Chapter Nine
ELECTRIC CURRENT AND CIRCUITS

KEY IDEAS

Electric current is defined as the time rate at which charge flows through a substance. Currents are established and maintained through a conductor by the application of a potential difference across the conductor.

The resistance of a substance is the ratio of the potential difference across the substance to the current through it. Resistance depends on the nature of the substance, its length, its cross-sectional area, and the temperature. In metallic conductors, the resistance at a given temperature is relatively constant, a fact that is the basis for a relationship known as Ohm's law. Certain substances lose all resistance at low temperatures, a phenomenon known as superconductivity.

Current and potential difference are usually measured in an electric circuit that is a closed path for the flow of charge. A circuit usually contains a source of potential difference and one or more resistances, and may include other devices as well. A circuit that has only one current path is known as a series circuit; a circuit with more than one current path, as a parallel circuit.

Complex circuits may consist of both series and parallel branches or be even more complicated. Such circuits need to be solved by means of Kirchhoff's two laws, which are mathematical statements of the laws of conservation of energy and of electric charge.

KEY OBJECTIVES

At the conclusion of this chapter you will be able to:

- Define the term *electric current*, and state the SI unit for it.
- Solve problems involving current, charge, and time.
- Distinguish between conventional current and electron flow.
- Define the term *resistance*, and state the SI unit for it.
- Solve problems that relate current, potential difference, and resistance.
- Relate the resistance of a material to its length, cross section, resistivity, and temperature.
- Solve problems that relate resistivity, cross section, and length.

- Define the terms *superconductor, series circuit,* and *parallel circuit.*
- State Ohm's law, relate it to metallic conductors, and solve Ohm's law problems.
- Define the term *electric circuit*, and list the components in a simple circuit.
- Draw the circuit symbols for a resistor and a source of potential difference.
- State the equations for determining power and energy output in electric circuits, and solve problems using these equations.
- State the relationships in a series circuit, and solve problems using these relationships.
- State the relationships in a parallel circuit, and solve problems using these relationships.
- Compare and contrast series and parallel circuits.
- State Kirchhoff's rules as they apply to electric circuits.

9.1 INTRODUCTION

In Chapter 8, we learned about static charges. In this chapter we learn about charges in motion and the way they function in electric circuits. Without electric circuits it would be impossible to operate most of the devices that we take for granted, such as televisions, telephones, blenders, and vacuum cleaners.

✱ 9.2 ELECTRIC CURRENT

The word *current* means "flow" and **electric current** means "flow of charge." When we use the term *electric current*, we are referring, not to the speed of the charged particles, but to the *quantity* of charge that passes a single point in time.

As an analogy, consider the movement of cars on a highway. We measure the speed of a single car by calculating the distance it travels in a given amount of time. We measure the flow of traffic by counting the number of cars passing a given point in a given amount of time. During rush hour, the flow of traffic will be high, but the speed of any individual car may be quite small. At 4 A.M., however, the reverse is likely to be true.

The SI unit of current is the *ampere* (A) which is equivalent to 1 *coulomb per second*. The symbol used to represent current is *I,* and we can write

PHYSICS CONCEPTS

✱ $$I = \frac{\Delta q}{t}$$

✱ indicates that material is part of the New York State core curriculum.

Substance	Resistivity ($\Omega \cdot$ m at 20°C)
Silver	1.59×10^{-8}
Copper	1.68×10^{-8}
Aluminum	2.65×10^{-8}
Iron	9.71×10^{-8}
Silicon (semiconductor)	0.1 – 60
Glass (insulator)	$10^{9} - 10^{12}$

The greater the resistivity of a material, the greater its resistance will be.

2. The resistance of a regularly shaped conductor is directly proportional to its length and inversely proportional to its cross-sectional area. This fact makes sense because making a conductor longer increases the likelihood that the electron will collide with the atoms of the conductor, thereby increasing the resistance. Making a conductor wider increases the number of *paths* that the electrons can take and, therefore, decreases the resistance.
3. Generally, the resistance of a metallic conductor increases with rising temperature. This fact also makes sense because increasing the temperature of a conductor increases the vibrational kinetic energy of its atoms, making collisions with electrons more likely. In other materials, such as semiconductors, resistance actually decreases with increasing temperature. In practice, a temperature such as 20°C is chosen as a standard temperature for comparing resistances.

We can combine all of these factors into one relationship:

Physics Concepts

$$R = \rho \cdot \frac{L}{A} \text{ (at a specified temperature)}$$

where L represents the length of the conductor and A is its cross-sectional area.

PROBLEM

Calculate the resistance at 20°C of an aluminum wire that is 0.200 meter long and has a cross-sectional area of 1.00×10^{-3} square meter.

SOLUTION

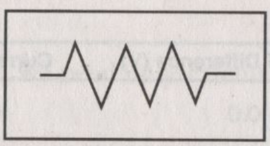
We know from the table given above that aluminum has a resistivity of 2.65×10^{-8} $\Omega \cdot$m. Therefore:

Certain substances, called **superconductors**, lose all of their resistance when cooled to very low temperatures. It is also interesting that superconductors do not conduct very well at higher temperatures. If a substance could be made to be superconducting at or near atmospheric temperatures, vast quantities of energy could be conserved because the heat lost by superconducting wires would be minimal.

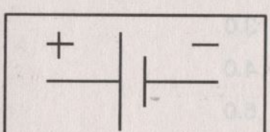
9.5 ELECTRIC CIRCUITS AND OHM'S LAW

The word *circuit* means "closed path." By an **electric circuit** we mean an arrangement where electric charges can flow in a closed path. The simplest electric circuit consists of a source of potential difference (a battery or a power source), a single resistance, and connecting wires (which are assumed to have negligible resistance).

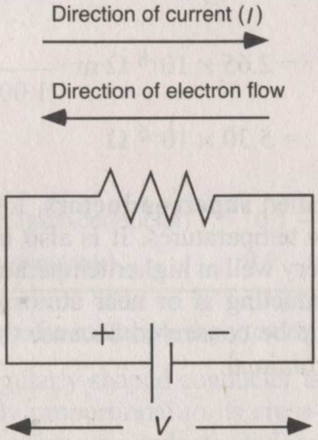
A device that provides resistance to a circuit is called a *resistor*, and its symbol is as follows:



The symbol for a source of potential difference is



The diagram below represents the completed circuit:

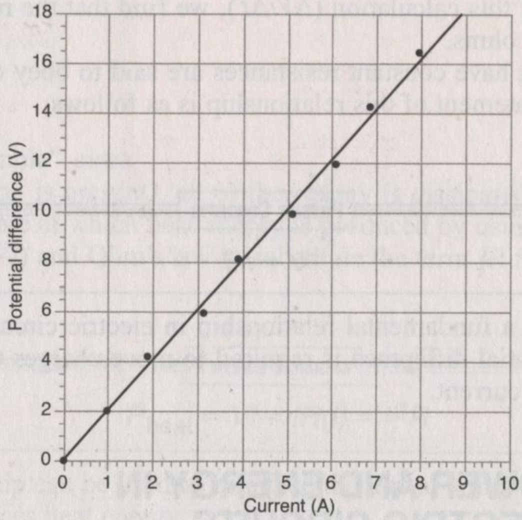


As the electrons begin to flow from the negative terminal, their energy is provided by the source of potential difference. When the electrons reach the positive terminal, all of their energy has been expended (i.e., converted to heat), and the power source must provide them with additional energy for the next trip around the circuit.

Suppose a student performs an experiment using the circuit diagrammed above. The student varies the potential difference across the circuit and measures the current in the circuit with each change. Her results are recorded in the table below.

Potential Difference (V)	Current (A)
0.0	0.0
1.0	1.8
2.0	4.3
3.0	5.7
4.0	8.2
5.0	9.9
6.0	11.8
7.0	14.3
8.0	16.4

To analyze the data, the student finds it useful to plot the data points and then draw a graph, as shown here.



We can see that the graph of the data points, within experimental error, is a straight line that passes through the origin. The *slope* of the graph is the ratio of potential difference to current (which we know as *resistance*). Since the slope is constant, it follows that the resistance of the material is also constant.

PROBLEM
Calculate the resistance of the conductor whose graph is given above.

SOLUTION
To calculate the resistance, we need only calculate the slope of the straight line on the graph.

As a result of this calculation ($\Delta V/\Delta I$), we find that the resistance of the conductor is 2.0 ohms.

Materials that have constant resistance are said to obey *Ohm's law*. The mathematical statement of this relationship is as follows:

PHYSICS CONCEPTS

$$V = IR$$

Ohm's law is a fundamental relationship in electric circuits. It describes how much potential difference is required to move charges through a resistance at a given current.

9.6 POWER AND ENERGY IN ELECTRIC CIRCUITS

Suppose we wish to calculate the *rate* at which energy is supplied to the simple circuit drawn in Section 9.5. We know that potential difference is related to work and energy and that current is related to time. If we multiply the potential difference across the circuit by the current in the circuit and examine the *units*, we find that

$$\text{Potential difference} \cdot \text{current} = \text{volts} \cdot \text{amperes}$$

$$= \frac{\text{joules}}{\text{coulomb}} \cdot \frac{\text{coulombs}}{\text{second}}$$

$$= \frac{\text{joules}}{\text{second}} = \text{watts} \equiv \text{power}$$

Therefore, the *power* supplied to a circuit by the source is given by this relationship:

PHYSICS CONCEPTS

$$P_{\text{source}} = VI$$

PROBLEM

Calculate the rate at which energy is supplied by a 120-volt source to a circuit if the current in the circuit is 5.5 amperes.

SOLUTION

Since our "simple" circuit contains only resistance (i.e., no other device, such as a motor, is present), all of the energy is dissipated as heat. We can calculate the rate at which heat energy is produced by using the power relationship ($P = VI$) and Ohm's law to substitute the term IR for V:

PHYSICS CONCEPTS

$$P_{\text{heat}} = VI = IR(I) = I^2R$$

This relationship can be used to calculate the rate at which any resistance in a circuit produces heat energy.

PROBLEM

A 150-ohm resistor carries a current of 2.0 amperes. Calculate the rate at which heat energy is produced by the resistor.

SOLUTION

If, however, another device is present in the circuit, the term VI will not be equal to the term I^2R because not all of the electric energy is converted to heat energy.

There is a third relationship for calculating power: $P = \frac{V^2}{R}$. See whether you can derive this relationship using Ohm's law and the power equation.

If we know how much power is developed by a circuit, it is an easy matter to calculate the amount of energy produced: we need only to multiply the power (in watts) by the time the circuit operates (in seconds). The result will be the amount of energy (in joules).

PHYSICS CONCEPTS

$$\text{Energy } (W) = \text{Power } (P) \cdot \text{time } (t)$$

$$= VIt = I^2Rt = \frac{V^2t}{R}$$

PROBLEM

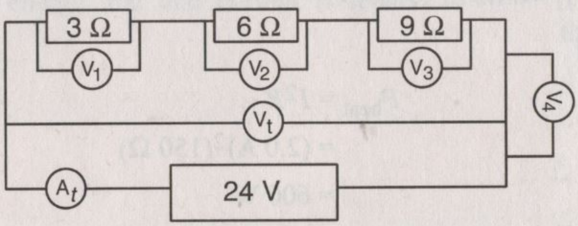
How much energy is produced by 50.-volt source that generates a current of 5.0 amperes for 2.0 minutes?

SOLUTION

9.7 SERIES CIRCUITS

At one time, small holiday lights were arranged so that, if one bulb burned out, the entire string of lights remained unlit. We call the type of electric circuit that produced this effect a series circuit. A **series circuit** has only one current path and if that path is interrupted, the *entire* circuit ceases to operate.

The diagram represents a circuit containing three resistors arranged in series. In addition a number of *meters* have been placed in order to measure various characteristics of the circuit.



Each symbol —(V)— represents a *voltmeter*; this very-high-resistance device measures the potential difference across two points in a circuit. The symbol V_t represents the total potential difference across the circuit. The symbol —(A)— represents an *ammeter*; this very-low-resistance device measures the current passing through any part of the circuit. The subscript t indicates that the ammeter in the diagram is measuring the total current through the circuit.

Since a series circuit contains only one current path, the current throughout the circuit is constant; therefore, an ammeter placed at any other position in the circuit would record the same value.

The situation is not the same with potential difference: The potential difference across two points depends on the work the source must do in order to move the charge between these two points. In the circuit shown in the diagram, the resistance across any two points will determine how much work needs to be done to transport the charges through the circuit. Our aim is to calculate the readings on all of the meters in the diagram.

We can solve this problem by being aware of the following relationships, which hold true for *any series circuit*:

PHYSICS CONCEPTS

- $I = I_1 = I_2 = I_3 = \ldots$
- $V = V_1 + V_2 + V_3 + \ldots$

$$V_n = I_n R_n$$

The first relationship states that the current through any resistance in a series circuit is constant throughout the circuit. The second relationship states that the potential difference across the entire circuit (V_t), supplied by the power source, is equal to the sum of the potential differences (V_1, V_2, . . .) across all the resistances. (This is really a statement of the law of conservation of energy and is known, in honor of German physicist Gustav Kirchhoff, as *Kirchhoff's first rule* or is called, more simply, the *loop rule*.) The third relationship states that Ohm's law holds for each resistance.

If we combine the three statements, we can develop a means of finding the resistance of the circuit as a whole:

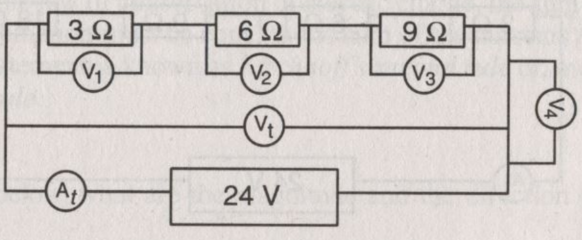
$$V = V_1 + V_2 + V_3 + \ldots$$
$$IR = I_1R_1 + I_2R_2 + I_3R_3 + \ldots$$
$$= I_tR_1 + I_tR_2 + I_tR_3 + \ldots$$
$$= I_t(R_1 + R_2 + R_3 + \ldots)$$

PHYSICS CONCEPTS

- $R_{eq} = R_1 + R_2 + R_3 + \ldots$

R_{eq} is known as the *equivalent resistance* of the circuit.

Now let us examine the series circuit diagram again and calculate all of the meter readings.



The equivalent resistance (R_{eq}) of the circuit is found from the relationship

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$
$$= 3\ \Omega + 6\ \Omega + 9\ \Omega = 18\ \Omega$$

The current through the circuit (I) is found from the relationship

$$V = IR_{eq}$$

We know that V_t equals 24 volts since the source supplies the entire circuit:

$$24\text{ V} = I(18\ \Omega)$$
$$I = 1.33\text{ A}$$

The potential difference across each resistance can be found by using Ohm's law:

$$V_1 = (1.33\text{ A})(3\ \Omega) = 4\text{ V}$$
$$V_2 = (1.33\text{ A})(6\ \Omega) = 8\text{ V}$$
$$V_3 = (1.33\text{ A})(9\ \Omega) = 12\text{ V}$$

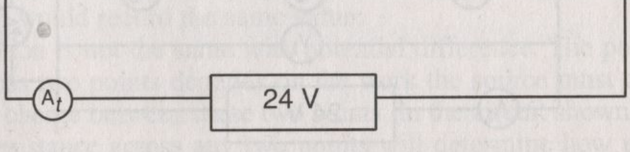
What does voltmeter V_4 read? Since we neglect the resistance of the connecting wires, we assume their resistance to be (nearly) 0 ohm, so the potential difference needed to move the charges across that section of the wire is (nearly) 0 volt.

Another fact about series circuits is important: As the *number* of resistances in a series circuit increases, the equivalent resistance of the circuit increases and the current through the circuit *decreases*. This effect is roughly equivalent to that obtained by increasing the length of a conductor. If the resistances were light bulbs, the bulbs would get dimmer as more were added to the circuit. The next problem illustrates this point.

PROBLEM

Suppose a fourth resistance of 18 ohms is added to the series circuit we have been considering. Calculate (a) the equivalent resistance of the circuit and (b) the current through the circuit.

SOLUTION



(a) The equival

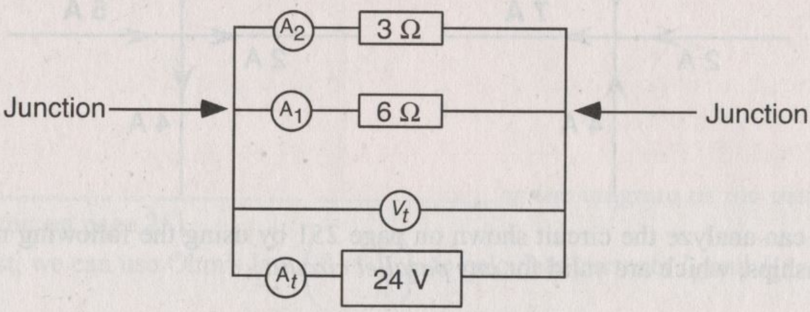
(b) The current m's law:

As stated above, the equivalent resistance has increased and the current through the circuit has decreased in comparison to the original circuit.

9.8 PARALLEL CIRCUITS

In contrast to a series circuit, a **parallel circuit** has more than one current path. If a segment of a parallel circuit is interrupted, the result will not necessarily be that the entire circuit ceases to operate. In a home, for example, the burning out of a single bulb does not usually darken the entire house.

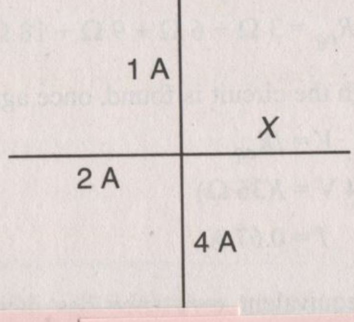
The diagram below represents a parallel circuit containing two resistances and a number of suitably placed meters.



In this type of circuit, the current separates into more than one path. The point (or points) where this separation occurs is known as a *junction*. As a consequence of the law of conservation of electric charge, the sum of the currents *entering* a junction must be equal to the sum of the currents *leaving* the junction. This statement is known as *Kirchhoff's second rule* or, more simply, as the *junction rule*.

PROBLEM

In the diagram below, what are the magnitude and the direction of the current in wire *X*?



We can analyze the circuit shown on page 251 by using the following relationships, which are valid for *any parallel circuit*:

PHYSICS CONCEPTS

- $$V = V_1 = V_2 = V_3 = \ldots = V_n$$
- $$I = I_1 + I_2 + I_3 + \ldots + I_n$$

$$V_n = I_n R_n$$

The first relationship states that the potential difference across a parallel circuit is constant and equal to the battery voltage. This relationship follows from the fact that each resistance comprises an independent path for the flowing charges and that, if one resistance ceases to operate, the others can continue to function. The second relationship states that the current through the entire circuit is equal to the sum of the currents through all the resistances. (This is really an application of Kirchhoff's second rule.) Once again, the third relationship states that Ohm's law holds for each resistance.

If we combine the three statements, we can develop a means of finding the resistance of the parallel circuit as a whole:

$$V_n = I_n R_n \Rightarrow I_n = \frac{V_n}{R_n}$$

$$I = I_1 + I_2 + I_3 + \ldots$$

$$\frac{V}{R_{eq}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \ldots$$

$$= \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} + \ldots$$

$$I = V\left(\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\right)$$

PHYSICS CONCEPTS

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n}$$

Here, R_t is the equivalent resistance of the parallel circuit. We will use these relationships to calculate the meter readings in the diagram of the parallel circuit on page 251.

First, we can use Ohm's law $\left(I = \frac{V}{R}\right)$ to calculate currents I_1 and I_2:

$$I_1 = \frac{V}{R_1} = \frac{24\text{ V}}{3\ \Omega} = 8\text{ A}$$

$$I_2 = \frac{V}{R_2} = \frac{24\text{ V}}{6\ \Omega} = 4\text{ A}$$

Next, we calculate the total current I by adding I_1 and I_2:

$$I = 8\text{ A} + 4\text{ A} = 12\text{ A}$$

We find the equivalent resistance R_{eq} from the relationship

$$\frac{1}{R_{eq}} = \frac{1}{R_1} = \frac{1}{R_2}$$

$$= \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} = \frac{1}{2\ \Omega}$$

$$R_{eq} = 2\ \Omega$$

We could also have used Ohm's law ($V = IR_{eq}$) to calculate the equivalent resistance of this circuit.

We note that the equivalent resistance is less than any single resistance in the circuit. This is characteristic of parallel circuits in general.

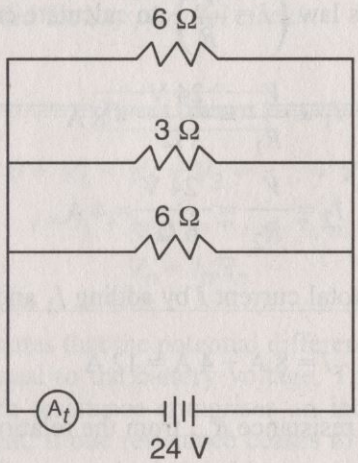
If more resistance is added in parallel, the equivalent resistance *decreases* and the total current *increases* because each new parallel resistance creates another independent path in which charges can flow. The result is roughly equivalent to that obtained by increasing the cross-sectional area of a conductor. For this reason, overloading a household circuit by connecting too many electrical appliances is dangerous. As the current in the house wires increases, the amount of heat energy also increases, a situation that may lead to fires in unprotected circuits. Fortunately, fuses and circuit-breakers are designed to prevent such fires from occurring. The next problem illustrates this effect.

PROBLEM

A 6-ohm resistor is added in parallel to the parallel circuit shown at the beginning of this section. Calculate (a) the equivalent resistance and (b) the total current of the altered circuit.

SOLUTION

The diagram of the modified circuit is as follows:



(a)

(b)

As we can see, the equivalent resistance has decreased to 1 Ω and the total current has increased to 36 A.

Most circuits represent more complex combinations of series and parallel arrangements than are shown in this chapter. In addition, they may include additional power sources, current loops, and junctions. These complex circuits will not be analyzed in this book. You should be aware, however, that Kirchhoff's rules and some fancy algebra can be used for these analyses. Ask your physics teacher to show you how to analyze one of these complex circuits. Here is a situation where your teacher earns his or her richly deserved pay!