Chapter Seven

WORK AND ENERGY

KEY IDEAS

Work is the product of force and the component of displacement in the direction of the force; work is a scalar quantity. Without motion there can be no work.

Power is the rate at which work is done, and it is also a scalar quantity. If work is done on an object, the work may be used to change the object's kinetic energy (the energy associated with its motion), its potential energy (the energy associated with its position), or its internal energy (the energy associated with its atoms and molecules).

An elastic collision is one in which momentum and kinetic energy are conserved. When gas molecules collide with the walls of a container, these collisions are very nearly elastic.

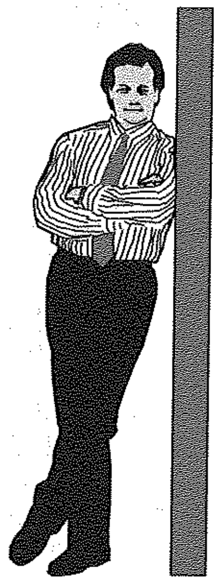
Heat energy is the energy associated with changes in internal energy.

KEY OBJECTIVES

At the conclusion of this chapter you will be able to:

- Define the following terms: *kinetic energy; gravitational potential energy; elastic potential energy; internal energy; partially inelastic collision; totally inelastic collision; elastic collision.*
- Define the term *work*, and state its SI unit.
- Solve problems involving force, displacement, and work.
- Define the term *power*, and state its SI unit.
- Solve problems involving power and work.
- State the equation for calculating kinetic energy, and solve problems using this equation.
- State the equation for calculating gravitational potential energy, and solve problems using this equation.
- Solve problems that relate changes in kinetic energy to changes in gravitational potential energy.
- State the equation for calculating the elastic potential energy of a spring, and solve problems using this equation.

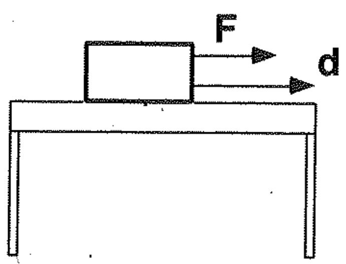
✱ 7.1 WORK



Would you pay a person $5 an hour to do this? The typical answer is "No!" because the person isn't doing any work. What exactly do we mean by the term *work*? In physics, **work** is defined as the product of force and displacement, assuming that both are in the *same direction*. As in the case of the man above, if the displacement of the object in the direction of the force is equal to zero, then the force does no work.

PHYSICS CONCEPTS

✱ $W = \mathbf{F} \cdot \mathbf{d}$



Even though force and displacement are vector quantities, work is a scalar quantity. The unit of work is the *newton · meter*, which is called a joule (**J**) in honor of English scientist James Prescott Joule.

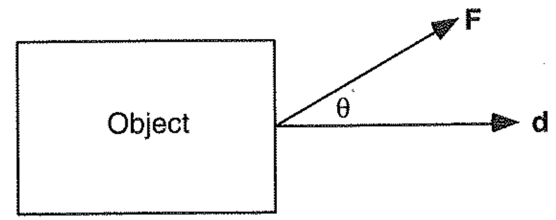
✱ indicates that material is part of the New York State core curriculum.

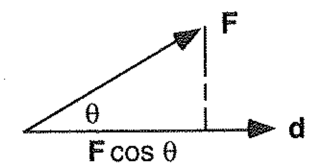
PROBLEM
How much work is done on an object if a force of 30 newtons [south] displaces the object 200 meters [south]?

SOLUTION

Now we will suppose that the force and the displacement are not in the same direction. We define work to be the product of the component of the force in the direction of the displacement and the displacement.

The diagram illustrates this relationship.





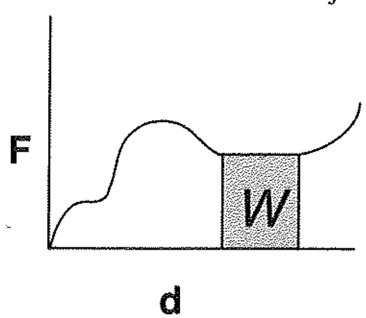
$$W = (\mathbf{F} \cos\theta) \cdot \mathbf{d}$$

PROBLEM
As Alex pulls his red wagon down the sidewalk, the handle of the wagon makes an angle of 60° with the pavement. If Alex exerts a force of 100 newtons along the direction of the handle, how much work is done when the displacement of the wagon is 20 meters along the ground?

SOLUTION

PROBLEM
A constant force of 50 newtons is applied over a distance of 10 meters.
(a) Prepare a graph of force versus displacement.
(b) Calculate the area underneath the curve.
(c) Calculate the work done.

It is evident that the area under the force-versus-displacement curve, shown below, is equal to the work done on the object.



In general, the area under *any* force-versus-displacement curve is equal to the work done.

7.2 POWER

Power is a term that is frequently misused. We define **power** as the *rate* at which work is done:

Physics Concepts

$$P = \frac{W}{t} = \frac{Fd}{t} = F\bar{v}$$

Power is also a scalar quantity, and its unit is joules per second (J/s), also known as the watt (W).

PROBLEM
If 3000 joules of work is performed on an object in 1.0 minute, what is the power expended on the object?

SOLUTION

PROBLEM
A 200-newton force is applied to an object that moves in the direction of the force. If the object travels with a constant velocity of 10 meters per second, calculate the power expended on the object.

SOLUTION

7.3 ENERGY

When work is done on an object, the "energy" of the object is changed. *Energy* is a very broad term related to work, and it has a variety of forms. In this chapter we will consider two of these forms, kinetic and potential energy. Together the kinetic energy and the potential (gravitational and elastic) energies are called the *mechanical energy* of the object.

Kinetic Energy

An object is traveling along a frictionless horizontal surface. A constant force is applied to the object in the direction of its displacement. What is the result of the work done on the object?

We know from Newton's second law that $F = ma$, therefore $W = ma \cdot d$. If the force on the object is also constant, its acceleration is constant and we can write

$$v_f^2 = v_i^2 + 2a \cdot d$$

If we solve this relationship for $a \cdot d$, we find that

$$a \cdot d = \frac{v_f^2 - v_i^2}{2}$$

We can substitute this solution for $\mathbf{a} \cdot d$ into our work relationship:

$$W = ma \cdot d$$
$$= m\left(\frac{v_f^2 - v_i^2}{2}\right)$$
$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The work done on the object changes a quantity called kinetic energy that is related to the mass and the square of the speed of the object. We define kinetic energy (KE) to be:

PHYSICS CONCEPTS

$$KE = \frac{1}{2}mv^2$$

Therefore, $W = KE_f - KE_i = \Delta KE$.

PROBLEM

A 10-kilogram object subjected to a 20.-newton force moves across a horizontal, frictionless surface in the direction of the force. Before the force was applied, the speed of the object was 2.0 meters per second. When the force is removed, the object is traveling at 6.0 meters per second. Calculate the following quantities: (a) KE_i, (b) KE_f, (c) ΔKE, (d) W, and (e) d.

(c)

(e)

Gravitational Potential Energy

Darya lifts a textbook vertically off a desk and holds it above her head. It is clear that Darya has done work on the book because she has applied a force through a distance. However, the change in the kinetic energy of the book is zero. What did Darya's work accomplish?

In this case, the work overcame the attraction of the gravitational field, and as a result the position of the book with respect to the Earth changed. We relate this change to a quantity we call **gravitational potential energy**.

To calculate the change in the gravitational potential energy (PE) of the object, we measure the work done on the object. The force needed to overcome gravity is simply $\mathbf{F}_g$, which is equal to mg. Therefore, since $W = \mathbf{F}_g \cdot d$, we define the change in the gravitational potential energy (ΔPE) as:

PHYSICS CONCEPTS

$$\Delta PE = mg\Delta h$$

Here we use Δh, rather than d, to represent the change in vertical displacement above the Earth. Since we are dealing with a scalar quantity, we will not consider the algebraic signs of the quantities involved. We will simply agree that an object decreases its gravitational potential energy as it moves closer to the Earth (and vice versa).

PROBLEM

A 2.00-kilogram mass is lifted to a height of 10.0 meters above the surface of the Earth. Calculate the change in the gravitational potential energy of the object.

SOLUTION

$$\Delta PE = mg\Delta h$$
$$= (2.00\ \text{kg})\ (9.8\ \text{m/s}^2)(10.0\ \text{m})$$
$$= 196\ \text{J}$$

Since the object has moved *away* from the Earth's surface, its gravitational potential energy has *increased* by 196 joules.

For a change in gravitational energy to occur, there must be a change in the *vertical* displacement of an object; if it is moved only *horizontally*, its gravitational potential energy change is zero. If an object is moved up an inclined plane, its potential energy change is measured by calculating only its *vertical* displacement; the horizontal part of its displacement does not change its potential energy.

Interaction of PE and KE (Conservation of Mechanical Energy)

Ketan tosses an object upward, and it returns to the Earth. Let's analyze the motion of this object using an "energy" point of view. For simplicity we will ignore air resistance.

Ketan's hand does work on the object. The work is transformed into kinetic energy. As the object rises, we observe that its speed decreases to zero. As a result, the kinetic energy of the object is decreasing while its potential energy is increasing. This represents a transformation of energy from kinetic to potential.

On the downward trip, the speed of the object increases. As the potential energy of the object decreases, its kinetic energy increases. This represents a transformation from potential to kinetic energy.

In the system, the sum of potential energy and kinetic energy (the total mechanical energy) has been conserved (i.e., is constant); a change in one is accompanied by an opposite change in the other.

$$\Delta PE = -\Delta KE$$

$$PE_i + KE_i = PE_f + KE_f$$

This can also be expressed as the law of conservation of energy, which states that energy can neither be created nor destroyed. In an ideal mechanical system (a closed system upon which no friction or other external forces act) the total mechanical energy is constant.

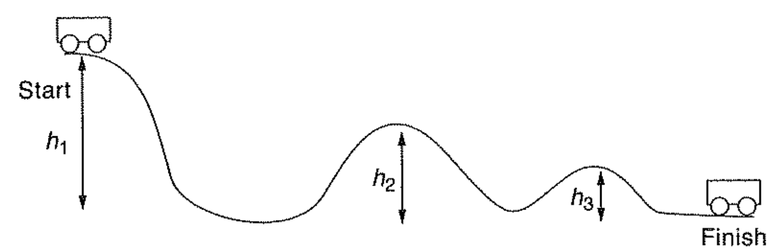
PHYSICS CONCEPTS

$$E_f = E_i$$
$$E_T = PE + KE$$

PROBLEM

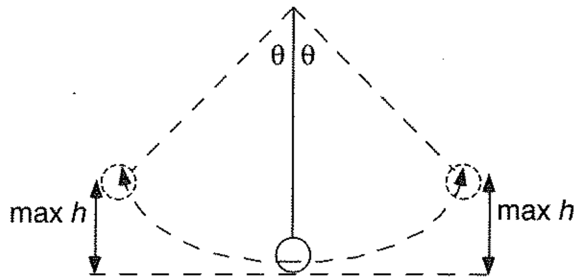
A 0.50-kilogram ball is projected vertically and rises to a height of 2.0 meters above the ground. Calculate: (a) the increase in the ball's potential energy, (b) the decrease in the ball's kinetic energy, (c) the initial kinetic energy, and (d) the intitial speed of the ball.

An amusement-park roller coaster is an example of the interchange of the kinetic and potential energies of the coaster car. As the car falls, its kinetic energy increases; as it rises, its kinetic energy decreases, as shown in the diagram.



Subsequent hills are made shorter and shorter so that the car will continue to have kinetic energy as it moves along the track.

A simple pendulum, shown in the following diagram, is another device that illustrates the transformation between kinetic and potential energies.



In the absence of friction, the swing of a pendulum back and forth will go on continuously. This type of motion, known as *simple harmonic motion* (SHM), occurs often in nature. For example, the oscillation of a spring and the vibration of a tuning fork are examples of SHM.

The time needed to complete one full swing of the pendulum is known as its period (T). The period of a pendulum (for small angles) is related to the length of the pendulum (ℓ) and to gravitational acceleration (g) according to the relationship

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

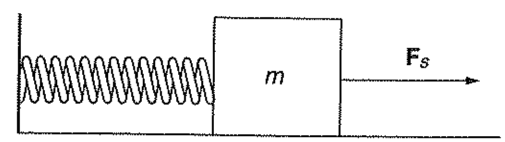
Note that the period of a simple pendulum is independent of the mass of the bob.

PROBLEM

A pendulum whose bob weighs 12 newtons is lifted a vertical height of 0.40 meter from its equilibrium position. Calculate: (a) the change in potential energy between maximum height and equilibrium height, (b) the gain in kinetic energy, and (c) the velocity at the equilibrium point.

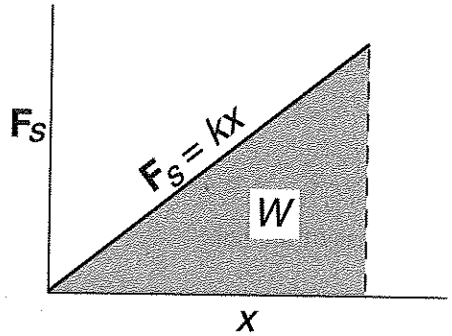
Elastic Potential Energy and Springs

Consider the arrangement shown in the diagram:



Here a spring is attached to a wall and to a mass resting on a horizontal, frictionless table. If we apply a force on the mass and displace it to the right, we have done work. This work has been converted into the spring's potential energy, a quantity we call **elastic potential energy**.

We can calculate the work done in stretching the spring as follows. We know that springs obey Hooke's law, a graph of which is shown below.



Since the area under this graph equals the work done, and we know that the area of a triangle is equal to ½ (base · height), we use the equations

$$W = \frac{1}{2}\, x \cdot \mathbf{F}_S \text{ and } \mathbf{F}_S = kx$$

Then we have

$$W = \frac{1}{2}\, x \cdot kx = \frac{1}{2}\, kx^2$$

and the potential energy of the spring (PE_s) is given by:

PHYSICS CONCEPTS

$$PE_s = \frac{1}{2}\, kx^2$$

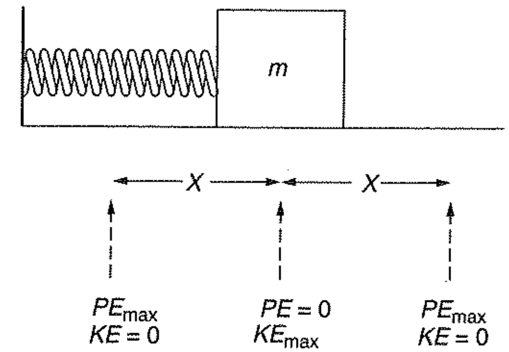
PROBLEM

A spring whose constant is 2.0 newtons per meter is stretched 0.40 meter from its equilibrium position. What is the increase in the elastic potential energy of the spring?

SOLUTION

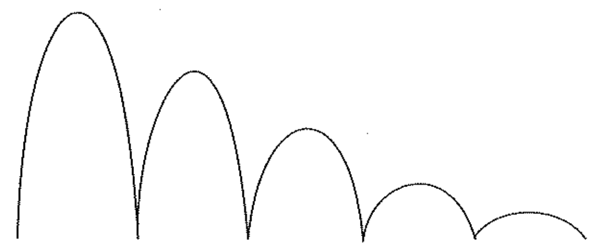
What would happen if we released the spring after stretching it? The force exerted on the mass by the spring (the restoring force) would displace the

mass toward the wall (to the left). The mass would then overshoot its equilibrium position and compress the spring. In turn, the spring would exert a force on the mass away from the wall (to the right). Since friction is absent, this back-and-forth motion, namely, SHM would continue indefinitely. As the spring moved back and forth, there would be a continual exchange between kinetic and potential energies, as shown in the diagram.



Elastic and Inelastic Collisions

Imagine a ball bouncing repeatedly on a sidewalk, as illustrated below.



After each successive bounce, the ball's height above the ground diminishes; eventually the ball comes to rest on the ground.

When the ball is on the ground, part of its kinetic energy is lost and there is an incomplete conversion to potential energy, a phenomenon known as a **partially inelastic collision**. If the ball stuck to the ground after its first bounce, the collision would be termed **totally inelastic**.

If, however, the ball rose repeatedly to the same height, the collisions would be termed **elastic**. In an elastic collision both kinetic energy and momentum are conserved, as outlined below:

$$\mathbf{p}_{1_i} + \mathbf{p}_{2_i} = \mathbf{p}_{1_f} + \mathbf{p}_{2_f}$$
$$KE_{1_i} + KE_{2_i} = KE_{1_f} + KE_{2_f}$$