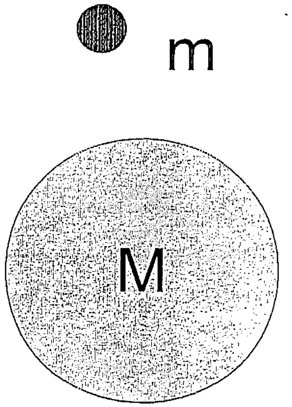
E-field: The Electric Force-field

1. What is a 'force-field'?
   - What is a field? - it is a map over all space

- For example, what is the gravitational force field?

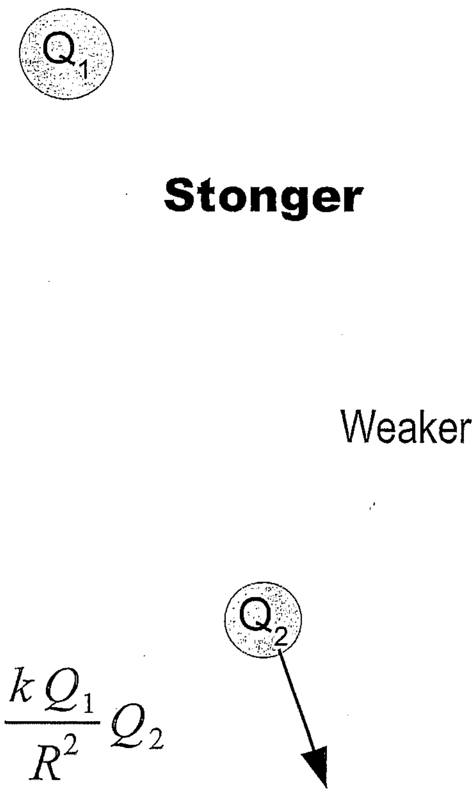
$$F = \frac{G M m}{R^2} = \frac{G M}{R^2} m = g m$$



We know there is a force field near Earth, because whenever we put a mass near it, it falls!

E-field

- Start with charge Q_1
- The space around charge Q_1 has an *Electric Force Field*
- We know this because if another charge Q_2 is brought near, Q_2 experiences a force, even though Q_1 and Q_2 are not touching



$$F = \frac{kQ_1Q_2}{R^2} = \frac{kQ_1}{R^2}Q_2$$

$$E = \frac{kQ_1}{R^2} \quad \text{and} \quad F = EQ_2$$

Q creates a force-field kQ/R^2 throughout all space

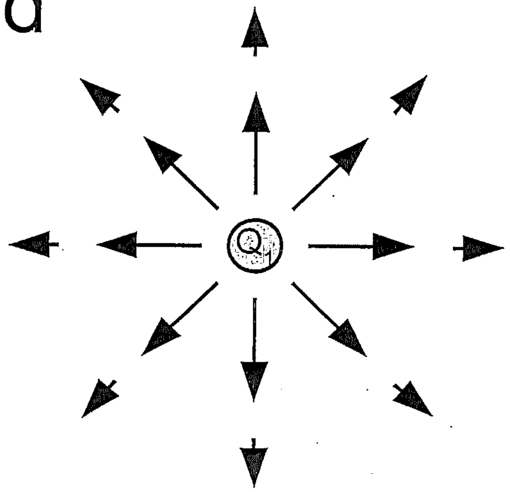
Define and Visualize Electric Field

- The electric field *E* is defined by
  - Units of E are 'N/C'
  - E is a vector
  - E is defined everywhere in space

$$\boxed{F = Q_2 \vec{E}}$$

Q_2 is a (1 C) test charge we carry around space, measure the force on it at each point, and equate that to the *E*-field

- Example: The *E*-field around a charge Q_1
  - Radial vectors
  - $1/R^2$ dependence



$$F = Q_2 \vec{E} = Q_2 \frac{k * Q_1}{R^2}$$

$$\boxed{\vec{E} = \frac{k * Q_1}{R^2}}$$

Electric field problems

1. I am 1 meter away from a 1.0 C charge. What is the electric field where I am?

- What is the electric field 2 meter from the 1.0 C charge?

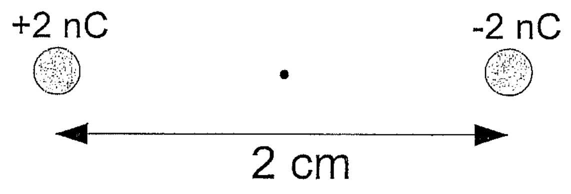
2. The electric field at a location near a point charge is 120 N/C. If the charge is increased by a factor of 4, what is the E-field at the same location?

3. The electric field near a point charge is 120 N/C. If our distance from the charge is increased by a factor of 3, what then is the electric field?

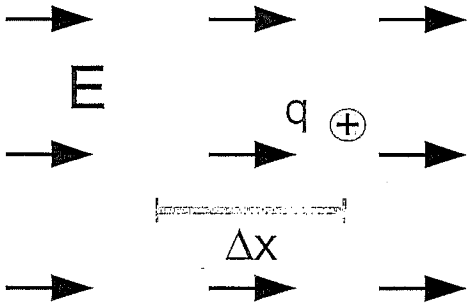
4. What is the E-field 6 cm to the East of a -50 nC point charge? (Give both magnitude and direction)

5. Two charges are arranged as shown:

- What is the E-field at the point half way in between them?



Uniform electric field



- Constant electric force on a charge q
  - If we push q to move it to the left Δx, the potential energy increases by $\Delta PE = F \cdot \Delta x = -qE \cdot \Delta x$
  - Then the voltage must also be changing: $\Delta PE = q\Delta V$
  - So E is related to slope of V
  - Therefore E has units of volts/meter

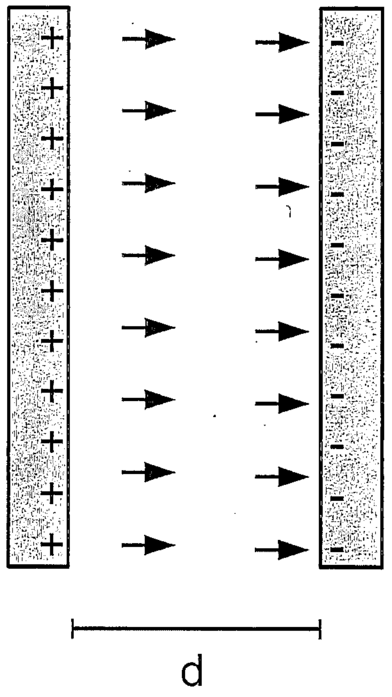
$$\boxed{E = -\Delta V / \Delta x}$$

There is this minus sign because V increases when Δx goes against the E-direction

N / C and V / m both are units for electric field E

How to create a uniform E field?

- Infinite, parallel plates
  - One charged (+), the other has an equal and opposite (–) charge
  - If the separation of the plates is d, what is the voltage difference between the plates?



Parallel metal plates form a 'capacitor'

Electric field and voltage Examples

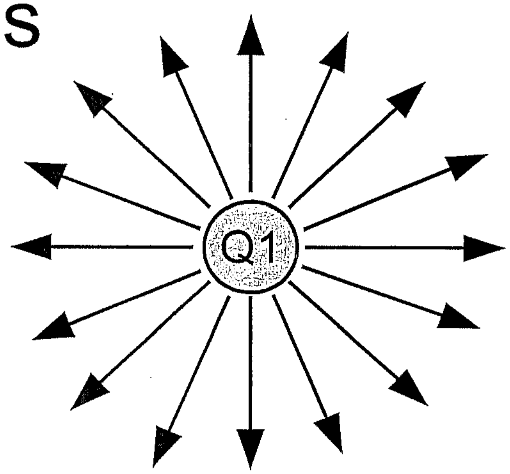
1. Two plates are separated by 1 cm. If one plate is at voltage +50 V, and the other plate is at −100 V, how large is the electric field between the plates?

2. The electric field at a location near a point charge is 120 N/C. Express this in Volts/meter.

3. If the electric field in an air gap increases to 3 x 10^6 Volts / meter, breakdown occurs and a spark jumps across the gap. What is the maximum voltage that can be applied without breakdown across a pair of flat plates separated by 1 cm?

Electric Lines of Force: a way to visualize E-field

1. Lines of Force are continuous lines which show the direction of the E-field throughout space
2. The magnitude of the E-field is represented by the density of the Lines of Force : Denser lines means stronger E-field



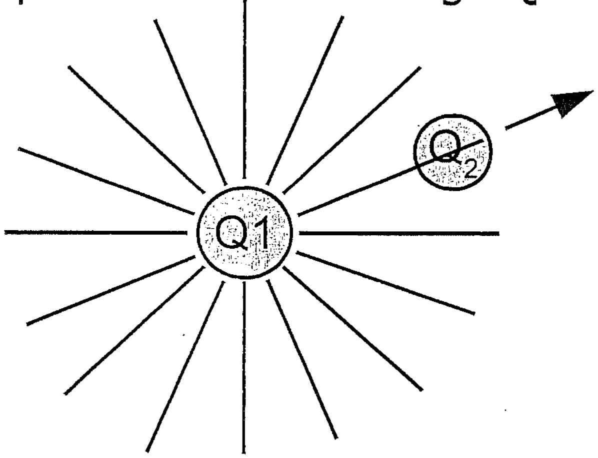
If the charge is 2x bigger, the number of lines of force coming out is 2x as many

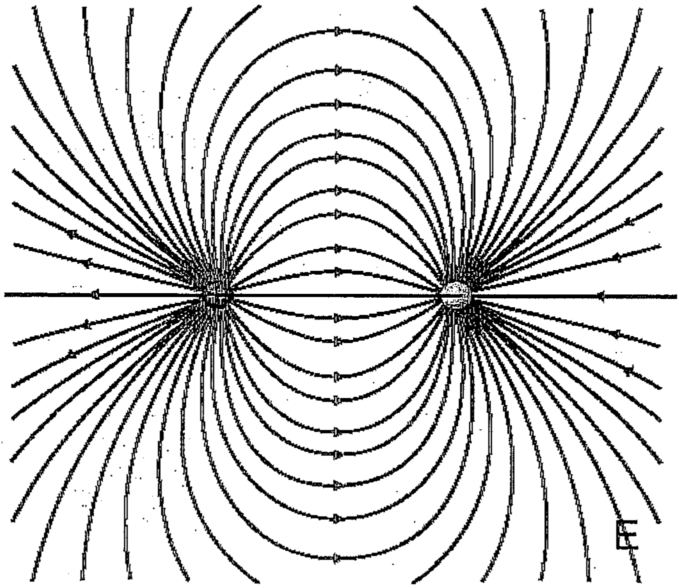
Lines of force similar in shape to strings emanating from the vdg

Rules for Electric Lines of Force

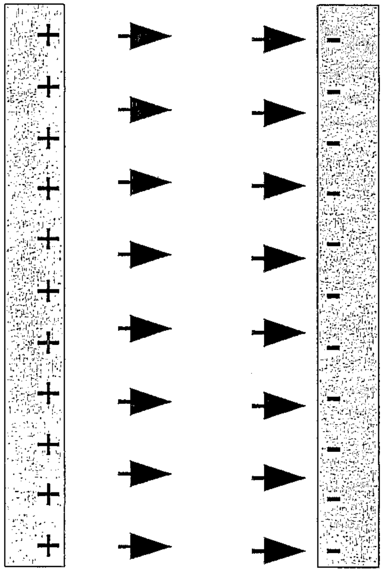
1. The force on test charge Q_2 is parallel to the line of force
   - The magnitude of the force at Q_2 is proportional to the density of the lines of force
2. <u>Lines of Force start on + charges and end on – charges</u>
   - They cannot cross or stop except on charges

Lines of force are equally spaced around a charge Q1



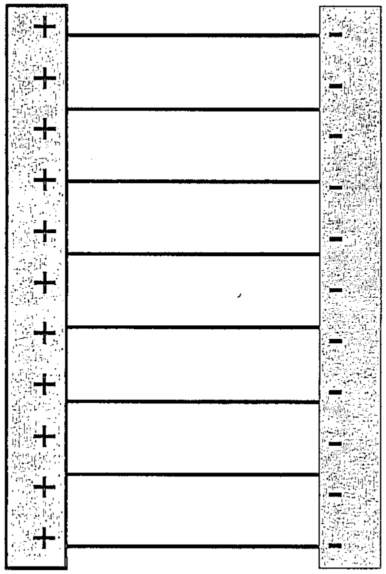


Rules for Electric Lines of Force (continued)



Electric field vectors

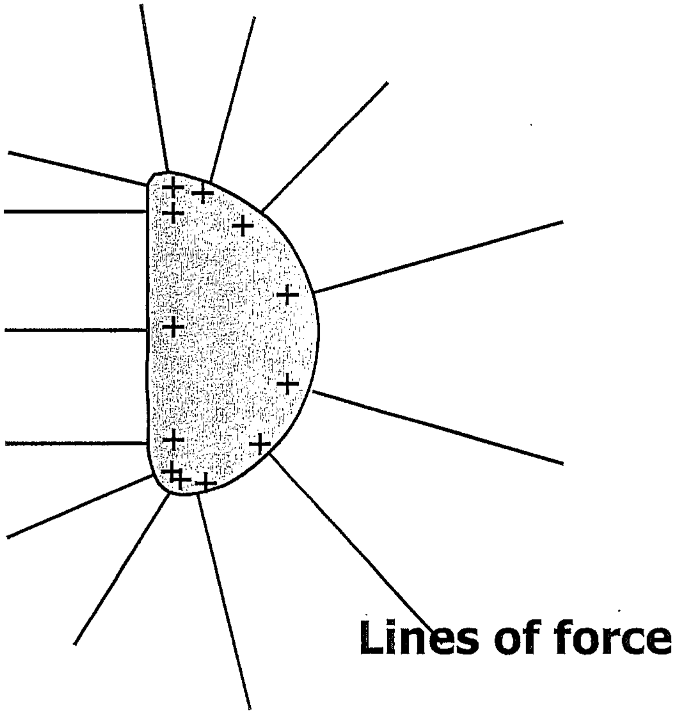
3. For a flat plate, the field is uniform and the lines are evenly spaced, straight, and perpendicular to the plate



Lines of force

Rules for Electric Lines of Force (continued)

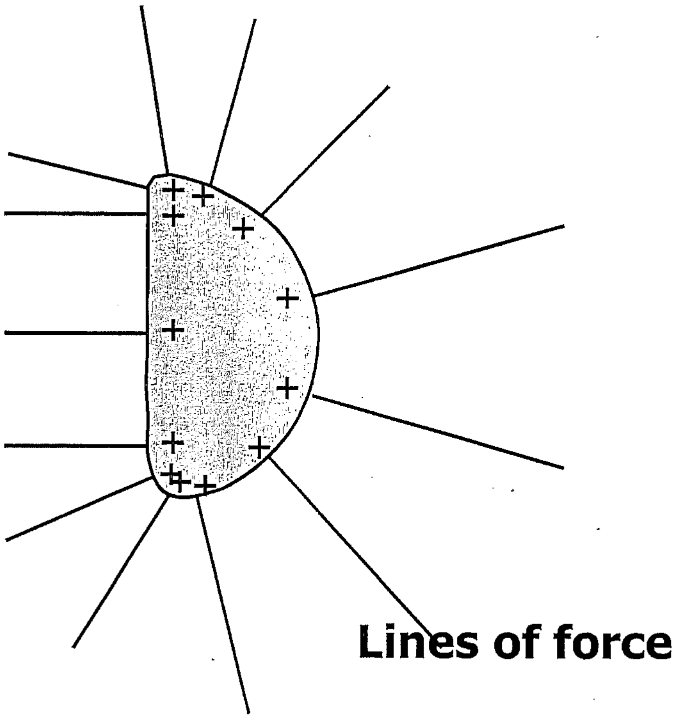
4. Lines of force cannot penetrate a metal object



E-field is zero within a conducting volume

Rules for Electric Lines of Force (continued)

5. Lines approach a metal surface at right angles

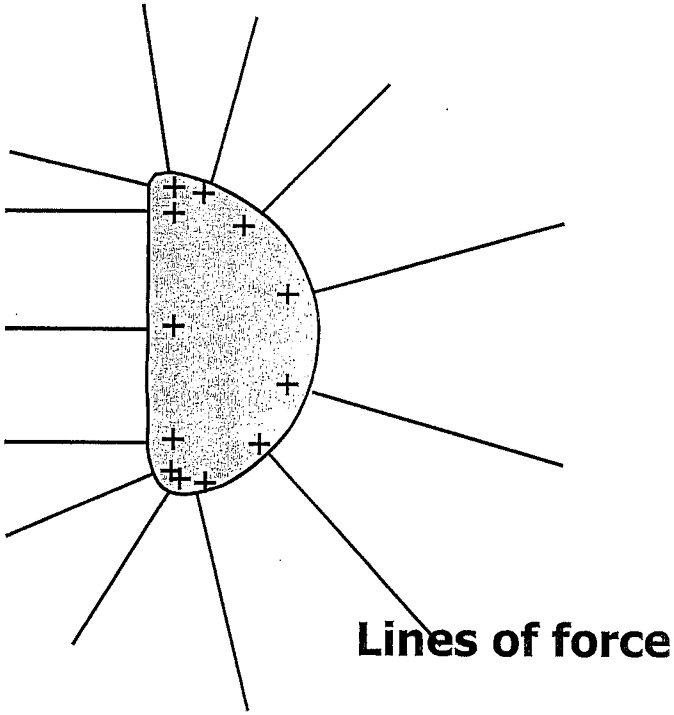


E-field parallel component must be zero at a metal surface

Rules for Electric Lines of Force (continued)

5.

6. On a conducting non-spherical object, charges and lines of force densify near a bulge or point

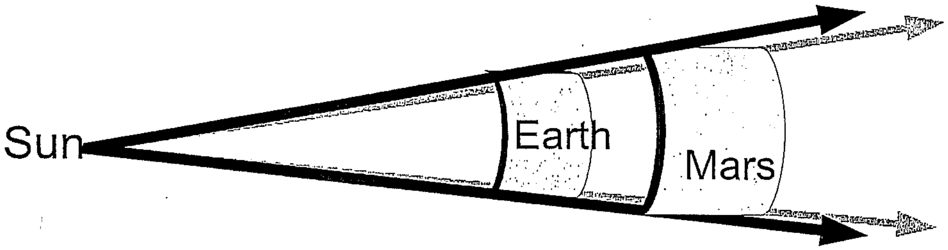


Repulsion moves like charges as far apart as possible in a metal

Magic of the inverse square law

sun's rays flow out from the sun in all directions

- The sun's radiation, calculated on Earth is 1000 W/m^2



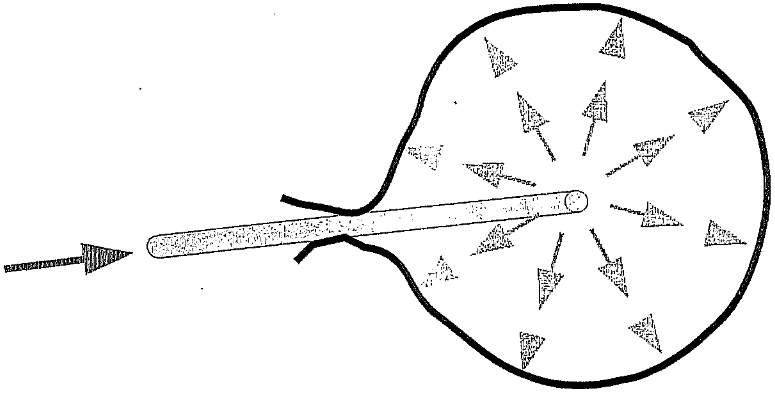
- The sun's radiation, calculated on Mars
  - Which is 1.5 x as far from the sun,
  - Is $1000/1.5^2$ W/m^2

- The rays' intensity falls off as $1/R^2$
- But the area of a sphere is $4\pi R^2$
- So (Area*Intensity) a fixed total number of rays no matter what the distance
  - Total energy emitted by the Sun is fixed

A point charge is a source emitting E-field rays called Lines of Force

Electric Field is similar to Flow in a Fluid – another visual analog

- Suppose water is coming out at a steady rate from a straw placed inside a balloon.
- E-field can represent the flow of the fluid
  - It moves outward radially from the source
  - At a larger radius, the fluid is flowing more slowly – just like $1/R^2$
  - The volume enclosed by the surface is increasing at a steady rate



To get the **exact** analogy to a point charge, the straw must be blowing into the middle of an *infinite* balloon

The electric lines of force and fluid streamlines obey identical rules

Number of lines of force coming out of a charge is proportional to Q. How many lines exactly?

- Imagine a spherical surface placed around the charge
  - The Area of the sphere is proportional to R^2
  - The value of the E-field is proportional to $1/R^2$
- So...the area of the sphere x the value of the E-field *is defined as...*

the # of lines of force N

This is constant everywhere and is equal to $4\pi kQ_1$

$$\vec{E} = \frac{k * Q_1}{R^2}$$

$$Area = 4\pi R^2$$

$$\boxed{\mathbf{N} = E * Area = 4\pi k Q_1}$$

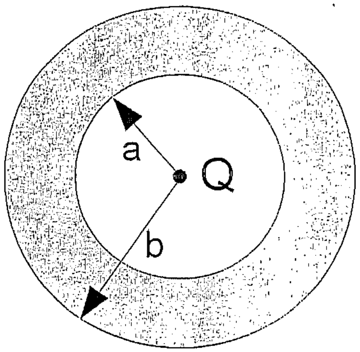
Spherical Charges & Gauss's Law

A hollow metal sphere has an inner radius a and an outer radius b. A charge Q is placed at the center of the sphere.

a. What is the electric field in the hollow part of the sphere, where radius $r < a$?

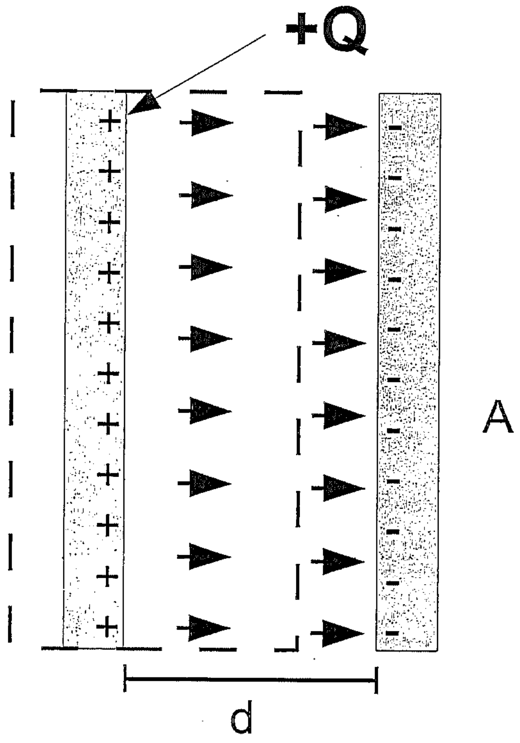
b. What is the electric field in the metal part of the sphere, where $a < r < b$?

c. What is the electric field outside the sphere, where $r > b$?



Uniform Field from Gauss' Law

- Dotted line shows surface around +Q
  - No contribution to N=EA on top and bottom sides
  - E is zero outside of plates



$$A * \vec{E} = 4\pi k Q$$

$$E = 4\pi k\left(\frac{Q}{A}\right)$$

- E is constant between the plates

$$Q = \frac{1}{4\pi k} A * \frac{V}{d} = C * V$$

C is a constant of proportionality called the 'capacitance'

Parallel plates give a constant uniform E-field, and store charge