H2

A *vector* is a combination of exactly two values: a magnitude (like the speed of an object in motion) and a direction (such as the direction of an object in motion). All kinds of things can be described with vectors, including constant motion, acceleration, displacement, magnetic fields, electric fields, and many more.

Vectors are defined by a *magnitude* (the length of the vector) and a direction. For example, take a look at the vector in Figure 3-1.

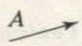


**Figure 3-1:** A vector.

In physics, vectors are written in bold type. The vector in Figure 3-1 — I'll call it **A** — represents the displacement of a golf ball from the tee. Its length is 100 yards, and its direction is 15° north of due east. That's all you need to have a vector — a magnitude and a direction.

Now take a look at the two vectors in Figure 3-2, **A** and **B.** These two vectors are considered equal, which is written as **A** = **B.**


**Figure 3-2:** Two vectors.

REMEMBER

Two vectors are considered equal if they have the same magnitude and direction. They do *not* need to start at the same point. The magnitude of a vector **A** — that is, its length — is written as A, not in bold type.

Figure 3-3 shows the standard coordinate system for vectors. Note the x and y axes, which vectors are measured against.

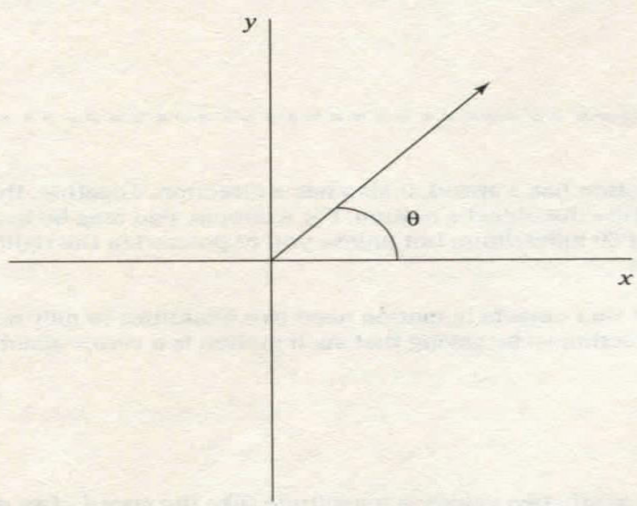


**Figure 3-3:** Vector coordinate system.

The x and y axes are measured using some standard physics units, such as centimeters. Positive x (also called *east*) is to the right, negative x is to the left; positive y (also called *north*) is up, negative y is down. The center of the graph, where the axes meet, is called the *origin*. A vector is commonly described by its length and its angle from the positive x axis (0° to 360°).

**1.** A marble starts at the origin and rolls 45 meters east. Describe where it ends up, in vector notation.

**Solve It**

**2.** A marble starts at the origin and rolls 45 meters east. Then it moves 90 meters west. Describe where it ends up, in vector notation.

**Solve It**

# Understanding Vector Components

In addition to specifying a vector with a magnitude and a direction, you can specify it with a pair of coordinates as measured from the origin. For example, take a look at the vector in Figure 3-4, where the measurements are in centimeters.

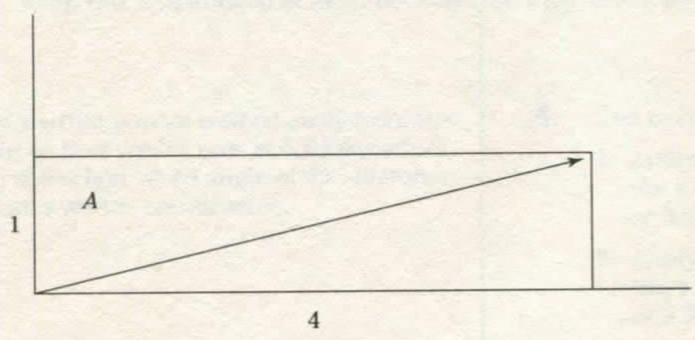


**Figure 3-4:** Resolving a vector.

You can describe the vector in Figure 3-4 with a length and an angle, of course, but you also can describe it by the coordinates of the tip of its arrow. In this case, that tip is at 4 centimeters to the right and 1 centimeter up from the origin. You notate that location as (4, 1), which is a valid way of expressing a vector.

**3.** A marble starts at the origin and moves to the right 5.0 centimeters. What is its new displacement, in vector coordinate terms?

**Solve It**

**4.** Suppose you move to the right of the origin by 3.5 meters and then up 5.6 meters. What is your final vector from the origin, in coordinate terms?

**Solve It**

# Finding a Vector's Components

You can convert from the magnitude/angle way of specifying a vector to the coordinate way of expression. Doing so is essential for the kinds of operations you can expect to execute on vectors, such as when adding vectors together.

For example, you have one vector at 15° and one at 19°, and you want to add them together. How the heck do you do that? If you were to convert them into their coordinates, (a, b) and (c, d), the answer would be trivial because you only have to add the x and y coordinates to get the answer: (a + c, b + d).

To see how to convert between the two ways of looking at vectors, take a look at vector **v** in Figure 3-5. The vector can be described as having a magnitude v at an angle of θ.

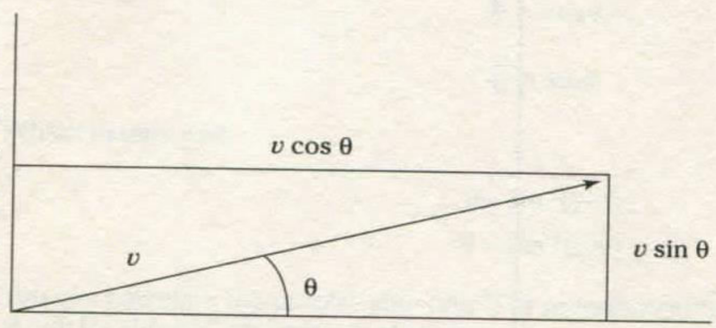


**Figure 3-5:** Finding a vector's components.

To convert this vector into the coordinate way of looking at vectors, you have to use the trigonometry shown in the figure. The x coordinate equals v cos θ, and the y coordinate equals v sin θ:

$$v_x = v \cos \theta$$

$$v_y = v \sin \theta$$

Keep this relationship in mind because you'll come across it often in physics questions.

**5.** Resolve a vector 3.0 meters long at 15° into its components.

**Solve It**

**6.** Resolve a vector 9.0 meters long at 35° into its components.

**Solve It**

**7.** Resolve a vector 6.0 meters long at 125° into its components.

**Solve It**

**8.** Resolve a vector 4.0 meters long at 255° into its components.

**Solve It**

# Finding a Vector's Magnitude and Direction

If you're given the coordinates of a vector, such as (3, 4), you can convert it easily to the magnitude/angle way of expressing vectors using trigonometry.

For example, take a look at the vector in Figure 3-5. Suppose that you're given the coordinates of the end of the vector and want to find its magnitude, v, and angle, θ. Because of your knowledge of trigonometry, you know that

$$x = v \cdot \cos\theta$$

$$y = v \cdot \sin\theta$$

In other words, you know that

$$\frac{x}{v} = \cos\theta$$

$$\frac{y}{v} = \sin\theta$$

Which means that

$$\theta = \sin^{-1}(y/v)$$

$$\theta = \cos^{-1}(x/v)$$

You can calculate the inverse sine ($\sin^{-1}$) or inverse cosine ($\cos^{-1}$) on your calculator. (Look for the $\sin^{-1}$ and $\cos^{-1}$ buttons.)

In Figure 3-5, you're given x and y, the coordinates, but not v, the magnitude. Dividing the expressions for y and x above gives you

$$\frac{y}{x} = \frac{\cancel{v}\sin\theta}{\cancel{v}\cos\theta} = \tan\theta$$

Where tan θ is the tangent of the angle. This means that

$$\theta = \tan^{-1}(y/x)$$

Suppose that the coordinates of the vector are (3, 4). You can find the angle θ as the $\tan^{-1}(4/3) = 53°$. And you can use the Pythagorean theorem to find the *hypotenuse* — the magnitude, v — of the triangle formed by x, y, and v:

$$v = \sqrt{x^2 + y^2}$$

Plug in the numbers for this example to get

$$v = \sqrt{3^2 + 4^2} = 5$$

So if you have a vector given by the coordinates (3, 4), its magnitude is 5, and its angle is 53°.

**Q.** Convert the vector given by the coordinates (1.0, 5.0) into magnitude/angle format.

**A.** The correct answer is magnitude 5.1, angle 78°.

1. Apply the equation $\theta = \tan^{-1}(y/x)$ to fin the angle. Plug in the numbers to get $\tan^{-1}(5.0/1.0) = 78°$.
2. Apply the Pythagorean theorem $v = \sqrt{x^2 + y^2}$ to find the magnitude. Plug in the numbers to get 5.1.

**9.** Convert the vector (5.0, 7.0) into magnitude/angle form.

**Solve It**

**10.** Convert the vector (13.0, 13.0) into magnitude/angle form.

**Solve It**

**11.** Convert the vector (–1.0, 1.0) into magnitude/angle form.

**Solve It**

**12.** Convert the vector (–5.0, –7.0) into magnitude/angle form.

**Solve It**

# Adding Vectors Together

You're frequently asked to add vectors together when solving physics problems. To add two vectors, you place them head to tail and then find the length and magnitude of the result. The order in which you add the two vectors doesn't matter. For example, suppose that you're headed to the big physics convention and have been told that you go 20 miles due north and then 20 miles due east to get there. At what angle is the convention center from your present location, and how far away is it?

You can write these two vectors like this (where east is along the x axis):

$$(0, 20)$$

$$(20, 0)$$

In this case, you need to add these two vectors together, and you can do that just by adding their x and y components separately:

$$\begin{array}{r} (0,20) \\ +(20,0) \\ \hline (20,20) \end{array}$$

Do the math, and your resultant vector is (20, 20). You've just completed a vector addition. But the question asks for the vector in magnitude/angle terms, not coordinate terms. So what is the magnitude of the vector from you to the physics convention? You can see the situation in Figure 3-6, where you have $v_x$ and $v_y$ and want to find v.

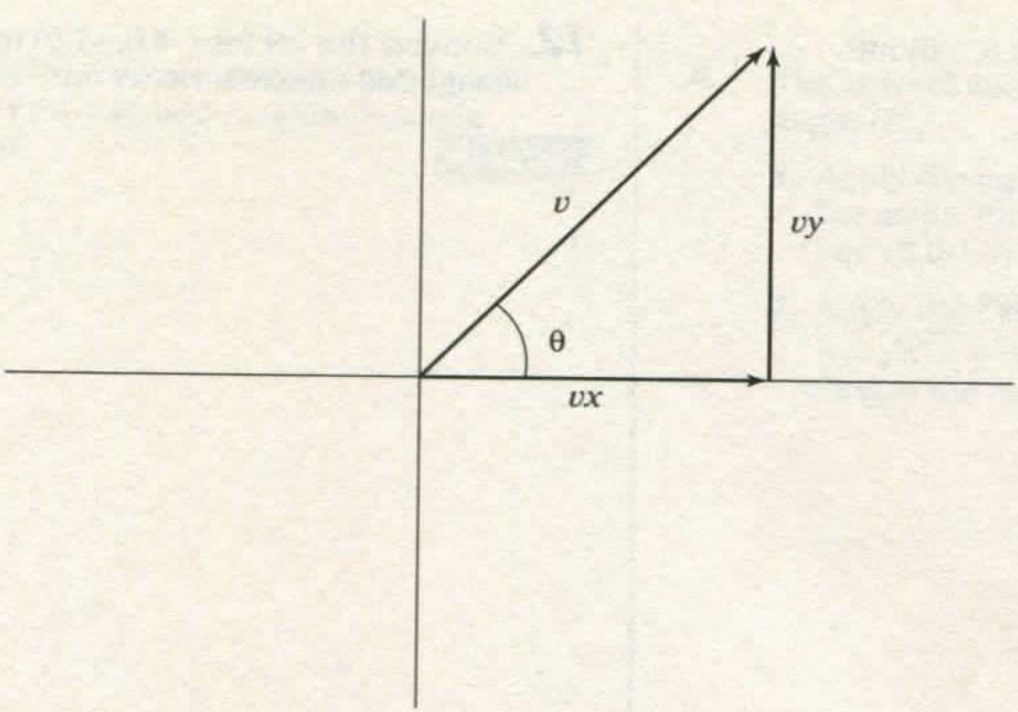


**Figure 3-6:** Adding two vectors.

Finding v isn't so hard because you can use the Pythagorean theorem:

$$R = \sqrt{X^2 + Y^2}$$

Plugging in the numbers to get

$$R = \sqrt{20^2 + 20^2} = 28.3 \text{ miles}$$

So the convention is 28.3 miles away. What about the angle θ? You know that

$$\theta = \sin^{-1}(y/h)$$
$$\theta = \cos^{-1}(x/h)$$

In this case, you can find the angle θ like so:

$$\theta = \sin^{-1}(y/h) = \sin^{-1}(20/28.3) = 45°$$

And that's it — you now know that the convention is 28.3 miles away at an angle of 45°.

**13.** Add a vector whose magnitude is 13.0 and angle is 27° to one whose magnitude is 11.0 and angle is 45°.

**Solve It**

**14.** Add a vector whose magnitude is 16.0 and angle is 56° to one whose magnitude is 10.0 and angle is 25°.

**Solve It**

**15.** Add two vectors: Vector one has a magnitude 22.0 and angle of 19°, and vector two has a magnitude 19.0 and an angle of 48°.

**Solve It**

**16.** Add a vector whose magnitude is 10.0 and angle is 257° to one whose magnitude is 11.0 and angle is 105°.

**Solve It**

Suppose you're in a car traveling east at 88 meters/second when you begin to accelerate north at 5.0 meters/second$^2$ for 10. seconds. What is your final speed?

You may think that you can use this equation to figure out the answer:

$$v_f = v_o + a \cdot t$$

But that's not a vector equation; the quantities here are called *scalars* (the magnitude of a vector is a scalar). This is a scalar equation, and it's not appropriate to use here because the acceleration and the initial speed aren't in the same direction. In fact, speed itself is a scalar, so you have to think in terms not of speed but of velocity.

Velocity is a vector, and as such, it has a magnitude and a direction associated with it. Here's the same equation as a vector equation:

$$\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$$

Note that the speeds are now velocities (speed is the magnitude of a velocity vector) and that everything here is a vector except time (which is always a scalar). This change means that the addition you perform in this equation is vector addition, which is what you want because vectors can handle addition in multiple dimensions, not just in a straight line.

Here are the equations of motion, written as vector equations:

$$\mathbf{v}_f - \mathbf{v}_o = \mathbf{a} \cdot t$$

$$\mathbf{v} = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{\mathbf{x}_f - \mathbf{x}_o}{t_f - t_o}$$

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_f - \mathbf{v}_o}{t_f - t_o}$$

$$\mathbf{s} = \mathbf{v}_o \cdot (t_f - t_o) + \frac{1}{2} \cdot \mathbf{a}(t_f - t_o)^2$$

**Q.** You're in a car traveling east at 88 meters/second; then you accelerate north at 5 meters/second$^2$ for 10 seconds. What is your final speed?

**A.** The correct answer is 91 meters/second.

1. Start with this vector equation:

$$\mathbf{v}_f = \mathbf{v}_o + \mathbf{a} \cdot t$$

2. This equation is simply vector addition, so treat the quantities involved as vectors. That is, $v_o = (88, 0)$ meters/second and $a = (0, 5)$ meters/second$^2$. Here's what the equation looks like when you plug in the numbers:

$$\mathbf{v}_f = (88, 0) + (0, 5)(10)$$

3. Do the math:

$$\mathbf{v}_f = (88, 0) + (0, 5)(10) = (88, 50)$$

4. You're asked to find the final speed, which is the magnitude of the velocity. Plug your numbers into the Pythagorean theorem.

5. You can also find the final direction. Apply the equation $\theta = \tan - 1(y/x)$ to find the angle, which is $\tan^{-1}(.57) = 29.6°$ in this case.

**17.** You're going 40 meters/second east, and then you accelerate 10 meters/second squared north for 10 seconds. What's your final velocity?

**Solve It**

**18.** You're going 44.0 meters/second at 35°, and then you accelerate due west at 4 meters/second$^2$ for 20 seconds. What's your final velocity?

**Solve It**

**19.** A hockey puck is going 100.0 meters/second at 250° when it's hit by a hockey stick, which accelerates it at 1000 meters/second$^2$ at 19° for 0.1 seconds. What's the final velocity?

**Solve It**

**20.** A car is driving along an icy road at 10 meters/second at 0° when it skids, accelerating at 15 meters/second$^2$ at 63° for 1.0 seconds. What's the final velocity?

**Solve It**