
CHAPTER FIVE

Direct-Current Compound Circuits

In Chapter Four you practiced solving circuit values in series and parallel circuits. However, few circuits in actual use are either pure series or pure parallel circuits; most are a combination of the two. They are called series-parallel, or compound, circuits.

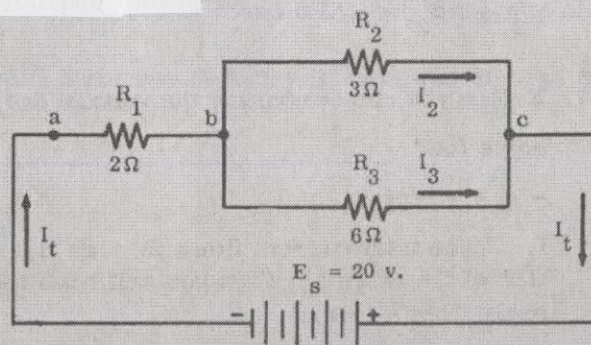
When you finish this chapter you will be able to:

- reduce compound circuits to their simplest form;
- solve for current, voltage, resistance, and power in compound circuits; and,
- solve for currents and voltages in voltage dividers.

Series-Parallel Combinations

1. At least three resistors are required to form a compound circuit. (Remember that a resistor could be any circuit component across which voltage is dropped—a fuse, a lamp, or some other device.) A one-resistor circuit must be series. A two-resistor circuit could be either series or parallel, but not a combination of the two. Why must a circuit containing only one resistor be a series circuit? _____
-
- _____

Figure 5-1. R_1 in series with parallel combination of R_2 and R_3 . Refer to Figure 5-1 for Frames 2 through 16.



2. To better understand a compound circuit, we identify portions of the circuit as either series or parallel. Look at the compound circuit in Figure 5-1. Would the portion of the circuit between points a and b be considered series or parallel? _____ Why? _____

3. The portion of the circuit between points b and c is parallel because _____

4. Is R_t equal to the sum of R_1 , R_2 , and R_3 in Figure 5-1? _____ Why or why not? _____

5. What is the combined resistance (R_t) of the resistors R_2 and R_3 ? _____

6. What is R_t for the complete circuit in Figure 5-1? _____

7. Before you can determine the voltage drop across R_1 , what must you solve for? _____

8. $I_t =$ _____

9. What is the voltage drop across R_1 ? _____

10. The branch current I_2 is $3\frac{1}{3}$ a. What is the voltage drop across R_2 ?

11. The branch current I_3 is $1\frac{2}{3}$ a. What is the voltage drop across R_3 ?

12. Now let's see how adding another resistance in parallel changes all the values of voltage and current in the circuit shown in Figure 5-1. If we add another resistor in parallel with R_2 and R_3 , the total resistance of the parallel branches will decrease. (R_1 will not change, of course, because it is a fixed value.) Since R_t of the circuit has decreased (R_1 plus a lower value of parallel resistance), I_t must increase, because of the equation $I = \frac{E}{R}$. The increased I_t will cause an increased voltage drop across R_1 because of the equation $E = IR$. To see how this works, we will add a 10-ohm resistor in parallel with R_2 and R_3 . For simplicity (and because this is standard practice) we will round off to two decimal places in our calculations.

First find R_t of the parallel branches, which now have resistances of 3Ω , 6Ω , and 10Ω .

$$\frac{1}{R_t} = \frac{1}{3} + \frac{1}{6} + \frac{1}{10}$$

$$\frac{1}{R_t} = 0.33 + 0.17 + 0.10$$

$$\frac{1}{R_t} = 0.6$$

$$R_t = \frac{1}{0.6}$$

$$R_t = 1.67 \Omega$$

R_t for the whole circuit is now 3.67Ω , since R_1 (2Ω) is added to the parallel resistance of 1.67Ω .

$$I_t = \frac{E_s}{R_t} = \frac{20}{3.67} = 5.45 \text{ a.}$$

The voltage drop across R_1 is found by the equation $E = IR$, or $5.45 \text{ a.} \times 2 \Omega = 10.9 \text{ v.}$

By Kirchhoff's Law of Voltages, the voltage drop across the parallel branches is $20 \text{ v.} - 10.9 \text{ v.}$ (the source voltage minus the voltage drop across R_1), or 9.1 v.

Thus, the voltage drop across the parallel portion of a circuit depends on the voltage drops in the series portion, but the voltage drop across each branch of the parallel portion is the same. A parallel portion of the circuit can be regarded as a single resistor whose value is the R_t of all the branches.

When a resistance is added in parallel with a parallel portion of a circuit, does the total resistance (R_t) of that portion increase or decrease?

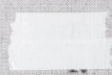
Why? _____

13. If the circuit shown in Figure 5-1 is changed as described in Frame 12, does the voltage drop across R_1 increase or decrease? _____

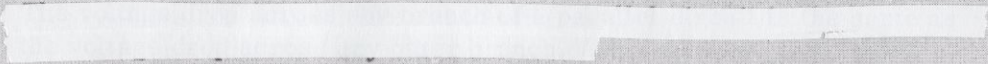
Why? _____

14. The voltage drop across one parallel branch is the same as the voltage drop across all other branches. However, the current through each branch depends on the resistance of that branch. (Remember that resistors are physical devices whose values are fixed.) When the resistance of any branch is changed, or when another branch is added, the current (I_t) flowing in the series portion of the circuit is divided among the branches according to the individual resistance of each branch. I_t flows into the parallel portion, and I_t flows out, in accordance with Kirchhoff's Current Law. Circle the Ohm's Law equation used to find the current of any branch of a parallel circuit.

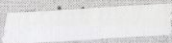
$$E = IR \qquad I = \frac{E}{R} \qquad R = \frac{E}{I}$$



15. You must keep in mind that all the source voltage in a series circuit must be accounted for in individual voltage drops around the circuit. A parallel portion of a series-parallel circuit can be regarded as a single resistor whose value R_t is the effective resistance of that portion. Can you make a general statement about the voltage drops across the branches of a parallel circuit, or across the parallel portion of a series-parallel circuit?



16. You can be sure that branch circuits are truly parallel if they have a common junction at either end. A common junction is either a point at which the total circuit current I_t divides into the branch currents or a point at which the branch currents rejoin to form the total current. One common junction for R_2 and R_3 is point b in Figure 5-1. (Point b is also common to one end of R_1 , but we are concerned with the parallel branches.) What is another common junction point? _____



17. Remember that the same difference in potential exists across all branches of a parallel circuit. However, the total current in the series-parallel circuit depends on the effective resistance of the parallel portion and on the other resistances in series with it. Now let's solve some problems in another series-parallel circuit.

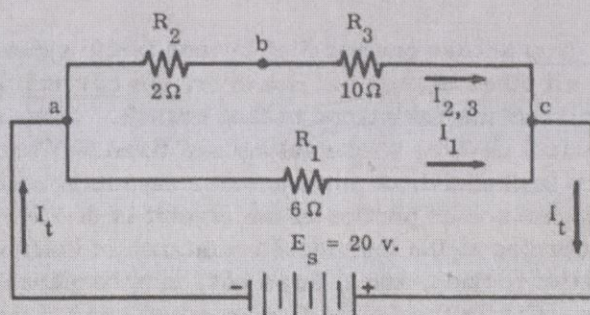


Figure 5-2. R_1 in parallel with the series combination of R_2 and R_3 .

Refer to Figure 5-2 for Frames 17 through 22.

Figure 5-2 shows a circuit in which two resistors in series (R_2 and R_3) form one branch of a parallel circuit. The total current I_t flows into the parallel circuit, splitting into two branches at point a. At what point do the branch currents rejoin to form I_t ? _____

18. The source voltage E_s (20 v.) is dropped between points a and c. The largest voltage drop is across which resistor? _____ Why? _____

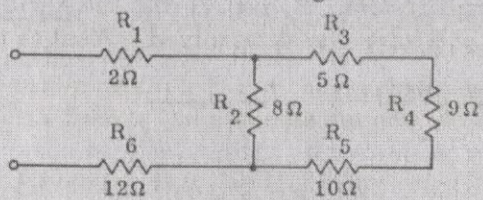
19. The next few frames will show you a systematic way to find the branch currents. What is the resistance of the top branch of the circuit in Figure 5-2? _____

20. $R_t =$ _____

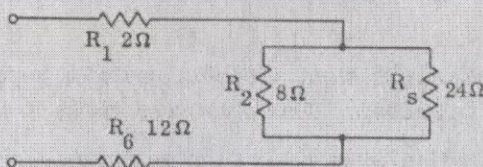
21. What is the current through the top branch ($I_{2,3}$)? _____

22. $P_t =$ _____ (You do not need to solve for I_t first.)

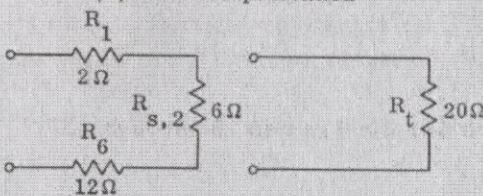
Refer to Figure 5-3 for Frames 23 through 27.



(A) Original Circuit



(B) First Simplification



(C)

Equivalent Resistance

(D)

Figure 5-3. Solving total resistance in a compound circuit.

23. Compound circuits may be made up of any number of resistors arranged in numerous series and parallel combinations. Before you can arrive at R_t , the equivalent resistance of the entire circuit, you must first reduce the circuit to its series and parallel equivalents. The original circuit in Figure 5-3 has six resistors. Name the four resistors that make up the parallel portion of the circuit. _____

24. The parallel portion of the circuit has how many branches? _____

25. In the original circuit of Figure 5-3, one branch of the parallel portion has a single resistor, R_2 , whose resistance is 8Ω . The other branch has three resistors in series. What is their combined resistance? _____
-

26. In the first simplification, R_s represents the series equivalent of R_3 , R_4 , and R_5 , which you have just determined. What is the total resistance of the parallel branches only? _____
-

27. In the second simplification, a 6-ohm resistor represents the resistance of the parallel branches. Thus, you are ready to reduce the series-parallel combination to an equivalent resistance, R_t , of _____.
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Refer to Figure 5-4 for Frames 28 through 32.

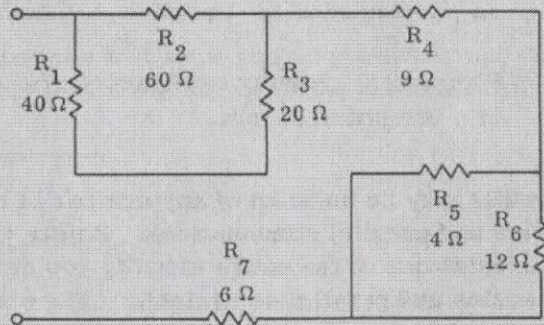


Figure 5-4. Series-parallel combination of resistors.

28. Now let's try a different kind of compound circuit. Figure 5-4 shows a circuit that has parallel portions alternating with single resistors. This circuit is more complicated, so it is important to be sure you can trace through the circuit to identify the series and parallel portions. The

placement of the resistors can be misleading, but you will stay on the right track if you remember exactly what constitutes a parallel portion. Think of the circuit in terms of current flow. The source voltage is not shown, so you have no way of knowing the direction of current flow. However, in this case it doesn't matter. Start at one of the terminals and trace around the circuit. Each time you find a point where the current (if there were a voltage source) would have to split into branches, you know you have one end of a parallel portion of the circuit. (It might help to put a large dot at that point.) When you get to a point where those same branch currents flow together again, you have the other end. Don't be confused by the number of resistors. Any parallel branch could have several resistors in series. You know they are in series if the same current must flow through each. After you have identified all the parallel portions of the circuit, the series portion of the entire circuit includes any resistor through which current flow is the same as the current flow entering the circuit from the voltage source, that is, the total current I_t . In Figure 5-4, name the resistors that make up the series portion of the entire circuit; that is, the resistors that are not found in any parallel branch.

- _____
-
- _____
29. The circuit has two separate parallel combinations. Resistors R_5 and R_6 form one parallel combination. Name the resistors that form the other.

- _____
-
- _____
30. The equivalent resistance of R_1 , R_2 , and R_3 is _____.

- _____
-
- _____
31. The equivalent resistance of R_5 and R_6 is _____.

- _____
-
- _____
32. The total circuit resistance, R_t , is _____.

Refer to Figure 5-5 for Frames 33 through 51.

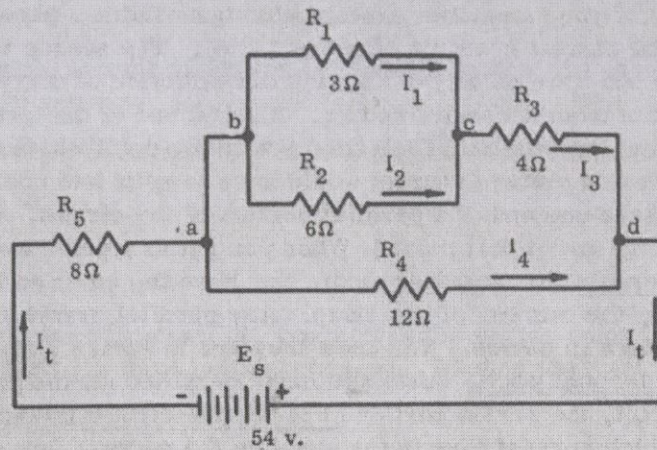


Figure 5-5. A compound circuit.

33. Figure 5-5 shows an even more complicated circuit, because it includes a parallel portion within a larger parallel portion. You have already observed that subscripts are used for labeling both circuit components and their associated values. (However, a number or letter that is a subscript in text might be hand-lettered that way on a schematic diagram.) Any convenient method of labeling may be used as long as it is descriptive. The voltage drop across R_1 , for example, could be designated E_{R_1} or E_{bc} . Either correctly identifies the points between which the voltage would be measured. The voltage drop across R_4 could be designated E_{R_4} or _____.
-

34. It is often necessary to make intermediate calculations before you can arrive at a specific unknown. For example, before you can solve for I_1 , you must first know the voltage drop across R_1 (E_{bc}). Before you can solve for I_t , you must know _____.
-

35. Computations can become very confusing unless you keep track of values as you solve them. It is a good idea to write in values on the circuit diagram or to write them on a separate piece of paper. The advantage of writing them on the diagram is that it makes any electrical situation easier to visualize. Before you can find R_t , you must simplify the circuit in successive steps. R_1 and R_2 are parallel branches between points b and c, so their equivalent resistance can be designated R_{bc} . But R_{bc} is part
-

of one of the parallel branches connected between points _____ and

_____.

36. $R_{bc} =$ _____

37. R_{bd} includes what three resistors? _____

38. $R_{bd} =$ _____

39. $R_{ad} =$ _____

40. You have now arrived at an equivalent circuit that includes the battery E_S , R_{ad} , and R_5 . Does the circuit need to be further simplified? Why?

41. $R_t =$ _____

42. By Ohm's Law, the line current (another term for I_t) is _____.

43. The line current flows through R_5 , so the voltage drop E_5 is _____.

44. According to Kirchhoff's Law of Voltages, the sum of the voltage drops around the circuit is equal to the source voltage. Therefore, E_{ad} is _____.

45. $I_4 =$ _____ (Hint: You know E_{ad} and R_4 .)

46. There are only two parallel branches between points a and d. One of these branches, however, includes another pair of parallel branches. You already know that I_t is 4.5 a. and I_4 is 1.5 a. Therefore, the current flowing into the junction at point b is _____.

47. According to Kirchhoff's Current Law, all the current flowing into the junction at point b flows out at point c. $I_3 =$ _____

48. E_3 (the voltage drop across R_3) is _____.

49. Since E_{ad} is 18 v. and E_3 is 12 v., what is E_{bc} ? _____

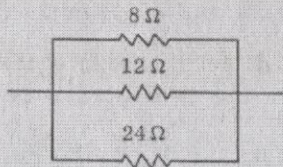
50. $I_1 =$ _____

51. $I_2 =$ _____

52. You have been able to use the "product over sum" shortcut in solving parallel resistance in the last several frames, but remember that you must sometimes use the regular equation, which involves reciprocals:

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots \text{(etc.)}$$

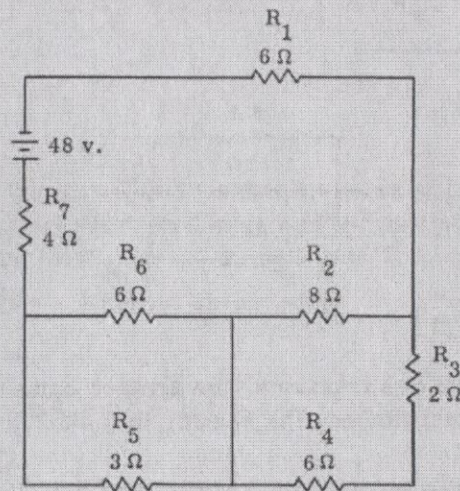
Solve R_t in the partial circuit diagram below. $R_t =$ _____



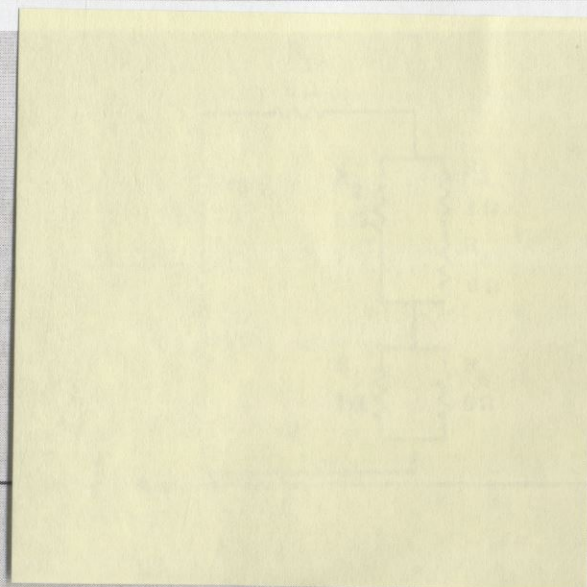
53. Can you think of another way to solve for R_t that avoids reciprocals?
- _____
- _____
-
- _____
- _____
- _____

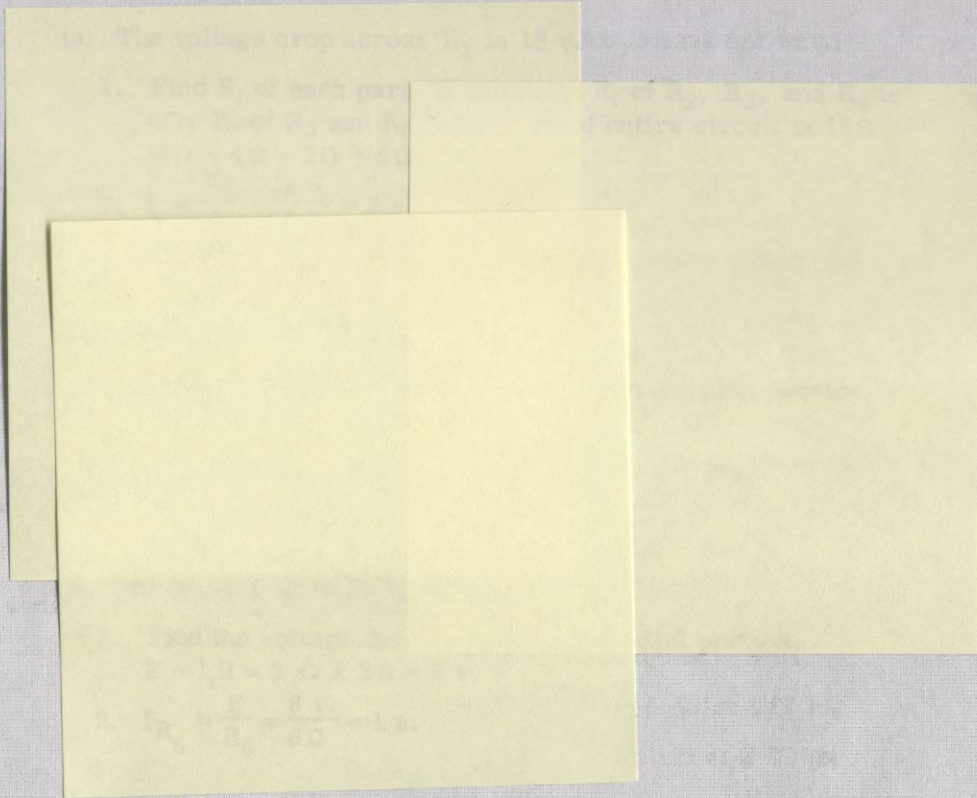
54. As you have learned in this section, any series-parallel circuit can be reduced to an equivalent series circuit. This equivalent circuit is useful to find R_t and I_t , as well as to determine the difference in potential across any parallel portion of a compound circuit. I_t splits into at least two paths when it enters a parallel portion; the voltage drop across that portion (which is the same across all branches) can be used to find each branch current, simply by dividing the branch resistance into the voltage across that branch.
-

Now let's put it all together by solving some problems in one more compound circuit. The draftsmen who draw schematic diagrams do not like to waste space or lines, so the parallel portions of a circuit are often far from obvious. Such a circuit appears below. It might be helpful to redraw the circuit to make the parallel portions more clear, or to place dots at the end junctions of parallel branches. You are asked to solve for only a few values; however, you will have to find other values before you have enough information to find the specific values requested. If you feel you need more practice, solve for some additional circuit values, such as other voltage drops and branch currents. You can check your own answers, because all values will be consistent (if they are correct) with the values given in the answer to this frame. (For example, the sum of branch currents for each parallel portion must be equal to I_t .)



- (a) What is the voltage drop across R_1 ?
- (b) What is the voltage drop across R_4 ?
- (c) What is the current through R_6 ?
-

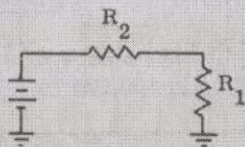




Voltage Dividers

55. In practically all electronic devices, such as radio receivers and transmitters, certain design requirements (which require different voltages) recur many times. It is both impractical and unnecessary to have a separate power supply for each voltage requirement, because the same result can be achieved by voltage dividers.

To understand the discussion that follows, you need to know a few new terms and a new symbol. A ground is merely a common connection point—usually the metal chassis of the equipment. For example, several wires might be connected to the sheet metal frame of a radio. All such connection points are electrically the same, so each point in the schematic could be represented by the "ground" symbol. If one side of the power supply is connected to ground, every point connected to ground is also electrically connected to that side of the power supply. Normally the grounded side is negative. In the diagram below, the negative side of the battery and R_1 are both grounded.



Draw the symbol for "ground."

(Note: The number of horizontal lines is not important, but each line is shorter than the one above it. The lowest line could be a dot.)

56. Many devices draw very small amounts of current. To reduce the current to the value required, large values of resistance must be used. Two common prefixes in electricity are "milli-," which means 1/1000th (or 0.001) and "kilo-," which means 1,000. One milliampere is written "1 ma." One kilohm is usually abbreviated 1 k Ω or simply 1 K. (Don't be confused by the interchangeability of small and capital letters in some abbreviations; conventions differ somewhat.)

- (a) If the source voltage is 100 v. and the total circuit resistance is 50 K, the current is 0.002 a., or _____ ma.
- (b) If the source voltage is 40 v. and the total circuit resistance is 10 K, the current is _____ ma.
- (c) The value 6 ma. is the same as _____ a.
- (d) 50 K is the same as _____ Ω .

(Note: For help in working with very large numbers and small decimals, you may wish to review the laws of exponents in Appendix III, page 278.)

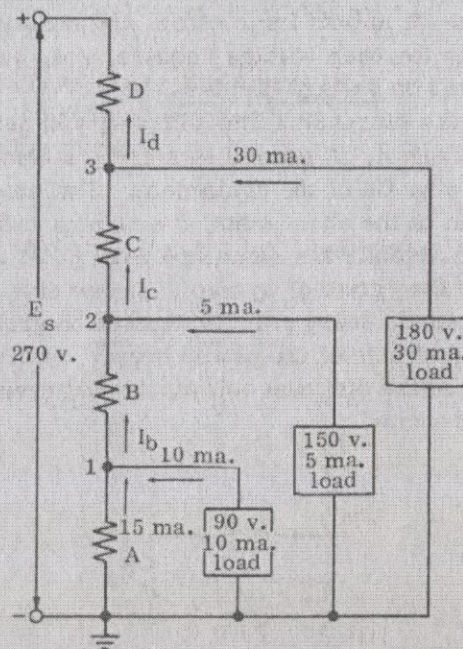


Figure 5-6. Voltage divider, to solve for R and P.

Refer to Figure 5-6 for Frames 57 through 66.

57. A typical voltage divider consists of two or more resistors connected in series across the primary power supply. The primary voltage E_s must be as high as or higher than any of the individual voltages it is to supply. As the primary voltage is dropped by successive steps through the series resistors, any desired fraction of the original voltage may be "tapped off" to supply individual requirements. The value of each series resistor used is determined by the voltage to be dropped across it. In Figure 5-6, a voltage divider is connected across a 270-volt source E_s . (The voltage source is often indicated on schematics by showing only the terminals.) The voltage divider in Figure 5-6 consists of four resistors, labeled A, B, C, and D. The points (1, 2, and 3) at which partial voltages are tapped off are called "taps." In this case, external loads are connected to all three taps, and the voltage and current requirements of each load are indicated on the schematic. The load placed across resistor A is 90 v. and 10 ma. The voltage drop across A, measured between ground and tap 1, is _____ v.

58. The Ohm's Law equations have so far been based on the basic units: ohms, amperes, and volts. But now we are starting to use milliamperes (called "milliamps") and kilohms. These should cause no problem as long as you are careful to use the appropriate decimals for milliamperes and the correct number of zeroes for kilohms. But since these two units of measurement are quite common, you should know a short cut. Since $E = IR$, you know that 1 a. flowing through a 1-ohm resistor drops 1 v. A current of 1 ma. flowing through a 1-kilohm resistor drops _____ v.

59. Thus, in an equation where all resistances are in kilohms and all currents are in milliamperes, voltage is in _____.

60. According to Kirchhoff's Law of Currents, the current flowing through resistor B must be the sum of the current flowing through A and that flowing through the load across A. I_b is _____.

61. The required voltage for the 150-volt load is obtained by selecting a tap on the voltage divider at which the potential difference (between the tap and ground) is 150 v. The combined voltage drop across resistor A and resistor B is also 150 v. Why? _____

62. The current I_c is 30 ma. How do we know? _____

63. What is current I_d ? _____

64. I_t in Figure 5-6 flows through only one resistor of the voltage divider. Which one? _____

65. The correct voltage requirements are supplied to the three loads in Figure 5-6 because of the varying differences in potential at taps 1, 2, and 3. These differences in potential are with respect to ground. For example, the voltage drop across resistor A is 90 v. This is also the difference in potential between tap 1 and ground. The voltage drop across resistor B is 60 volts. The difference in potential, with respect to ground, between tap 2 and ground is 150 v. Thus, the two voltage drops of 90 v. and 60 v. represent a difference in potential of 150 v. The voltage drop across resistor C (between taps 2 and 3) is 30 v. What is the difference in potential between tap 3 and ground? _____

66. We now know all the values of voltage and current necessary to find the values of resistance in the circuit. For example, we can find the value of resistor A because we know the current through it and the voltage dropped across it.

(a) $R_A = \underline{\hspace{2cm}}$

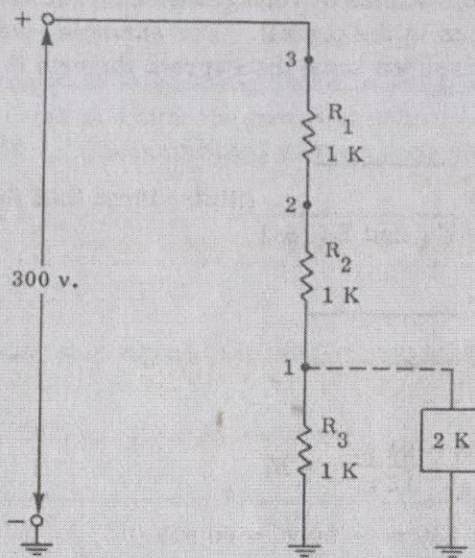
(b) $R_B = \underline{\hspace{2cm}}$ (Hint: First find E_B . You already know E_A and E_{A+B} .)

(c) $R_C = \underline{\hspace{2cm}}$

(d) $R_D = \underline{\hspace{2cm}}$

67. We have seen, in working the problems associated with Figure 5-6, that the voltage divider is an excellent application of the properties of parallel circuits. Each external device, or "load," connected to one of the taps on the voltage divider is a branch in parallel with part of the voltage divider. As we move to a tap farther away from ground, which is the negative end of the voltage source in Figure 5-6, we select a larger difference in potential between that tap and ground. Since all branches of a parallel circuit "see" the same difference in potential, we can select a voltage appropriate to the external load. Remember that adding another resistance in parallel with part of the circuit will decrease the resistance of that part of the circuit. This will in turn decrease the difference in potential across the parallel circuit. Thus, the voltage across the parallel circuit is lower, but the voltage across the series resistors is correspondingly greater. This is consistent with Kirchhoff's Law of Voltages. It makes sense, because the total current flowing in the circuit has also increased (since the total resistance is lower).

The circuit shown on the following page shows a voltage divider consisting of three one-kilohm resistors connected to a 300-volt source. Before the load is connected, the total circuit current is 100 ma. (0.1 a.) and each resistor sees a voltage drop of 100 v. The dashed line indicates that the two-kilohm load has not been connected to the tap.



After the load is connected:

(a) What is the difference in potential between tap 3 and ground? _____

(b) What is the resistance between tap 1 and ground? _____

(c) What is the current through R_2 ? _____

(d) What is the difference in potential between tap 1 and ground? _____

(e) What is the current through the two-kilohm load? _____

68. Now let's see how currents are distributed when external loads are connected to a voltage divider. In Figure 5-7, on the following page, resistances are given for R_4 , R_5 , R_6 , and R_7 (the voltage divider), but not for the three loads, R_1 , R_2 , and R_3 . The source voltage is known ($E_s = 510$ v.), and enough information about currents is given to enable us to work out all the voltage drops in the circuit; but the only way to go about it is to apply Kirchhoff's Law of Currents.

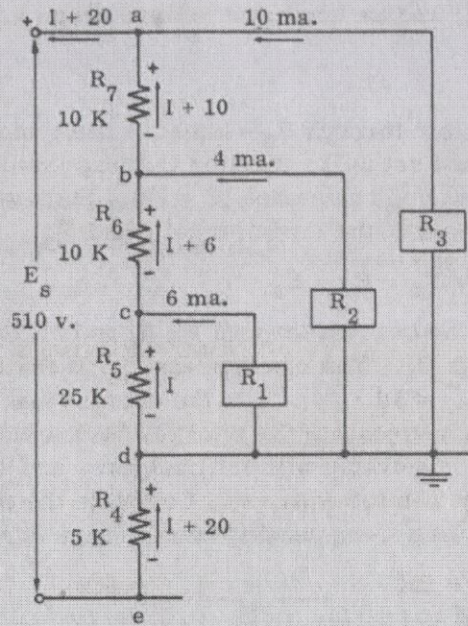


Figure 5-7. Voltage divider, to solve for E and R.

Refer to Figure 5-7 for Frames 68 through 82.

The only resistor through which all circuit current flows is _____.

69. The load currents, as shown in the diagram, are 6 ma., 4 ma., and 10 ma.

The only resistor through which no external load current flows is _____.

70. Thus, if we first solve for I (the current through R_5), we can easily find the current through the other resistors of the voltage divider (R_6 and R_7). The current through R_6 is I plus 6 ma. (the current through R_1); and the current through R_7 is I plus 6 ma. plus 4 ma. (the current through R_2). Naturally, once we know both the current and the resistance for any resistor, we can find the voltage drop across that resistor. We shall also know the voltage across each external load. Why? _____

Each load is in parallel with a portion of the circuit for which the voltage drop can be found, and we know that voltage drops across parallel branches are equal.

71. To find I , the current through R_5 —since we don't know the voltage drop across R_5 —we must set up an equation that expresses all voltage drops in terms of current and resistance ($E = IR$). Start with your knowledge that all voltage drops in the circuit must equal E_S :

$$E_4 + E_5 + E_6 + E_7 = E_S$$

You know that the current through R_5 (I) and all load currents (20 ma. in all) flow through R_4 . You can express E_4 , then, in terms of current and resistance: $E_4 = 5(I + 20)$. I is the current that does not flow in the loads, and 20 ma. represents the total load current. The resistance of R_4 is 5 K. You are working with milliamperes and kilohms, so the resistance is simply 5 in the equation. Complete the equation that will express all voltage drops, expressing each voltage drop in terms of R and I .

$$5(I + 20) + 25I + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 510$$

72. Now collect your terms, and your equation is simplified as:

$$5I + 100 + 25I + 10I + 60 + 10I + 100 = 510$$

$$50I + 260 = 510$$

$$50I = 250$$

$$I = \underline{\hspace{2cm}}$$

73. The current through R_5 is _____.
-
- _____

74. The current through R_4 is _____.
-
- _____

75. $E_4 =$ _____
-
-

76. $E_5 =$ _____

77. What is the voltage drop across R_1 ? _____

78. What is the resistance of R_1 ? _____

79. What is the voltage across R_2 ? (Hint: First you have to find E_G .) _____

80. How much power is absorbed by R_3 ? _____

81. How much power is absorbed by R_2 ? _____

82. $P_t =$ _____

The circuits described in this chapter are the kind most frequently encountered in actual practice; that is, they have both series and parallel elements. It is necessary to look at the circuit as a whole to find some of the values. For example, you need to know I_t before you can solve for the voltage across parallel branches—and this voltage is needed to solve for the current through a specific branch.

You must approach the direct-current compound circuit systematically, first identifying the parallel portions and then reducing the entire circuit to its simplest form: an equivalent series circuit. Once you have done this, it is not difficult to solve any problem of current, voltage, resistance, or power.

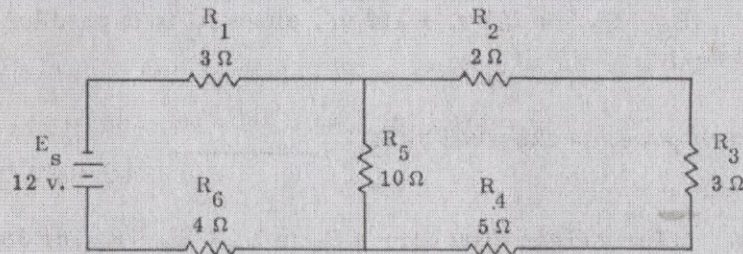
The voltage divider is merely one application of a compound circuit. Each external load tapped into the voltage divider is merely a branch of a parallel circuit. However, its resistance changes the equivalent resistance of one portion of the circuit, and thus, the values of voltage and current throughout the circuit.

If you feel you understand the basic concepts of the compound circuit and the voltage divider, go on to the Self-Test.

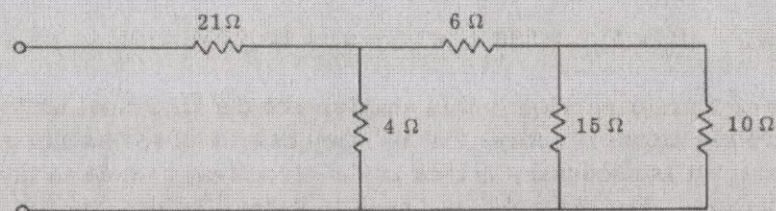
Self-Test

These questions will test your understanding of Chapter Five. Write your answers on a separate sheet of paper and check them with the answers provided following the Self-Test.

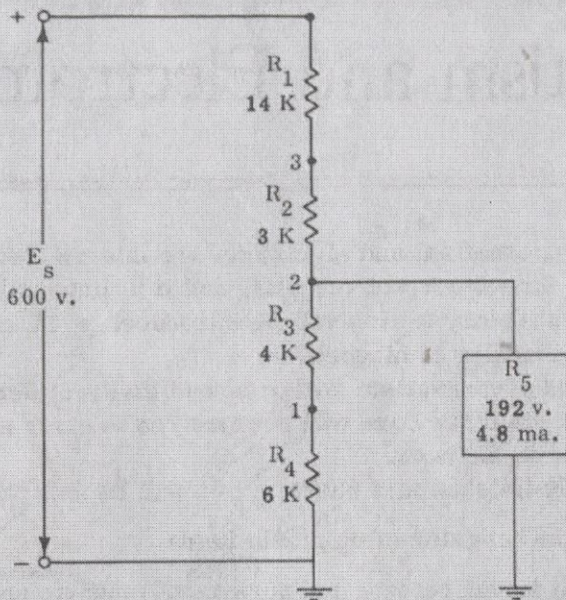
Refer to the diagram below for questions 1 through 4.



1. What is the total resistance R_t of the circuit?
2. What is the total current I_t in the circuit?
3. What is the voltage across R_3 ?
4. What is the total power consumed in the circuit?
5. What is R_t , the equivalent resistance, of the circuit below?



6. Refer to the diagram below.



- What is the current through R_1 ?
- What is the voltage across R_4 ?
- What is the difference in potential between tap 3 and ground?
- How much current would flow in the circuit if R_5 were disconnected?