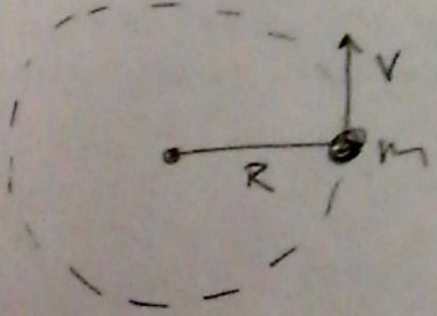

Summary

$I = mR^2$

$K_{trans.} = \frac{1}{2} m v^2$

$K_{Rot.} = \frac{1}{2} I \omega^2 = \frac{1}{2}(mR^2)\left(\frac{v^2}{R^2}\right) = \frac{1}{2} m v^2$

$\vec{L} = I\vec{\omega} = \vec{r} \times \vec{p}$

$\vec{L} = (mR^2)\left(\frac{v}{R}\right) = Rmv$

$\vec{F}_{net} = \frac{d\vec{p}}{dt}$ $\qquad \tau_{net} = \frac{d\vec{L}}{dt}$

if $F_{net} = 0$ $\qquad$ if $\tau_{net} = 0$

$\vec{P}_i = \vec{P}_f$ $\qquad$ Calculus $\qquad$ $\vec{L}_i = \vec{L}_f$