## AP ${ }^{\oplus}$ PHYSICS <br> 2011 SCORING GUIDELINES

## General Notes About 2011 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be earned. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.
3. Implicit statements of concepts normally earn credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still earned. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections - Student Presentation" in the AP Physics Course Description.
4. The scoring guidelines typically show numerical results using the value $g=9.8 \mathrm{~m} / \mathrm{s}^{2}$, but use of $10 \mathrm{~m} / \mathrm{s}^{2}$ is also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically earn full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278 ). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

# AP ${ }^{\circledR}$ PHYSICS C: MECHANICS <br> 2011 SCORING GUIDELINES 

## Question 3

15 points total

Distribution
of points
(a) 3 points

For a statement of Newton's second law for rotation
1 point
$\Sigma \tau=I \alpha$
For substituting the given torque expression for the net torque $\Sigma \tau \quad 1$ point
$I \alpha=-\beta \theta$
For substituting the second derivative of angular position for angular acceleration
1 point
$I \frac{d^{2} \theta}{d t^{2}}=-\beta \theta$
(b) 3 points

Applying Newton's second law for translation to a mass on a spring gives

$$
m \frac{d^{2} x}{d t^{2}}=-k x, \text { and } \omega=\sqrt{\frac{k}{m}} .
$$

For this torsion pendulum, $I \frac{d^{2} \theta}{d t^{2}}=-\beta \theta$.
Comparing differential equations, $I$ is analogous to $m$ and $\beta$ is analogous to $k$.
For the correct expression for $\omega$
1 point
$\omega=\sqrt{\frac{\beta}{I}}$
For the correct relationship between $\omega$ and $T$
1 point
$T=\frac{2 \pi}{\omega}$

For the correct answer
1 point
$T=2 \pi \sqrt{\frac{I}{\beta}}$
Alternate Solution
Alternate points
The period of a mass on a spring is $T=2 \pi \sqrt{\frac{m}{k}}$.
For recognizing that I is analogous to $m$
1 point
For recognizing that $\beta$ is analogous to $k$
For the correct answer
1 point
1 point
$T=2 \pi \sqrt{\frac{I}{\beta}}$

Question 3 (continued)

## Distribution <br> of points

(c)


For correctly plotting the data
1 point
For drawing a reasonable, best-fit straight line 1 point
Note: For correctly plotted data, a reasonable, best-fit straight line does NOT pass through the origin.
(d) 3 points

The general equation for a straight line is $y(x)=m x+b$, where $m$ is the slope and $b$ is the $y$-intercept.
$T^{2}=m I+b$
$m=\Delta\left(T^{2}\right) / \Delta I$
For using two points from the best-fit line to calculate the slope
1 point
Example from the graph shown: $m=\frac{\left(11.5 \mathrm{~s}^{2}-2.0 \mathrm{~s}^{2}\right)}{\left(0.07 \mathrm{~kg} \cdot \mathrm{~m}^{2}-0.00 \mathrm{~kg} \cdot \mathrm{~m}^{2}\right)}$
$m=135 \mathrm{~s}^{2} / \mathrm{kg} \cdot \mathrm{m}^{2}$
For an intercept calculated or directly read from the graph
1 point
$b=2.0 \mathrm{~s}^{2}$
For using the variables $T^{2}$ and $I$ in the equation
1 point
$T^{2}=\left(135 \mathrm{~s}^{2} / \mathrm{kg} \cdot \mathrm{m}^{2}\right) I+2.0 \mathrm{~s}^{2}$

# AP ${ }^{\circledR}$ PHYSICS C: MECHANICS <br> 2011 SCORING GUIDELINES 

## Question 3 (continued)

## Distribution

of points
(e) 3 points

Using the equation from part (b)
$T=2 \pi \sqrt{\frac{I}{\beta}}$
$T^{2}=4 \pi^{2} \frac{I}{\beta}=\frac{4 \pi^{2}}{\beta} I$
For comparing this to part (d) and noting that $\frac{4 \pi^{2}}{\beta}$ is the slope of the line
1 point
$\frac{4 \pi^{2}}{\beta}=m$
For using the value of the slope determined in part (d)
1 point
$\beta=\frac{4 \pi^{2}}{m}=\frac{4 \pi^{2}}{135 \mathrm{~s}^{2} / \mathrm{kg} \cdot \mathrm{m}^{2}}$
$\beta=0.292 \mathrm{~kg} \cdot \mathrm{~m}^{2} / \mathrm{s}^{2}$
For the correct units on the numerical answer
1 point
(f)

1 point
For a correct physical explanation for the intercept that mentions the effect of the
1 point flexible rod
Example: The intercept is the square of the period of oscillation of the flexible rod.


Mech. 3.
The torsion pendulum shown above consists of a disk of rotational inertia $I$ suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle $\theta$, the twisted rod exerts a restoring torque $\tau$ that is proportional to the angular displacement: $\tau=-\beta \theta$, where $\beta$ is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.
(a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement $\boldsymbol{\theta}$ of the torsion pendulum as a function of time $t$.

$$
\begin{aligned}
& I \alpha=-\beta \theta \\
& I \ddot{\theta}=-\beta \theta
\end{aligned}
$$

(b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.

$$
T=2 \pi \sqrt{\frac{I}{\beta}}
$$

To determine the torsion constant $\beta$ of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

| Rotational Inertia $I$ of Disk (kg $\mathrm{m}^{2}$ ) | Average | Time for Ten Oscillations (s) | Period T (s) | $T^{2}\left(\mathrm{~s}^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.025 |  | 22.4 | 2.24 | 5.0 |
| 0.036 |  | 26.8 | 2.68 | 7.2 |
| 0.049 |  | 29.5 | 2.95 | 8.7 |
| 0.064 |  | 33.3 | 3.33 | 11.1 |
| 0.081 |  | 35.9 | 3.59 | 12.9 |

(c) Na the graph below, plot the data points. Draw a straight line that best represents the data.


$$
\begin{aligned}
& \text { (d) Determine the equation for your line. } \\
& \text { slope }=\frac{13.0 \mathrm{~s}^{2}-4.5 \mathrm{~s}^{2}}{0.08 \mathrm{~kg} \cdot \mathrm{~m}^{2}-0.02 \mathrm{~kg} \cdot \mathrm{~m}^{2}}=142 \frac{\mathrm{~s}^{2}}{\mathrm{~kg} \cdot \mathrm{~m}^{2}} \\
& T^{2}=2.0+142 \mathrm{I}
\end{aligned}
$$

(e) Calculate the torsion constant $\beta$ of the rod from your line.

$$
\begin{aligned}
& T=2 \pi \sqrt{\frac{I}{\beta}} \\
& T^{2}=4 \pi^{2} \frac{I}{B} \\
& \beta=\frac{4 \pi^{2} I^{2}}{T^{2}}=\frac{4 \pi^{2}}{142 \frac{\mathrm{~s}^{\mathrm{kg} \cdot \mathrm{~m}^{2}}}{}}=0.28 \frac{\mathrm{~kg} \cdot \mathrm{~m}^{2}}{\mathrm{~s}^{2}}
\end{aligned}
$$

(f) What is the physical significance of the intercept of your line with the vertical axis?
$I=0$, so $T^{2}$ at the vertical axis would be the square of the period of the rod alone.


Mech. 3.
The torsion pendulum shown above consists of a disk of rotational inertia $I$ suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle $\theta$, the twisted rod exerts a restoring torque $\tau$ that is proportional to the angular displacement: $\tau=-\beta \theta$, where $\beta$ is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.
(a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement $\theta$ of the torsion pendulum as a function of time $t$.

$$
\begin{aligned}
& \theta=\frac{-T}{\beta} ; T \geqslant m \frac{d^{2} \theta}{d t^{2}} \\
& \theta=-\frac{m}{\beta} \frac{d^{2} \theta}{d t^{2}} \\
& \iint d t^{2}=\int \frac{-m d^{2} \theta}{\beta \theta^{2}}
\end{aligned}
$$

(b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.


To determine the torsion constant $\beta$ of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

| Rotational Inertia $I$ of Disk $\left(\mathrm{kg} \cdot \mathrm{m}^{2}\right)$ | Average Time for Ten Oscillations (s) | Period $\boldsymbol{T} \cdot(\mathrm{s})$ | $T^{2}\left(\mathrm{~s}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| 0.025 | 22.4 | 2.24 | 5.0 |
| 0.036 | 26.8 | 2.68 | 7.2 |
| 0.049 | 29.5 | 2.95 | 8.7 |
| 0.064 | 33.3 | 3.33 | 11.1 |
| 0.081 | 35.9 | 3.59 | 12.9 |

(c) On the graph below, plot the data points. Draw a straight line that best represents the data.

(d) Determine the equation for your line.

$$
\begin{aligned}
& \text { Lattice points are circled } \\
& \frac{y_{2}-y_{1}}{x_{2}-x_{1}}=\frac{11.5-6}{.07-.05}=\frac{5.5}{.04}=137.5 \\
& y_{2}-y_{1}=m\left(x_{2}-x_{1}\right) \\
& y-6=137.5(x-.03) \\
& y=137.5 x+1.875 \\
& \text { Calculate the torsion constant } \beta \text { of the rod from your line. }
\end{aligned}
$$

(e) Calculate the torsion constant $\beta$ of the rod from your line.

$$
\begin{gathered}
T^{2}=\frac{4 \pi^{2} I}{B} \\
\frac{4 \pi^{2}}{\beta}=\operatorname{slopc}=137.5 \\
4 \pi^{2}=137.5 \beta \\
\beta=1287
\end{gathered}
$$

(f) What is the physical significance of the intercept of your line with the vertical axis?

The $y$ coordinate of this point is the square of the pesid of the oscillations that ser when the rod is twisted with no. disk attested.


Mech. 3.
The torsion pendulum shown above consists of a disk of rotational inertia $I$ suspended by a flexible rod attached to a rigid support. When the disk is twisted through a small angle $\theta$, the twisted rod exerts a restoring torque $\tau$ that is proportional to the angular displacement: $\tau=-\beta \theta$, where $\beta$ is a constant. The motion of a torsion pendulum is analogous to the motion of a mass oscillating on a spring.
(a) In terms of the quantities given above, write but do NOT solve the differential equation that could be used to determine the angular displacement $\theta$ of the torsion pendulum as a function of time $t$.

(b) Using the analogy to a mass oscillating on a spring, determine the period of the torsion pendulum in terms of the given quantities and fundamental constants, as appropriate.


To determine the torsion constant $\beta$ of the rod, disks of different, known values of rotational inertia are attached to the rod, and the data below are obtained from the resulting oscillations.

| Rotational Inertia $I$ of Disk $\left(\mathrm{kg} \cdot \mathrm{m}^{2}\right)$ | Average Time for Ten Oscillations. s$)$ | Period $T(\mathrm{~s})$ | $T^{2}\left(\mathrm{~s}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| 0.025 | 22.4 | 2.24 | 5.0 |
| 0.036 | 26.8 | 2.68 | 7.2 |
| 0.049 | 29.5 | 2.95 | 8.7 |
| 0.064 | 33.3 | 3.33. | 11.1 |
| 0.081 | 35.9 | 3.59 | 12.9 |

(c) On the graph below, plot the data points. Draw a straight line that best represents the data.

(d) Determine the equation for your line.

$$
y-S=15416(x-.025)
$$

(e) Calculate the torsion constant $\beta$ of the rod from your line.

$$
\beta=1875
$$

(f) What is the physical significance of the intercept of your line with the vertical axis? The pendulum is always moving

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# AP ${ }^{\circledR}$ PHYSICS C: MECHANICS 2011 SCORING COMMENTARY 

## Question 3

## Overview

This lab question assessed students' understanding of simple harmonic motion and the standard linearization method of data analysis by analyzing the oscillation of a torsion pendulum.

## Sample: M3A

Score: 15
This is a very nicely written response with a well-stated answer to part (f). Note that both $\frac{d^{2} \theta}{d t^{2}}$ and $\ddot{\theta}$ were acceptable in part (a). Part (b) did not require that student work be shown; however, it is always good to do so because partial credit could not be earned without it.

Sample: M3B

## Score: 11

This response earned only 1 point in part (a) for substituting $\beta \theta$ for the net torque. Full credit was earned for parts (b) and (c), but a point was lost in part (d) for not using $T^{2}$ and $I$ in the equation of the line. The response did not earn 1 point in part (e) because there are no units in the answer. Part (f) earned full credit.

## Sample: M3C <br> Score: 4

Parts (a) and (b) earned no credit. Full credit was earned for the graphing in part (c). Part (d) lost 1 point for not using $T^{2}$ and $I$ in the equation of the line. Data points are used to calculate the slope, but because the best-fit line goes through these points, the point was earned. Parts (e) and (f) earned no credit.

