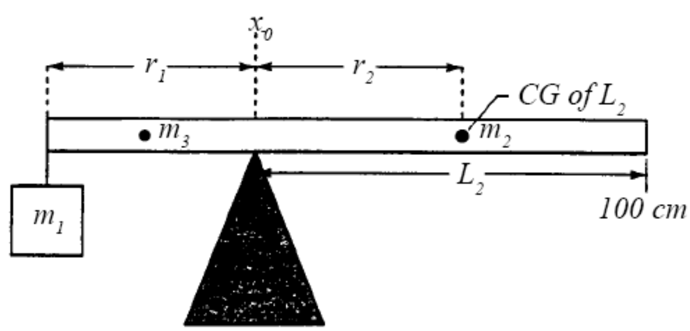
Torque, Equilibrium & Center of Gravity



Produced by the Physics Staff at Collin College

Purpose

In this experiment, you will investigate torques on rigid bodies and static equilibrium.

Equipment

- 1 Lab Balance
- 1 Meter stick
- 1 Balance Stand for meter stick
- 1 Set of Mass Hangers for meter stick
- 1 Hooked Mass Set
- 1 Small, Unknown Metal Mass

Introduction

Consider, for example, an ordinary chair. It appears to be a rigid body. Is it in equilibrium as it sits on the floor? If, as often happens, one of its legs is a bit short, it will not be in equilibrium; instead, it will wobble about the two diagonally-opposite longer legs. But if an elephant sits on the chair, it will most definitely be in equilibrium with all four legs resting solidly on the floor.

What has changed? Only the chair's shape. The elephant has distorted it. It is not a rigid body under these conditions.

In this experiment, you will investigate truly rigid bodies and static equilibrium. The static equilibrium condition is very important in civil engineering, applying to bridges, dams, buildings, statues, and balconies, and in our daily lives, applying to our ability to stand up, to drive around corners without overturning, and to slide a stein of beer the length of the bar without spilling it.

Analyzing static equilibrium conditions is an essential part of architectural engineering. The designer needs to identify all the forces and torques that act on a structural element, and to ensure through design and materials selection that the element will safely tolerate the loads to be exerted on it.

Two conditions must be met for a rigid object subject to a combination of external forces to be in mechanical equilibrium:

1. The vector sum of all the external forces acting on it must be zero. Translational equilibrium: $\sum \vec{F} = 0$
2. The vector sum of all the torques about an arbitrary axis must be zero. Rotational equilibrium: $\sum \vec{\tau} = 0$. To be in static equilibrium, a rigid object must also be in rotational equilibrium.

The concept of center of mass is what allows us to study the motion and equilibrium of extended (real world) objects as if they were point objects. By considering the translational motion of an object's center of mass (the motion of a point mass), and the rotational motion of the object about its center of mass, we can determine the complex motion of any extended object.

In this experiment, you will examine torques, rotational equilibrium, and center of mass as they apply to a rigid object. The rigid object will be an ordinary wooden meter stick. By measuring the forces and calculating the torques acting on this meter stick in different situations, you will experimentally verify the two equilibrium equations. In doing this, you will learn to

1. Describe mechanical equilibrium of a rigid object
2. Explain the center of mass concept
3. Explain how a laboratory balance measures mass

Theory

Equilibrium

A rigid body in static equilibrium must necessarily be in rotational equilibrium. Torque about some axis of rotation (also called *moment of force*) results from a force being exerted at a point ***not on the axis***. Torque is defined as the vector product of the force and the displacement to the axis:

$$\vec{\tau} = \vec{F} \times \vec{r}$$

Therefore, the magnitude of the torque is

$$\tau = Fr\sin\theta$$

The measurement unit for torque is the newton-meter (Nm), which you should not confuse with the unit of work (1 newton-meter = 1 joule).

Torque is a vector quantity. Its direction is normal to the plane containing ***r*** and ***F***. When you cross ***r*** into ***F*** using the right-hand rule, your thumb points in the direction of the torque. For convenience, torques are designated by the circular directions of motion that they tend to cause (clockwise and counter-clockwise)

A rigid body can rotate about a specific axis in only two directions, clockwise or counter-clockwise. Clockwise torques produce clockwise rotational motion and counterclockwise torques cause counterclockwise rotational motion. Rotation will not begin or change if the applied torques are balanced (if the system is in rotational equilibrium). The condition for rotational equilibrium is

$$\sum \vec{\tau} = \sum \vec{\tau}_{ccw} + \sum \vec{\tau}_{cw} = 0$$

where $\vec{\tau}_{ccw}$ and $\vec{\tau}_{cw}$ are the counterclockwise and clockwise torques. Conventionally, the counterclockwise direction is positive, and the clockwise direction is negative.

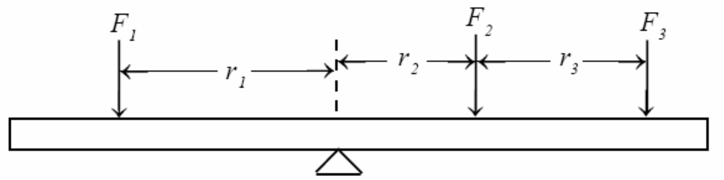


Figure 8.1. Torques applied to bar

For the system in Figure 8.1, the condition for rotational equilibrium becomes:

$$F_1 r_1 = F_2 r_2 + F_3 r_3$$

In this case the counterclockwise torque is $F_1 r_1$ and the clockwise torques are $F_2 r_2$ and $F_3 r_3$. Note that the forces F_i may also be expressed as weights according to Newton's Second Law ($W_i = F_i = mg$). In this equation, the weight W has units of N, the mass m is in kg, and the gravitational acceleration g is in m/s^2.

You can use the equation of balanced torques to find an unknown quantity such as a moment arm. Assume the bar above is in static equilibrium and the forces exerted on the bar are due to three masses hanging at the indicated positions. Then

$$m_1 r_1 = m_2 r_2 + m_3 r_3$$

or $r_1 = \dfrac{m_2 r_2 + m_3 r_3}{m_1}$

If $m_2 = m_3 = 100\text{g}, m_1 = 200\text{g}, r_2 = 40\text{cm}, \text{and } r_3 = 60\text{cm}, r_1$ must be

$$r_1 = \frac{100\text{g}(40\text{cm}) + 100\text{g}(60\text{cm})}{200\text{g}} = 50\text{cm}$$

Center of Gravity

The *center of gravity* of an object is the point where all of the weight of that object (*mg*) may be concentrated for the purpose of determining the torque gravity exerts on it. The weights of the infinitesimal mass particles making up a rigid object create torques about the object's center of gravity. A wooden meter stick of uniform cross section, for example, may be considered as made up of many point masses that are in rotational equilibrium about its center of gravity at the 50 cm point. The ruler can be balanced (supported in rotational equilibrium) on a fulcrum located at its center of gravity.

The *center of mass* of this same ruler would be at the same location as its center of gravity as long as the acceleration due to gravity *g* is uniform. For a symmetrical stick with a uniform mass distribution, its center of mass and its center of gravity will both be located at its center of symmetry.

The concept of linear mass density *u* is closely related to the concept of uniform mass distribution. The linear mass density of a rigid object is defined as its mass per unit length:

$$\mu = \frac{m}{L}$$

If a meter stick has a mass of 150 g, its linear mass density becomes 150 g /100 cm = 1.50 g/cm = 0.15 kg/m. Assuming a uniform mass distribution for the ruler, every centimeter of it has a mass of 1.50 g, and 60 cm would have 90 g of mass. Keep in mind, however, that laboratory 1-m rulers do not necessarily have uniform mass distribution, especially older ones that are worn on their edges, and such an assumption gives only approximate values.

Procedure

You will use minimal equipment and ***no computer sensors*** in this experiment. In hanging masses from the meter stick, you may suspend them from sliding clamps with hangers or from loops of string. Suspending the hooked masses from loops of string (or rubber bands) is easier since you can consider the strings to be massless. If you use the clamps with hangers, be sure that you add their masses to the hanging mass.

A. Rigid Rod Supported at its Center of Gravity

1. Measure the mass of the meter stick and record its value above Data Table 8.1.

Case 1. Two masses of known value

2. Supporting the meter stick at its center of mass (point where the meter stick is balanced by itself), hang a 100 g mass at the 20 cm point. Record this mass m_1 and its moment arm r_1 in Table 8.1.
3. Balance the ruler by placing a 200 g mass at an appropriate location on the other side of the fulcrum. Record this mass m_2 and its moment arm r_2 in Table 8.1.
4. Calculate and record the CW and CCW torques and the percent difference between them.

Case 2. Three masses of known value

5. With the ruler still supported at its center of mass, hang a 100 g mass m_1 at the 15 cm mark and a 200 g mass m_2 at the 75 cm mark. Hang a 50 g mass m_3 at the appropriate position so the ruler is in static equilibrium. Record all masses and moment arms in Table 8.1.
6. Compute and record the torques and percent differences as you did in step 4.

B. Rigid Rod Supported at Different Points

In Part A, you did not take the mass of the ruler itself into account because the support was located at its center of gravity, and the torque due to its weight was zero.

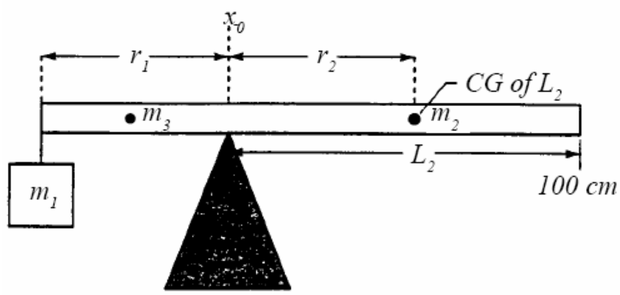


Figure 8.2. Unbalanced rod

In this part, the ruler will not be supported at its center of gravity but at other pivot points indicated in general in Figure 8.2. For these cases, you will have to consider the weight of the ruler and compute appropriate torques for each case due to m_2 and m_3.

1. Hang a 100 g mass m_1 near the zero end of the ruler and slide the support to the appropriate position to achieve static equilibrium. Record the mass and moment arm in Table 8.2.
2. Assuming uniform mass distribution of the ruler, calculate and record the mass m_2 of L_2.
3. Calculate and record the moment arm r_2.
4. For the same setup, calculate and record the moment arm r_3 due to the mass m_3 of that portion of the ruler on the other side of the support.
5. Calculate and record the torques and percent differences as you did in Part A.

C. Center of Gravity

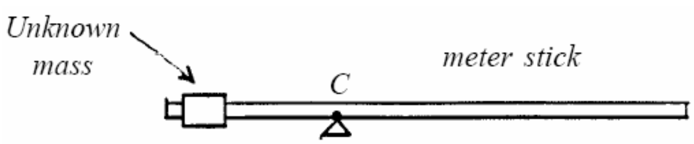


Figure 8.3. Loaded ruler balanced at C

1. Tape an unknown mass near one end of the ruler as shown in Figure 8.3. Keep the mass at the same position for the rest of this part.
2. Measure the mass m_r of the loaded ruler and record it under Data Table 8.3.
3. The center of gravity of the loaded ruler is now at point *C*. Balance the loaded ruler and record the exact position of point *C* in Table 8.3.
4. Hang a mass m_1 of arbitrary value from point *A* near the other end of the ruler. Adjust the support point of the loaded ruler from *C* to *B* until the ruler is again in static equilibrium (see Figure 8.4). Record the value of m_1 and the moment arms *L* and *x* in Table 8.3 (the weight *W* of the loaded ruler is considered as acting at point *C* a distance *x* from the pivot).

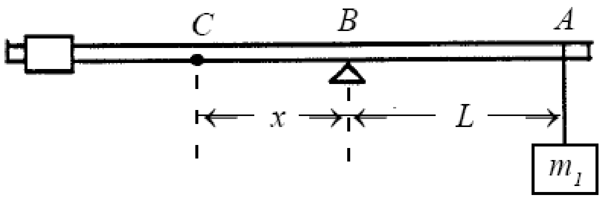


Figure 8.4. Loaded ruler balanced at B

5. Calculate and record the torques and percent differences as you did in Part A.
6. Disassemble the equipment and return it to the lab cart. Clean up around your lab table area.