Chapter Five

CIRCULAR MOTION AND GRAVITATION

KEY IDEAS

An object that travels in a circular path experiences an acceleration directed toward the center of the circle and known as a centripetal acceleration. The unbalanced force responsible for this acceleration is called a centripetal force and is also directed toward the center of the circle.

The study of planetary motion was greatly advanced by Kepler, who deduced three laws that provided a mathematical basis for this motion. Kepler's three laws were later proved by Newton when he developed his law of universal gravitation: two masses attract each other with a force proportional to their masses and inversely proportional to the square of the distance between them.

KEY OBJECTIVES

At the conclusion of this chapter you will be able to:

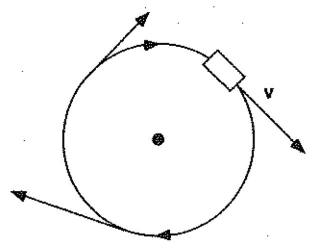
- Identify the direction of an object's velocity when it is undergoing uniform circular motion.
- Define the terms *centripetal acceleration* and *centripetal force*, and identify the directions of these quantities when an object undergoes uniform circular motion.
- State the equations for calculating centripetal force and centripetal acceleration.
- Solve problems involving uniform circular motion.
- Define the term *period of revolution* and relate it to the equations of uniform circular motion.
- State Newton's law of universal gravitation and solve problems related to it.
- Solve simple problems involving satellites in (a circular) orbit.
- Define the term *geosynchronous orbit.*
- Relate weight to gravitational force.
- Describe the field concept of gravitation.
- Relate the strength of a gravitational field with the acceleration due to gravity.

5.1 INTRODUCTION

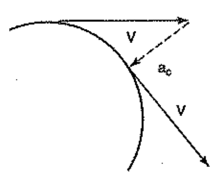
In this chapter we study another aspect of motion in a plane, namely, motion in a circle. The subject of circular motion leads, in turn, to a study of gravitation because both natural and artificial satellites travel in nearly circular paths.

❦ 5.2 CIRCULAR MOTION

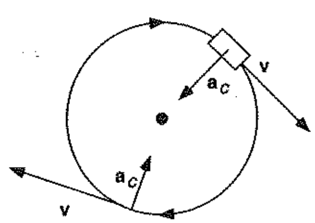
An object moves in a circular path at constant speed as indicated in the diagram below.



At various points in the path, the direction of the velocity, **v**, of the object is tangent to the circle, as shown in the diagram. Since the *direction* of the object's motion is changing, the object must be subjected to an unbalanced force and is, therefore, accelerating. Here is an example of an object that accelerates even though its speed does not change. This is due to the fact that acceleration is a change in the *velocity* of an object, and this change can be in the magnitude and/or direction of the velocity.



As the object moves in its circular path, its acceleration always points toward the center of the circle. For this reason we call the acceleration a **centripetal** ("center-seeking") acceleration ($\mathbf{a}_c$).



❦ indicates that material is part of the New York State core curriculum.

The centripetal acceleration of an object is calculated by means of this equation:

PHYSICS CONCEPTS

❦ $$\mathbf{a}_c = \frac{v^2}{r}$$

where **v** is the tangential speed of the object and r is the radius of the circular path.

The unbalanced force that causes the centripetal acceleration is called the **centripetal force** ($\mathbf{F}_c$). It also points toward the center of the circular path and is given by this relationship:

PHYSICS CONCEPTS

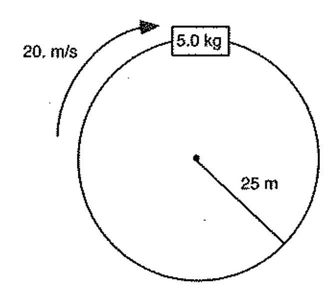
❦ $$\mathbf{F}_c = m\mathbf{a}_c = \frac{mv^2}{r}$$

Note that the format of the last equation above results from the substitution of the $\mathbf{a}_c$ formula into the $\mathbf{F}_c$ formula. The formula in this format does not appear on the Reference Tables; however, it is the most common form of the formula necessary for solving problems involving circular motion.

We can think of the centripetal force as the force needed to keep the object in its circular path. If the mass of the object were increased, or if the speed of the object were increased, more force would be needed to keep the object in its circular path. If, however, the radius of the path were increased, less force would be required for this purpose.

PROBLEM

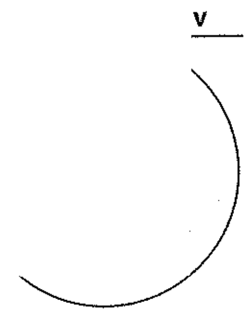
A 5.0-kilogram object travels clockwise in a horizontal circle with a speed of 20. meters per second, as shown in the diagram. The radius of the circular path is 25 meters.



(a) Calculate the centripetal acceleration and the centripetal force on the object.

(b) In the position shown, indicate the *direction* of the velocity and of the centripetal force of the object.

SOLUTION



Measuring the speed of an object in circular motion is not always easy, but we can measure the speed of the object indirectly by making use of a quantity known as the period (T). The period of revolution is the time an object takes to complete one revolution in a circular path. In one revolution, the distance the object travels equals the circumference of the circle ($2\pi r$). Using the equation $v = \frac{d}{t}$, we substitute and obtain

PHYSICS CONCEPTS

$$v = \frac{2\pi r}{T}$$

Note that the above formula does not appear on the Reference Tables but is frequently needed to solve problems involving circular motion.

PROBLEM
An object traveling in a circular path makes 1200 revolutions in 1.0 hour. If the radius of the path is 10. meters, calculate the speed of the object.

SOLUTION
If the object makes 1200 revolutions in 1.0 h (3600 s), the time (T) for one revolution is

The speed of the object is

$$v =$$

The equation for the centripetal acceleration can be rewritten in terms of the period of revolution rather than the speed of the object:

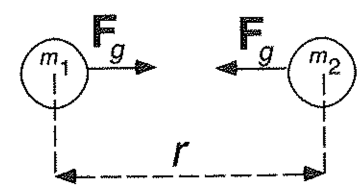
PHYSICS CONCEPTS

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

5.3 NEWTON'S LAW OF UNIVERSAL GRAVITATION

The motions of the planets in the heavens were of great interest in the sixteenth century. The German astronomer Johannes Kepler summarized the laws of planetary motion. More than half a century after Kepler proposed his laws, Isaac Newton was able to prove them mathematically by means of his law of universal gravitation. Newton recognized that the force responsible for pulling an apple toward the Earth has the same origin as the force that keeps the Moon in its orbit about the Earth. This force, which we call *gravitation*, is present between all bodies of mass in the universe, from the largest galaxies to the smallest atoms.

Newton's law of universal gravitation can be explained as follows. Consider two point masses, m_1 and m_2, separated by a distance r as shown in the diagram (the distance between object centers).



The force of gravitation is an *attractive* force. The force on either mass ($\mathbf{F}_g$) is given by this relationship:

PHYSICS CONCEPTS

$$\mathbf{F}_g = \frac{Gm_1 m_2}{r^2}$$

According to *Newton's universal law:*

The gravitational force between two masses is directly proportional to each mass and inversely proportional to the square of the distance between the masses.

This is known as an *inverse square* law. The constant G is called the *universal gravitational constant*. Its measured value is 6.67×10^{-11} newton·meter^2 per kilogram^2. The table below illustrates how the relative gravitational force between two objects depends on their relative masses and the relative distance between them.

Change	Relative $Mass_1$	Relative $Mass_2$	Relative Distance	Relative Force
Original condition	m_1	m_2	r	F
Double m_1	$2m_1$	m_2	r	2F
Triple m_2	m_1	$3m_2$	r	3F
Double m_1, Triple m_2	$2m_1$	$3m_2$	r	6F
Double r	m_1	m_2	$2r$	¼F
Triple r	m_1	m_2	$3r$	⅑F
Quarter r	m_1	m_2	¼r	16F
⅕ of r	m_1	m_2	⅕r	25F

PROBLEM

Calculate the gravitational force of attraction between the Earth and the Moon, given that the mass of the Earth is 6.0×10^{24} kilograms, the mass of the Moon is 7.4×10^{22} kilograms, and the average Earth–Moon distance is 3.8×10^8 meters.

5.4 SATELLITE MOTION

Suppose a satellite is traveling around the Earth in a circular orbit at a distance r from the center of the planet. The satellite is kept in orbit by a centripetal force whose magnitude is given by the relationship:

$$F_c = \frac{m_s v^2}{r}$$

where m_s is the mass of the satellite, $\mathbf{v}$ is its speed in orbit, and r is the distance from the center of the Earth. We know that the origin of this centripetal force is gravitational force; therefore, we can equate the centripetal force relationship with Newton's law of universal gravitation:

$$F_c = \frac{m_s v^2}{r} = \frac{Gm_{\text{Earth}} m_s}{r^2} = F_g$$

$$\frac{v^2}{r} = \frac{Gm_{\text{Earth}}}{r^2}$$

$$v = \sqrt{\frac{Gm_{\text{Earth}}}{r}}$$

We can use the relationship to find the speed necessary for the satellite to be in a given orbit. If the satellite were to move faster, its orbit would be closer to the Earth. If the satellite were to move slower, its orbit would be further from the Earth.

There is a specific distance at which the period of a satellite matches the period of the Earth's rotation (1 day). This type of orbit is known as a **geosynchronous orbit**. Satellites in geosynchronous orbits are especially useful for communication purposes because their positions with respect to the ground do not vary.

5.5 WEIGHT AND GRAVITATIONAL FORCE

We have measured an object's weight by the relationship $\mathbf{F}_g = m\mathbf{g}$, and we can measure an object's gravitational attraction to the Earth by the equation

$$\mathbf{F}_g = \frac{Gm_1 m_2}{r^2}$$

Weight and gravitational attraction, however, are one and the same force since an object's weight is gravitational in origin. Accordingly, we can set the two terms equal to each other:

$$\mathbf{F}_g = m\mathbf{g} = \frac{Gmm_{\text{Earth}}}{r^2} = \mathbf{F}_g$$

$$\mathbf{g} = \frac{Gm_{\text{Earth}}}{r^2}$$

We can now appreciate the fact that the gravitational acceleration of an object is independent of its mass; it depends only on the mass of the Earth and the distance of the object from the Earth's center.

PROBLEM
Calculate the gravitational acceleration of a satellite that is in orbit at a distance of 1.0×10^8 meters from the center of the Earth.

SOLUTI(

5.6 THE GRAVITATIONAL FIELD

A gravitational field is a region of space that attracts masses with a gravitational force. The gravitational-field concept assumes that one mass somehow changes the space around it and a second mass then interacts with the field. The result is that an attractive gravitational force is exerted on the second mass. One type of interaction is represented in the diagram below.