PHYSICS

A MODEL PREDICTING THE THEORETICAL FLIGHT OF A GOLF BALL

What forces must be taken into consideration in order to create a realistic model of the flight of a golf ball?

Word count: 3990

ABSTRACT

The purpose of the essay is to create and test a program giving the theoretical trajectory of a golf ball at any given initial velocity and launch angle. The aim is to find out if the correct forces have been taken into consideration for the creation of the model. The model is based on the forces acting on the ball that are deemed significant. The basic concepts that apply to the trajectory of a golf ball are introduced; this includes simple projectile motion and the forces that act on a golf ball (gravity, drag and lift). These theories are then applied, and using a technique of differential equations, the model is created on a Microsoft Excel 2008 spreadsheet.

The value and accuracy of the model is then tested using several methods. Firstly, uncertainties of the model are shown graphically and the importance of the forces due to air resistance is demonstrated. The graph obtained by the model is compared with two graphs of shots taken by the author that were analysed using Logger Pro software. The results obtained from the graph are then compared to non visual data from a Tiger Woods' shot, an Ian Poulter (PGA pro) shot and three of the author's shots.

Despite uncertainties stemming from drag and lift coefficients and the fact that many parameters are unaccounted for, the theoretical trajectory model graphs are seen to represent shots taken in the real game with a high level of precision and accuracy. It is therefore confirmed that the forces that act on a golf ball that are deemed significant in projecting the theoretical trajectory are gravity, the drag force and the lift force due to the Magnus effect.

Word count: 281

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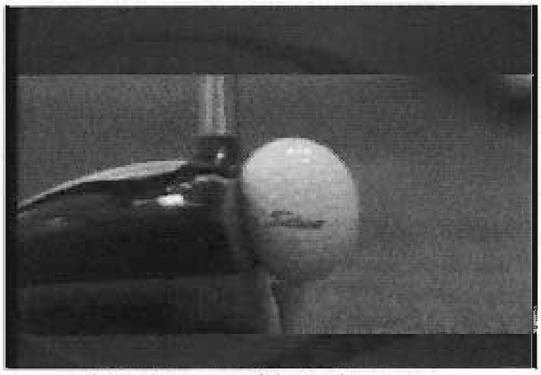
IB EXTENDED ESSAY

A model projecting the theoretical flight of a golf ball

What forces must be taken into consideration in order to create a realistic model of the flight of a golf ball?



www.golf-simulators.com/physics.htm 1

Introduction

Although the most studied motion in the sport of golf is the swing, the most significant aspect of the sport is the ball's trajectory, or rather where it ends up. The swing is the tool, the flight is the result. This essay will examine how to model the flight of a golf ball and why its motion is not a basic parabola. The first step is to establish what projectile motion really is and what formulae can be used to compute the trajectory of an object in motion. Then, case specific formulae of forces acting on a golf ball must be incorporated. Once all these concepts are mastered they can be put together to create the model of the theoretical trajectory of a golf ball. To be sure that the correct forces were taken into consideration and the correct formulae used the model will be compared to several real life golf shots. Unfortunately, there are many variables in the flight of a golf ball that cannot be computed in the model. Therefore, it is crucial that limitations be correctly assessed. Complications such as bounce, side spin and wind are beyond the scope of this essay.

Simple projectile motion- force due to gravity

Simple projectile motion is a parabolic curve that would predict the motion of a golf ball assuming that only the force of gravity was applied. Although this is obviously not the case, it serves as the basis of any flying object, and thus introduces the topic of the trajectory of a golf ball perfectly. At this point the forces due to air are not being considered.

In order to calculate the position of a projectile, the launch angle and launch velocity must be known. The initial position can be assumed to be the origin (0,0). The easiest way to analyse the motion is to understand that the ball is undergoing two simultaneous motions- one in the vertical y direction, the other in the horizontal x direction. This fact will form one of the bases of the trajectory calculations later in the essay.

In the horizontal x direction, the velocity will remain unchanged. It will therefore be the x component of the initial velocity, which can be written as

$$v_x = v_0 * cos\theta$$

As only gravity is being considered, which applies its force in the vertical direction, no forces act horizontally. The acceleration in the x direction is therefore zero. Thus, if the displacement in the horizontal direction is x, then after t seconds, the displacement will be:

$$x = v_0 * cos\theta * t$$

The velocity in the y direction is constantly changing because of the acceleration due to gravity. The initial vertical velocity is v_0*sinθ. From this, it can be determined that the vertical velocity at any time t is given by the following equation:

$$v_y = v_0 * sin\theta - gt$$

gt represents the change of velocity due to acceleration. In order to find the vertical displacement y, a differential equation must be used. Firstly, it is known that $v_y = \frac{dy}{dt}$. So substituting, we get the equation

$$\frac{dy}{dt} = v_0 * sin\theta - gt$$

Solving this equation for y using the variable separable method gives

$$\int dy = \int (v_0 * sin\theta - gt)\, dt$$

$$y = (v_0 * sin\theta)t - \frac{1}{2}gt^2 + c$$

When t=0, y=0. By substitution, c can therefore be solved to give 0 also. The equation for the y component displacement at any time t is

$$y = (v_0 * sin\theta)t - \frac{1}{2}gt^2$$

To find the path of the projectile, the equations for x and y displacements can be combined to yield the following equation of a parabola:

$$y = x\, tan\theta - \frac{g}{2{v_0}^2 cos^2\theta}x^2$$

This is forms the bases of the calculations that will follow where forces due to air are considered. Unfortunately, when these forces are used the graph does not have a simple quadratic equation like this one.

Before moving on to the full flight analysis of a golf ball, the other forces acting on the ball will be explored.

The Drag force

The drag force is more commonly known as air resistance or air friction. This force acts only when the golf ball is moving. As the ball penetrates through the air, it must push the air molecules out of the way. As stated by Newton's third law of motion, the air molecules will therefore apply a force equal in magnitude but opposite in direction opposing the motion of the ball. An equation can be written to represent the drag force on a golf ball:

$$F_{drag} = 0.5 * \rho * A * v^2 * C_d$$

Where:

- ρ is the density of the air at the altitude and temperature of the location where the golf shot is being hit
- A is the cross sectional area of the golf ball, and can be found using $A=\pi r^2$
- v is the instantaneous velocity of the golf ball (the drag force is therefore changing constantly)
- C_d is the drag coefficient. It has no units and varies depending on the spin, velocity and surface conditions. For a dimpled golf ball travelling at normal speeds with normal spin, according to the US patent for a golf ball, the drag coefficient lies between 0.21 and 0.26 ([1]). The drag coefficient can be found from graphs of experimental data but this is complicated and requires a large amount of time.

The drag force changes with the size, shape and speed of an object. For a golf ball, the formula listed above is appropriate. For the density of air, the assumption will be made that the ball is being hit in normal conditions that are 25°C at one atmosphere (typically sea level). The value is thus 1.22kg/m³ ([2]). The radius of a golf ball is 2.10cm ([3]). Through the equation $A=\pi r^2$, the cross sectional area is found to be 0.00139m². Substituting these values into the formula, gives the following numerical equation:

$$F_{drag} = 0.000844 * C_d * v^2$$

The Magnus force (=lift force)

The final force acting on a golf ball that will be investigated and used in the flight model is the Magnus force, which acts only when an object is spinning and moving. It causes objects that are spinning to curve in the direction of their spin, and is thus the cause of fade (or slice) and draw (or hook). However,

[1] "US Patent 5935023 - Golf ball." <http://www.patentstorm.us/patents/5935023.html>
[2] Culp, Randy. <http://my.execpc.com/~culp/rockets/descent.html>
[3] Wall, Alexander <http://www.freepatentsonline.com/6386994.html>

as this model will only consider the two dimensional motion of a golf ball with backspin, the only role of this force is the lift force associated with it. For simplicity, it will be called the lift force from here on. Backspin creates lift because it deforms the airflow that surrounds the ball, creating an area of higher pressure below the ball (see figure 1[4] – note that the ball is travelling from left to right). It can be calculated using the following equation:

$$F_{lift} = 0.5 * \rho * A * v^2 * C_l$$

Where:

- ρ is the density of the air
- A is the cross sectional area of the golf ball
- v is the instantaneous velocity of the golf ball
- C_l is the lift coefficient. It can be found in the same way as the drag coefficient. For a dimpled golf ball hit within the normal range of speed and spin, the lift coefficient is between 0.14 and 0.19 according to the US patent for a golf ball[5] (the higher the spin rate, the greater the coefficient).

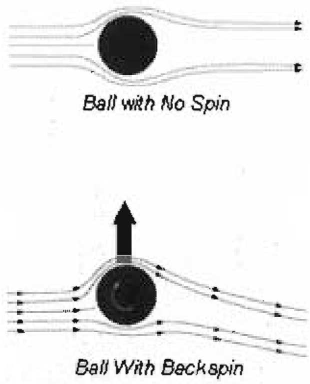


Figure 1

Normal conditions are once again considered, so the density of air and the cross sectional area will be the same as for the drag force, giving the following numerical formula:

$$F_{lift} = 0.000844 * C_l * v^2$$

It is worth noting that in the model of the theoretical trajectory of a golf ball described below, the drag coefficient will be maintained constant throughout the golf shot (As spin changes, so should the coefficient. However an average value will be taken and this should be close enough but remains a limitation). The velocity, which is changing significantly, will be recalculated every hundredth of a second. This is true for the drag force also.

The Buoyant force represents the weight of air that is displaced by an object. This force acts whether the ball is in motion or at rest. It acts only vertically and opposes gravity. It is caused by air molecules colliding with the surface of the ball, as gravity pushes it down. The buoyant force can be modeled by the following equation:

$$F_{buoy} = \rho_{air} * V_{ball} * g \quad ()$$

Where ρ_{air} is the density of air and V_{ball} is the volume of the ball.

However, due to the small volume of a golf ball, this force is negligible for the purpose of the following trajectory calculations. With a magnitude in the 10^{-5}N range, it will have little effect in opposing the gravitational force.

[4] Figure 1: http://tutelman.com/golf/design/swing3.php?ref=equip2golf

[5] "US Patent 5935023 - Golf ball." <http://www.patentstorm.us/patents/5935023.html>

Figure 2 [6] summarizes the forces acting on the ball as well as their direction:

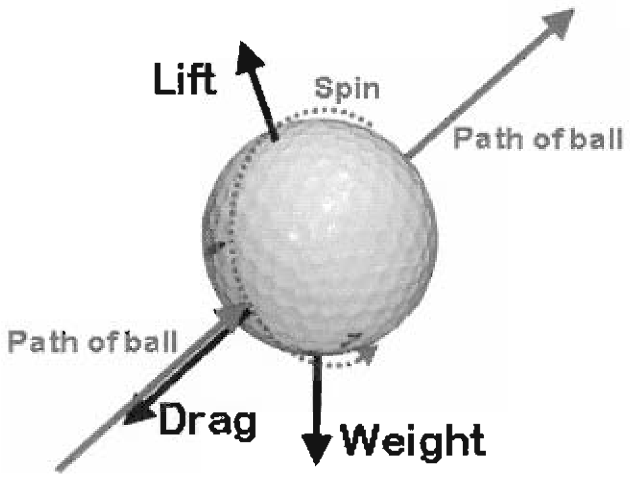


Figure 2

Finding the theoretical trajectory of a golf ball

Taking all those forces and their effects into consideration, a golf ball flight predictor was created using a simple MS Excel 2008 spreadsheet. In the following section, the logic used to develop the spreadsheet will be presented. Unfortunately, the motion of a golf ball cannot be modeled by a simple equation as velocity is constantly changing, and therefore the forces acting on it (remember that both lift and drag are proportional to the velocity squared). Luckily, it is possible to find its position by looking at small time increments. From its initial velocity and angle, we can find the velocity a short time later (0.01s is used in this essay as anything more precise would be unnecessary due to the uncertainties). In other words, an analytical method is impossible but a numerical method can be devised.

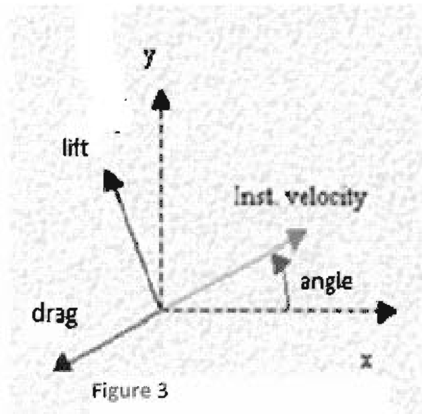


Figure 3

As with simple projectile motion, the flight of the golf ball will be analyzed separately in the horizontal and vertical directions. The first stage in finding the trajectory of a golf ball is to find the forces acting in both the x and y directions. Both the lift and drag forces have x and y components. The angle between the x-axis and F_{drag} is equal to θ, the angle between the x-axis and the instantaneous velocity. The x component of F_{drag} is thus $F_{dx}=F_d*\cos\theta$, while in the y direction, $F_{dy}=F_d*\sin\theta$. Meanwhile, The angle between the lift force and x-axis is 90-θ. As $\sin\theta=\cos(90°-\theta)$ and vice versa, it implies that $F_{lx}=F_l*\sin\theta$ and $F_{ly}=F_l*\cos\theta$ (see figure 3[7]). Gravity applies a force only in the downward y direction. Using the equations found previously, the following is true:

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[6] Figure 2: http://tutelman.com/golf/design/swing3.php?ref=equip2golf

[7] Figure 3: www.golf-simulators.com/physics.htm (edited for use in essay)

$$F_x = -F_{lift} * \sin\theta - F_{drag} * cos\theta$$

$$= -(0.000844 * Cl * v^2) * sin\theta - (0.000844 * Cd * v^2) * cos\theta$$

$$F_y = F_{lift} * cos\theta - F_{drag} * sin\theta - F_g$$

$$= (0.000844 * Cl * v^2) * cos\theta - (0.000844 * Cd * v^2) * sin\theta - 9.81(0.046)$$

However, forces alone are not very useful, as the x and y positions are trying to be found. The next stage is thus to transform these force equations into acceleration equations. This can be achieved using F_{net}=m*a. From this, velocities can be calculated, and from these, displacements.

Following are the transformations from the force in Newtons that gravity, lift and drag apply to the accelerations in ms^{-2} that they cause. To find the acceleration, the force is simply divided by the mass of the golf ball (0.0459kg)

Acceleration due to gravity: $a_g = 9.81$

Acceleration due to lift force: $a_{lift} = \frac{F_{lift}}{m} = \frac{0.000844 * Cl * v^2}{0.0459}$

Acceleration due to drag force: $a_{drag} = \frac{F_{drag}}{m} = \frac{0.000844 * Cd * v^2}{0.0459}$

Combining these with the force components found previously, it can be concluded that:

$$a_x = -a_{lift} * \sin\theta - a_{drag} * cos\theta$$

$$a_y = a_{lift} * cos\theta - a_{drag} * sin\theta - a_g$$

This will now be used to find velocities. Firstly, since $a_x = \frac{dv_x}{dt}$, then:

$$dv_x = a_x * dt$$

The change in velocity was found by finding the instantaneous acceleration at the desired velocity and multiplying it by a small time interval of 0.01s. The changes in velocity between 0s and 8.5s (as all humanly feasible shots will have landed within this time period) every hundredth of a second were found in this way. This was done for both the x and y directions.

By integrating the above equation, an expression for v_x (same for v_y)is found:

$$\int dv_x = \int a_x \, dt$$

$v_x = a_x * t + c$

At time t=0, $v_x = v_0$ = initial velocity. Substitute this into equation to find c:

$v_0 = a_x * 0 + c$

$c = v_0$

Therefore, the equation for velocity is:

$v_x = a_x * t + v_0$

Throughout, Δt is small and v_0 is considered to be the velocity at t-0.01s, so the following equation can be written:

$v_x = \Delta v_x + v_0$

In other words, to find the velocity at time n+0.01s, the change in velocity must be added to the velocity at time n. This is the technique used in the trajectory calculating spreadsheet. A similar process is used to find the change in x and y positions of the golf ball over time. The fact that $v_x = \frac{dx}{dt}$ is used to write:

$\Delta x = v_x * \Delta t$

The differential equation can now be solved by integration (variable separable method):

$$\frac{dx}{dt} = v_x = a_x * t + v_0$$

$$\int dx = \int (a_x t + v_0)\, dt$$

At this point it is not mathematically correct to simply integrate a_x as it also a function of time. However, if we only look at small changes in time then the following equation models the displacement for short time increments:

$$x = \frac{1}{2} a_x (\Delta t)^2 + v_0 \Delta t + c$$

C is equal to the displacement at time t=n-0.01, written as x_0. Also, $v_0 \Delta t = \Delta x$. The equation can therefore be written as:

$$x = \frac{1}{2} a_x (\Delta t)^2 + \Delta x + x_0$$

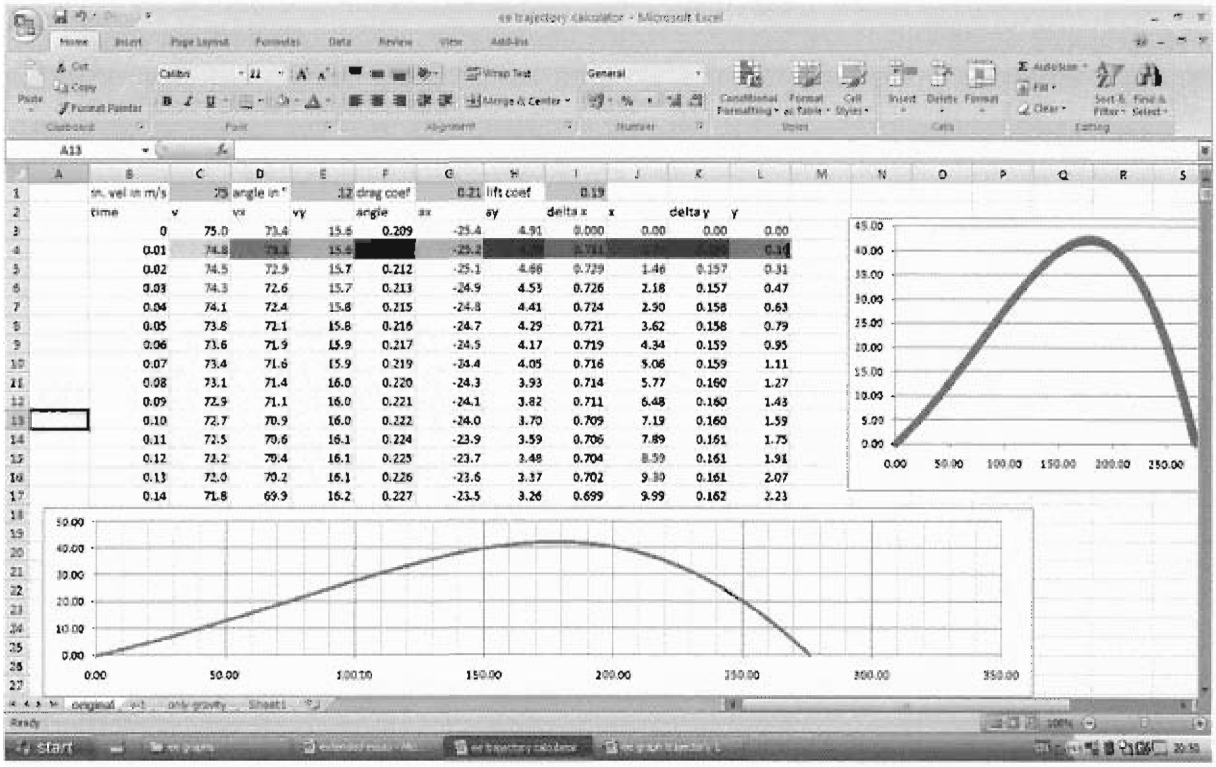
This is the formula and logic used in the creation of the model. The same technique is used to find the displacement in the y direction. When both are solved over the same time interval, a graph of Y against X can easily be plotted to show the trajectory of the golf ball.

The vertical height and horizontal distance from the origin of the golf ball is now available every hundredth of a second at any time between t=0s and t=8.5s. In all humanly feasible conditions, the ball will have landed prior to 8.5s after initial launch time.

The 850 data points were calculated using an excel spreadsheet, and automatically graph themselves, to give a graph of y against x, and thus the theoretical trajectory of the golf ball (only carry - bounces are not taken into consideration).

The spread sheet allows entering of variable values of the launch angle, the launch velocity, the drag coefficient and the lift coefficient.

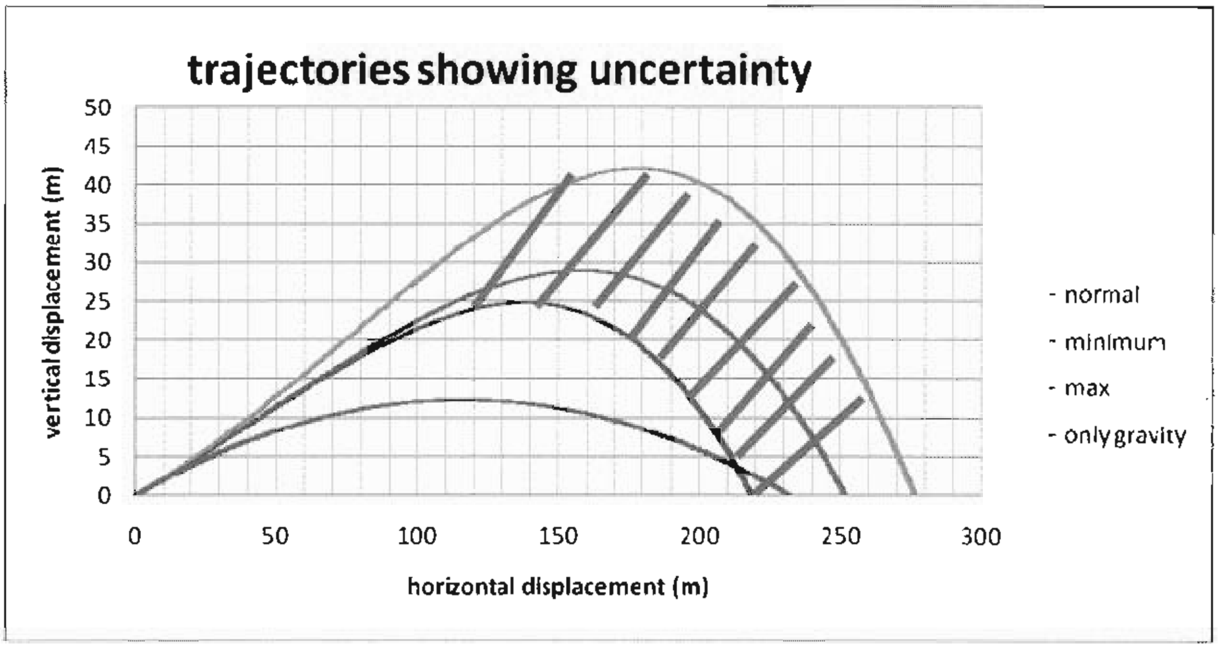
Following is a screenshot of the spreadsheet to better show how the model functions. Details of the coding in the cells producing the graph are in the appendix.



	A	B	C	D	E	F	G	H	I	J	K	L
1		in. vel in m/s	75	angle in °	12	drag coef	0.21	lift coef	0.19			
2		time	v	vx	vy	angle	ax	ay	delta x	x	delta y	y
3		0	75.0	73.4	15.6	0.209	-25.4	4.91	0.000	0.00	0.00	0.00
4		0.01	74.8	[illegible]	15.6	[illegible]	-25.2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5		0.02	74.5	72.9	15.7	0.212	-25.1	4.66	0.729	1.46	0.157	0.31
6		0.03	74.3	72.6	15.7	0.213	-24.9	4.53	0.726	2.18	0.157	0.47
7		0.04	74.1	72.4	15.8	0.215	-24.8	4.41	0.724	2.90	0.158	0.63
8		0.05	73.8	72.1	15.8	0.216	-24.7	4.29	0.721	3.62	0.158	0.79
9		0.06	73.6	71.9	15.9	0.217	-24.5	4.17	0.719	4.34	0.159	0.95
10		0.07	73.4	71.6	15.9	0.219	-24.4	4.05	0.716	5.06	0.159	1.11
11		0.08	73.1	71.4	16.0	0.220	-24.3	3.93	0.714	5.77	0.160	1.27
12		0.09	72.9	71.1	16.0	0.221	-24.1	3.82	0.711	6.48	0.160	1.43
13		0.10	72.7	70.9	16.0	0.222	-24.0	3.70	0.709	7.19	0.160	1.59
14		0.11	72.5	70.6	16.1	0.224	-23.9	3.59	0.706	7.89	0.161	1.75
15		0.12	72.2	70.4	16.1	0.225	-23.7	3.48	0.704	8.59	0.161	1.91
16		0.13	72.0	70.2	16.1	0.226	-23.6	3.37	0.702	9.30	0.161	2.07
17		0.14	71.8	69.9	16.2	0.227	-23.5	3.26	0.699	9.99	0.162	2.23

Uncertainties

The main source of uncertainty comes from the fact that the drag and lift coefficients are not known for each golf shot hit. These numbers are approximated with an uncertainty range of ±0.03 (this comes from the defined accepted range). As the coefficients are constants, this becomes a systematic error. The extent of the error will be determined graphically. On the first three curves only the drag and lift coefficients have been changed. The uncertainty is the shaded area between the green and red curves. (The purple curve is the case of simple projectile motion: it therefore shows the importance of considering forces due to air)



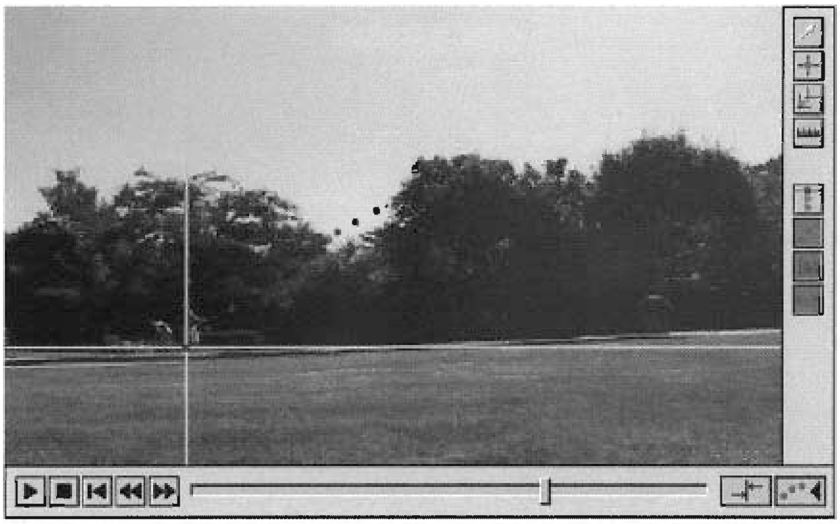
In terms of horizontal distance (this is the golfer's main concern), the value lies between 217m and 275m with the blue curve at 251m. This is equivalent to a horizontal displacement uncertainty of ±12%.

Comparisons between theoretical model proposed and experimental data

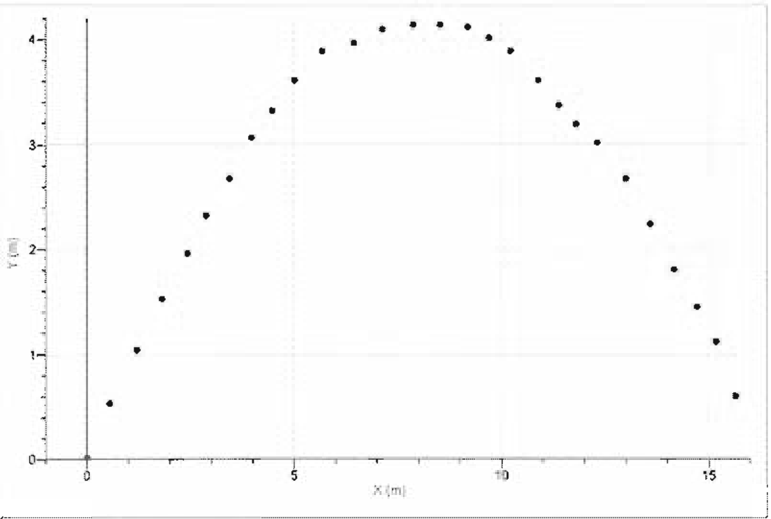
To have a worthy evaluation, the model will now be tested based on experimental findings. Firstly, it will be compared to a real life golf shot, then to shot statistics.

Here is a picture of a shot taken with a 58° lob wedge. The video was taken with a Nikon Coolpix P5000 10 mega pixel camera. It has a video speed of 24 frames per second. The video was then analysed, frame by frame, using logger pro software[8].

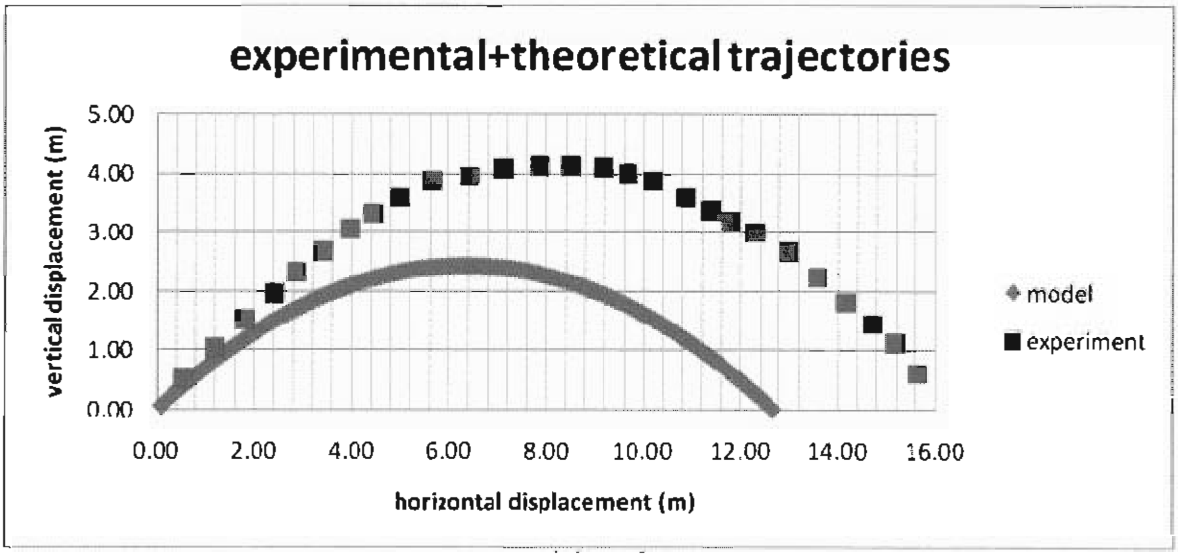
[8] Logger Pro 3.6.0 Copywrite © 2007 Vernier Software and Technology



Once a scale was given, and an origin set, Logger pro then generated the following graph, a model of the previous picture on a scaled axis. The software also gave the instantaneous x and y positions and velocities for every frame. This data was used to calculate the parameters that would later be entered into the model spreadsheet.

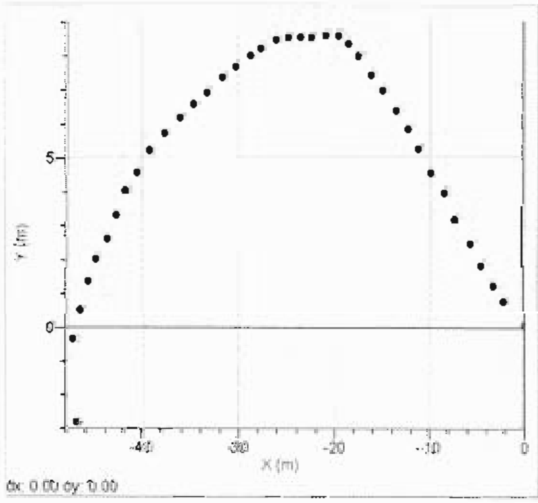


This will now be compared to the theoretical graph from the spreadsheet. Firstly, the launch angle must be calculated. This can be done by constructing a triangle with the x and y positions after two frames (as there is a great uncertainty on the first frame). Therefore, angle $\theta = \arctan\frac{y}{x}$. Then, the launch speed must be found. If the initial launch velocity in the x direction is v_x and the initial launch velocity in the y direction is v_y, then velocity $v = \sqrt{v_x^2 + v_y^2}$. In this case, the launch velocity was found to be 11.4ms^{-1} and the launch angle was found to be 38°. With these parameters, the spreadsheet produced the following graph. The reason that the initial velocities look different is there was a slight scaling error on the experimental graph (not exactly the same zero).

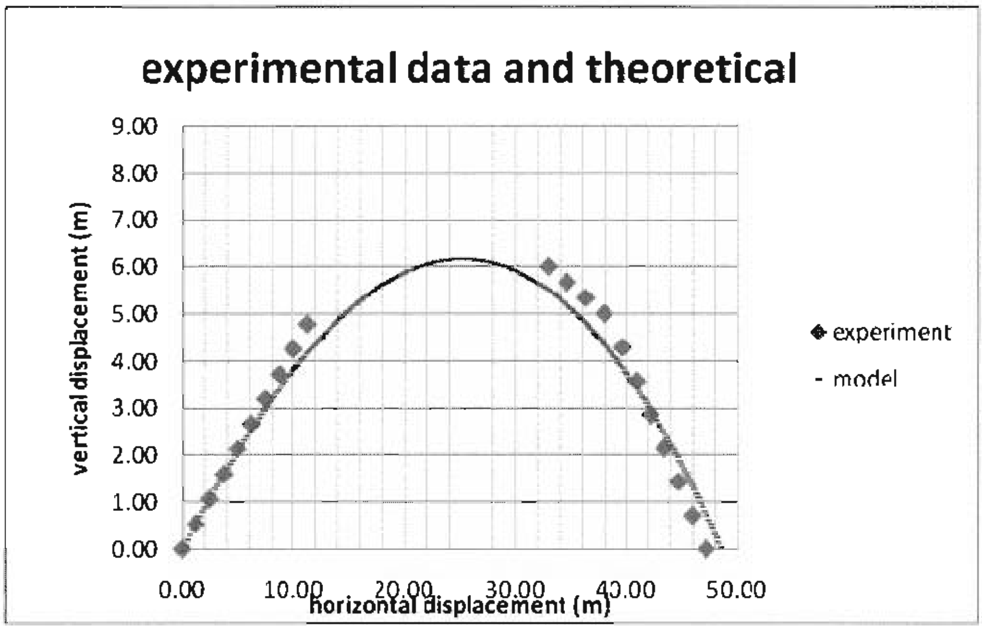


As, can be seen, this predicts a 12.4m shot with a vertical height of approximately 2.5 metres, compared to 4 metres high and 16 metres long in reality. A probable reason for this difference is the higher spin rate created by a lofted club, causing greater lift.

A slightly longer shot will be compared, using the same techniques described previously. However it must be recognised that there are great uncertainties in drawing the graph of the experimental shot. Due to a lack of high speed video quality, the exact location of the ball was not known in every frame and had to be approximated. Following are the picture of the shot and the associated graph. Note that the shot is hit from right to left.



With a launch angle of 25ms^{-1} and launch angle of 25°, the model suggests the brown ball flight. The blue shows the experimental data where the ball was visible (the ball could not be seen against the trees at the top of its trajectory).



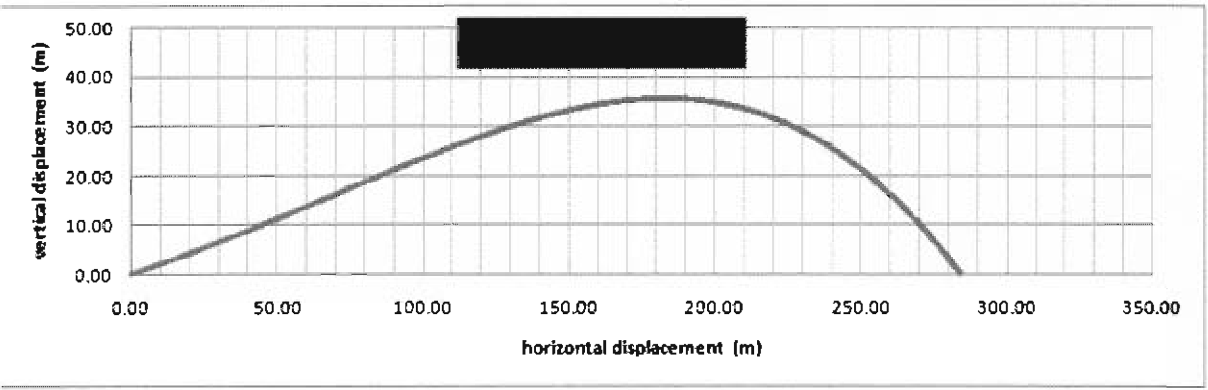
Convincingly, both models portray a shot distance of approximately 48 metres. However, the model suggests a vertical height of 6 metres whereas the reality was closer to 8 metres. Although the experimental graph is approximately correct, it must be remembered that the ball was only visible every few frames, and so parts of the trajectory had to be guessed. This explains the unnatural shape of the curve. Where the ball could be seen, the reality matches the model perfectly and is well inside the uncertainty range.

Although no visual data is available, the spreadsheet graph will be compared to a Tiger Woods drive.

The following is known:

- Launch angle: 11.2°
- launch velocity: 187mph
- carry distance: 297 yards

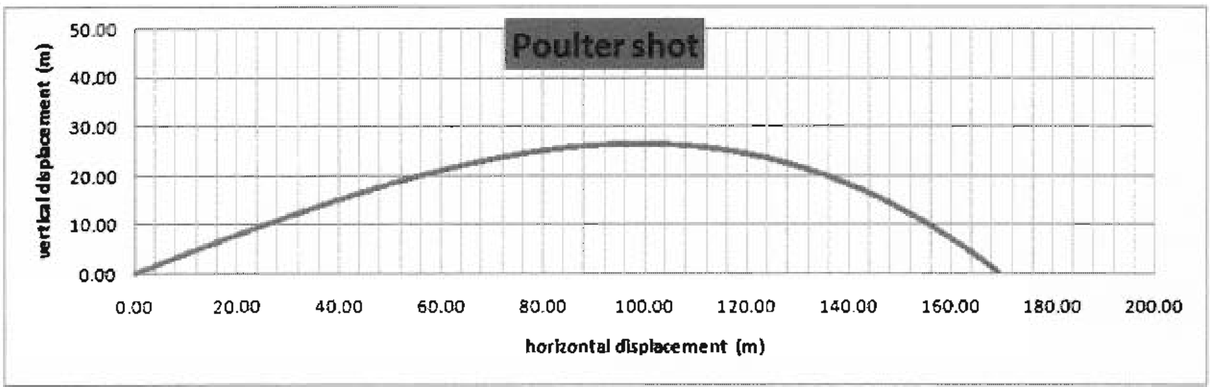
In the spreadsheet, a shot of $83ms^{-1}$ (=187mph) and launch angle of 11.2° with low coefficients of lift and drag produces the following results:



This total distance of 284m=312 yards is within 15 yards of the actual distance shot by Tiger Woods. This would suggest that the model has an uncertainty of 5% in terms of the horizontal displacement. Unfortunately, it is not known whether or not his ball took the trajectory proposed by the model.

An Ian Poulter shot at the 2008 Open Championship at Royal Birkdale will now be compared to the model. The data was obtained from the BBC coverage of the event. The shot was taken at the par 3 twelfth hole.

Poulter hit an 8 iron with an initial ball speed of 123mph (54.6m/s), that reached an apex of 30 yards (27metres) and carried 175 yards (157m). According to KenTannar[9], a typical launch angle with an 8-iron is 22°. The spin rate was 6739rpm, which is considered low according to Ken Tannar's predictions. With that data, the following graph was obtained.
+ 2,1 cm



Unfortunately, there was a lot of wind which would have a big effect not only on the carry, but also with uncertainty in launch angle (the players probably punched the ball under the wind). Indeed both wind and lower launch angle would reduce the theoretical carry. However, the graph compares quite nicely with the data, especially with regard to the apex.

[9] http://www.probablegolfinstruction.com/clubrangecompare.htm

The carry distance with a driver is a good method of finding the accuracy of the model. I therefore went down to the driving range with a 10° driver and doppler ball speed radar device (see figure 4[10]) to test it for myself! The following data was obtained, and compared against the model's theoretical carry distance. As the ball is caught in the upswing and placed on a tee, it must be noted that the average player hits a driver 3° steeper than his driver loft. As driving range balls were being used, generally considered to carry in the order of 20m less than a real golf ball, the lift coefficient was kept low (0.14) and the drag coefficient set high (0.25).

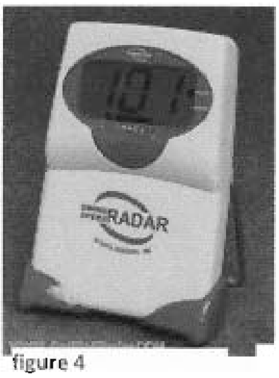


figure 4

Angle (± 2°)	Initial ball speed (±3 ms^{-1})	Experimental carry (±5 m)	Theoretical carry (±10m from angle and velocity uncertainties)
13	64.8	185	186
13	61.6	175	172
13	56.3	160	150

Differences come mainly from factors not taken into consideration for the theoretical model, mainly wind, side spin and the fact that normal air density is assumed, not necessarily the case in reality. Unfortunately, the carry distance is only known with an uncertainty of ±5m. Also, it was very difficult to reproduce the same shot for data confirmation as I (9 handicap amateur) was taking the shots. Although testing the carry does little to prove whether the trajectory modeled is correct, it will at least prove the utility of such a model to a golfer, as the golfer's main concern is where his ball will land, and less how it will get there.

Assumptions

Following is a list of all the **assumptions** made when creating the model. Only situations that match these assumptions can be modelled.

- The ball is travelling with backspin and no side spin, so therefore has no left right movement (i.e. the axis of rotation is perfectly horizontal).
- The drag and lift coefficients are constant. In reality the drag force would cause the spin rate to decrease and therefore so would the coefficients.
- The shot is being taken at standard pressure (1atm) and temperature (25°C). Due to lower air densities at high altitudes, the ball travels significantly further.
- The radius of the golf ball is 2.1cm and the mass of the golf ball is 0.0459g (approved size and mass by the USGA)
- The golf ball is being hit at speeds that obey the coefficients for lift and drag – hopefully it is but as it cannot be proved it remains an uncertainty

[10] Figure 4 : http://probablegolfinstruction.com/SwingSpeed/unit101.jpg

- There is no wind
- The shot is hit on a flat surface, i.e. the landing spot is at the same height as where the ball starts.

Limitations

The main limitation of this model is that it is not of much use if the player does not know what the drag and lift coefficients of his shot are. In an ideal situation, it would be found experimentally for all different types of shots. However this was not possible within the resources and time available. E-mails were sent to three big ball manufacturers (Titleist, Srixon and Callaway) requesting information concerning the lift and drag coefficients (and how these would vary with different velocities and spin rates). While Srixon did not reply, the other two stated that the information was strictly confidential. Therefore, I had to make do with approximating a value based on the normal range.

Conclusion and evaluation

It can be concluded that the uncertainty of the model in the horizontal direction is in the order of ±12% (with the experimental data being closer to 5%, but this may have happened by chance). Due to the number of variables, no uncertainty could be found for the vertical displacement. The uncertainty does not stem from the forces acting on the ball: so answering the research question, it is safe to say that gravity, lift and drag are the three forces acting on the ball that must be considered in order to have an accurate model.

Unfortunately, there are two major weaknesses. Firstly, the drag and lift coefficients are constants in the model, whereas they are variables in reality. Having said that, they vary within a confined range and the trajectory "calculator" takes an average. The second major limitation is that the average value of the coefficients is not known with any degree of certainty for any shot. For the purpose of this essay, a recommended range was used. However, should time be found to explore the exact variation of these coefficients, the current model could easily be adapted and taken to a much higher degree of precision. The bounce is a very significant aspect of golf that unfortunately was not explored in this model. For me, this would be the next concept to investigate in order to increase the value of my model.

APPENDIX

	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1	25	angle in °	25	drag coef	0.3	lift coef	0.2												
2	v	vx	vy	angle	ax	ay	delta x	x	delta y	y									
3	25	22.65769	10.56546	0.436332	-4.10019	-9.18331	0	0	0	0		E4/SIN(F4)							
4	24.92412	22.61669	10.47362	[illegible]	-4.07368	[illegible]	0.226167	[illegible]	[illegible]	0.104277		G3*0.01+D3							
5	24.84868	22.57596	10.38186	0.431026	-4.04736	-9.16939	0.22576	0.45152	0.103819	0.207637		H3*0.01+E3							
6	24.77369	22.53548	10.29017	0.428346	-4.02125	-9.16247	0.225355	0.676674	0.102902	0.31008		ATAN(E4/D4)							
7	24.69914	22.49527	10.19854	0.425648	-3.99533	-9.15558	0.224953	0.901427	0.101985	0.411607		(-0.00084485*I1*C4^2*SIN(F4)/0.0459)-(0.00084485*G1*C4^2*COS(F4)/0.0459)							
8	24.62504	22.45532	10.10699	0.422931	-3.96961	-9.14871	0.224553	1.125782	0.10107	0.512219		(0.00084485*I1*C4^2*COS(F4)/0.0459)-(0.00084485*G1*C4^2*SIN(F4)/0.0459)-9.81							
9	24.55138	22.41562	10.0155	0.420197	-3.94409	-9.14188	0.224156	1.349741	0.100155	0.611917		D4*0.01							
10	24.47817	22.37618	9.92408	0.417445	-3.91876	-9.13507	0.223762	1.573307	0.099241	0.710701		J3+H4+(0.5*G4*(0.01)^2)							
11	24.4054	22.33699	9.832729	0.414674	-3.89362	-9.12828	0.22337	1.796482	0.098327	0.808571		E4*0.01							
12	24.33309	22.29806	9.741446	0.411885	-3.86868	-9.12153	0.222981	2.019269	0.097414	0.905529		(L3+K4)+(0.5*H3*(0.01)^2)							

These codes were simply copied all the way down the column, to get x and y displacements every 0.01s up to 8.5s from launch. As the golf shot is assumed to take place on level ground, all displacements below zero are not included in the graph.

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