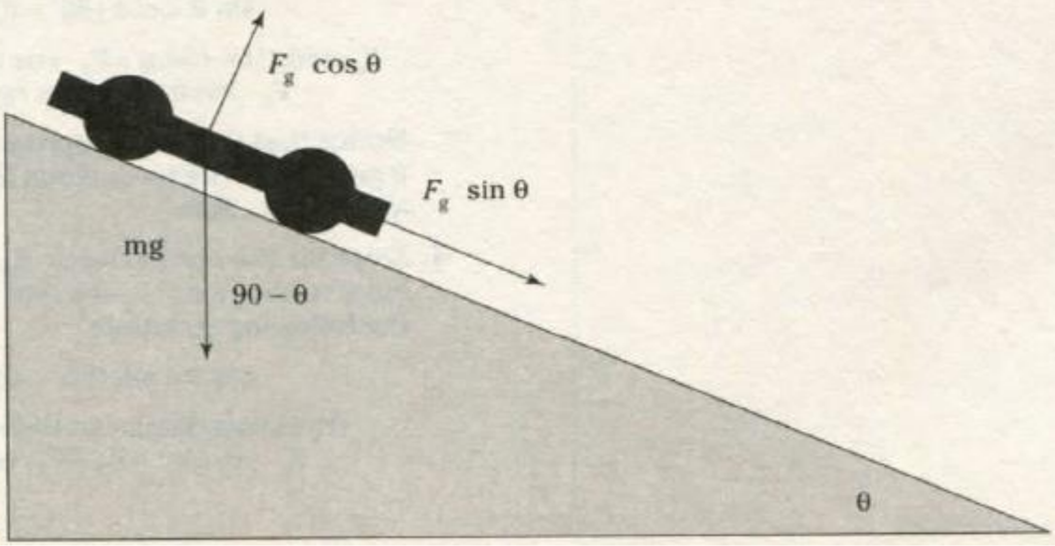

Figure 5-1: A cart on an inclined plane.

The force on the cart is the force due to gravity, $F_g = m \cdot g$. So how fast will the cart accelerate along the ramp? To get the answer, you should resolve the gravitational force — not in the x and y directions, however, but along the ramp's inclined plane and perpendicular to that plane.

The reason you resolve the gravitational force in these directions is because the force along the plane provides the cart's acceleration while the force perpendicular to the ramp, called the *normal* force, does not. (When you start introducing friction into the picture, you'll see that the force of friction is proportional to the normal force — that is, it's proportional to the force with which the object going down the ramp presses against the ramp.)

In Figure 5-1, what are the forces along the ramp and normal to the ramp?

1. Suppose that the cart in Figure 5-1 has a mass of 1.0 kg and the angle $\theta = 30°$. What are the forces on the cart along and normal to the ramp?

Solve It

2. Suppose that the cart in Figure 5-1 has a mass of 3.0 kg and the angle $\theta = 45°$. What are the forces on the cart along and normal to the ramp?

Solve It

3. You have a block of ice with a mass of 10.0 kg on a ramp at an angle of 23°. What are the forces on the ice along and normal to the ramp?

Solve It

4. You have a refrigerator with a mass of 100 kg on a ramp at an angle of 19°. What are the forces on the refrigerator along and normal to the ramp?

Solve It

When you have a block of ice (read: frictionless) moving down a ramp, it's being acted on by forces, which means that it's accelerated. How fast is it being accelerated? When you know that F = ma, you can solve for the acceleration.

After you solve for the force along the ramp, you can get the acceleration (a = F / m) along the ramp. Your block of ice is going to slide down the ramp — and accelerate.

Q. Suppose that you have a block of ice on a ramp at 40°, and it slides down. What is its acceleration?

A. The correct answer is 6.3 m/sec^2.

1. What's important here is the force along the ramp: $F_g \sin\theta = m \cdot g \cdot \sin\theta$.
2. The acceleration of the ice is $F/m = m \cdot g \cdot \sin\theta / m = g \sin\theta$. In other words, the acceleration is the component of g acting along the ramp. Note that this result is independent of mass.
3. Plug in the numbers: $g \sin\theta = 6.3\ m/sec^2$.

5. Suppose that a block of ice is on a ramp with an angle of 60°. What is its acceleration?

Solve It

6. You're unloading a couch on a cart from a moving van. The couch gets away from you on the 27° ramp. Neglecting friction, what is its acceleration?

Solve It

7. You have a block of ice with a mass of 10.0 kg on a ramp with an angle of 23° when it slips away from you. What is its acceleration down the ramp?

Solve It

8. You're sliding down a toboggan run at 35°. What is your acceleration?

Solve It

When objects slide (frictionlessly) down a ramp, they're acted on by a force, which means that they're accelerated and therefore their speed changes. The equation to use in physics problems like these is

$$v_f^2 - v_o^2 = 2 \cdot a \cdot s$$

Finding the object's final speed under these circumstances is easy when you remember that $a = g \cdot \sin\theta$, s is the length of the ramp, and v_o is usually 0.

Q. Say you have a block of ice on a ramp at 20°, and it slides down a ramp of 5.0 meters. What is its final speed at the bottom of the ramp?

A. The correct answer is 5.8 m/sec².

1. The force along the ramp is $F_g \sin\theta = m \cdot g \cdot \sin\theta$.
2. The acceleration of the ice is $F/m = m \cdot g \sin\theta / m = g \sin\theta$.
3. Use the equation $v_f^2 = 2 \cdot a \cdot s = 2 \cdot g \cdot s \cdot \sin\theta$. Plug in the numbers: $v_f^2 = 34$, which means $v_f = 5.8$ m/sec².

9. Starting from rest, you go down a 100 m ski jump of 60°. What is your speed at takeoff?

Solve It

10. You're heading down a toboggan run of 1 km at an angle of 18°. What is your final speed?

Solve It

11. You have a block of ice on a ramp with an angle of 23° when it slips away from you. What is its speed at the bottom of the 6.0 m ramp?

Solve It

12. A cart starts at the top of a 50 m slope at an angle 38°. What is the cart's speed at the bottom?

Solve It

In the real world, when things slide down ramps, friction is involved, and the force of friction opposes the motion down the ramp. The force of friction is proportional to the force driving the two forces together; the stronger that force, the more friction is involved.

When something goes down a ramp, the force driving the two surfaces together is the normal force because it's the force perpendicular to the ramp's surface. The constant of proportionality is something you have to measure yourself; if the ramp is made of steel, there's a different amount of friction than if it's made of sandpaper. In the equation relating the normal force to the force of friction, F_f, the constant μ is called the *coefficient of friction* (a dimensionless number between 0.0 and 1.0):

$$F_f = \mu F_n$$

13. You're pushing a 15 kg box of books across the carpet and need to apply 100 N. What is the coefficient of friction?

Solve It

14. You're pushing a 70 kg easy chair from one room to the next. If you need to apply 200 N, what is the coefficient of friction?

Solve It

The two coefficients of friction correspond to two different physical processes. The first, called the *static coefficient of friction*, applies when you start pushing something at rest to get it moving. When you already have something moving and need to keep applying a force to keep it in motion, that's called the *kinetic coefficient of friction* (see the next section for more).

The static coefficient of friction, μ_s, is usually larger than the kinetic coefficient of friction, μ_k, and both are between 0.0 and 1.0.

15. You're standing at the top of a ski slope and need 15 N of force to get yourself moving. If your mass is 60 kg, what is the static coefficient of friction, μ_s?

Solve It

16. You've started to pull a garbage can out to the curb. If the can has a mass of 20 kg and you need to apply 70 N to get the can moving, what is the static coefficient of friction, μ_s?

Solve It

The second kind of friction is kinetic friction, which is usually less than the force of friction you need to overcome static friction. Kinetic friction occurs when you're pushing or dragging an object that's already in motion.

Static friction is the force needed to get something to move, and it's usually larger than kinetic friction, the force needed to keep it moving. The equation for kinetic friction is $F = \mu_k \cdot F_n$.

17. You're cross-country skiing and need 20 N of force to keep going. If you have a mass of 70 kg, what is the kinetic coefficient of friction, μ_k?

Solve It

18. You're skating and need 17 N to keep going. If you have a mass of 80 kg, what is the kinetic coefficient of friction, μ_k?

Solve It

Static Friction along Ramps

Figure 5-2 shows a box on a ramp. Suppose that the box contains a new widescreen TV that you're pushing up the ramp and into your house.

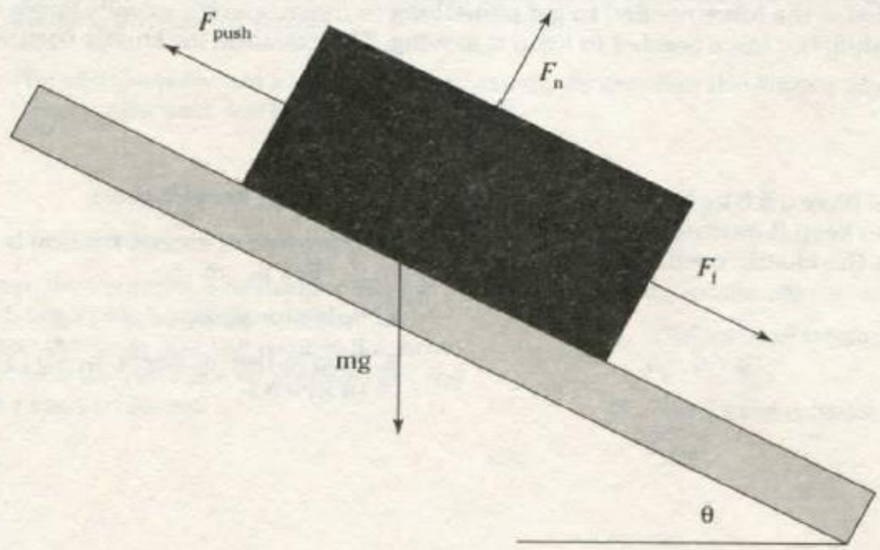

Figure 5-2: An object on a ramp.

A number of forces are acting on the box, in particular both gravity and friction, and you need to take both into account. There's also the force exerted upon the box as you push it up the ramp. So how do you balance all the forces? How much force is needed to get the box moving up the ramp into the house?

In order to figure that out, clearly you need to calculate the forces along the ramp. There's the force due to gravity, which is $m \cdot g \sin \theta$. But what is the force due to static friction along the ramp?

To find that, you use the equation $F = \mu_s F_n$. What is the normal force, F_n? You already know the answer — it's $F_n = mg \cos \theta$. That makes the force due to static friction $\mu_s mg \cos \theta$.

The force of static friction points along the ramp, and you've calculated the force due to gravity along the ramp. Both of these forces point down the ramp and need to be overcome by the force pushing up the ramp. So in other words, F_{push} is

$$F_{push} = F_g + F_s$$

Where F_g is the force due to gravity, and F_s is the force due to static friction. Plugging in these forces gives you:

$$F_{push} = F_g + F_s = m \cdot g \sin \theta + \mu_s \cdot m \cdot g \cos \theta$$

19. You're dragging your little brother up the 25° wheelchair ramp at the doctor's office. If he has a mass of 40 kg and the coefficient of static friction, μ_s, is 0.15, how much force will you need to apply to get him moving?

Solve It

20. Suppose that you're struggling to keep a 20.0 kg block of ice from sliding down a 40.0° ramp. If the coefficient of static friction, μ_s, is a low 0.050, how much force will you need to apply to overcome the weight pulling the block down the ramp and static friction?

Solve It

You will need to be able to handle problems involving objects in static equilibrium on a ramp. Consider if there were no push force on the TV box, yet the box remained in static equilibrium on the ramp. In order to determine μ_s between the ramp surface and the box, you only need to know the maximum angle of incline that does not allow the box to begin to slide. At the maximum angle of incline, the static friction force is equal in magnitude to the parallel weight component down the ramp. Determine the literal equation for μ_s in terms of angle Θ of the ramp incline and any necessary trigonometric identities. Show your work:

Two kinds of friction — static and kinetic — mean that you also have to handle ramp problems where kinetic friction is involved, as when an object on a ramp is sliding *down* that ramp. Because the object is moving, kinetic friction applies. That means you can solve problems with kinetic friction as easily as those that involve static friction.

Here's the equation for the force needed to get an object moving, and thus overcoming static friction:

$$F = F_g + F_k = m \cdot g \sin\theta + \mu_s \cdot m \cdot g \cos\theta$$

To convert this equation to using kinetic friction, all you have to do is to change from using the static coefficient of friction, μ_s, to using the kinetic coefficient of friction, μ_k:

$$F = F_g + F_k = m \cdot g \sin\theta + \mu_k \cdot m \cdot g \cos\theta$$

That's all there is to it.

21. You're dragging your little sister up the 25° wheelchair ramp at the doctor's office. If she has a mass of 30.0 kg and the coefficient of kinetic friction, μ_s, is 0.10, how much force will you need to apply to keep her going?

Solve It

22. You're pushing a box of books with a mass of 25 kg up a 40° ramp. If the coefficient of kinetic friction, μ_s, is 0.27, how much force will you need to apply to keep the box moving up the ramp?

Solve It

Suppose that you have a block of ice that has been mistakenly placed too near the top of a long ramp, and it starts sliding down that ramp. The preceding section helps you calculate how much force acts on that block of ice, so how about calculating its acceleration down the incline?

The object is sliding down the ramp — you're not pushing it — which means that the kinetic force of friction is opposing (not adding to) the component of gravity along the ramp. So the force on the block of ice is

$$F = F_g - F_k = m \cdot g \sin\theta - \mu_k \cdot m \cdot g \cos\theta$$

Because a = F / m, the acceleration of the block is

$$a = g \sin\theta - \mu_k \cdot g \cos\theta$$

25. You're dragging a suitcase up a ramp into a luxury hotel when it gets away from you. If the angle of the ramp is 31° and the kinetic coefficient of friction is 0.1, what is the suitcase's acceleration down the ramp?

Solve It

26. You drop a 5.0 kg box on a ramp of 12°, and the kinetic coefficient of friction is 0.15. Will the box slide down the ramp?

Solve It

27. You drop a 1.0 kg book on a 15° ramp, and the kinetic coefficient of friction is 0.30. Will the book slide down the ramp?

Solve It

28. A refrigerator breaks away from the movers and slides down a 23° ramp that has a coefficient of kinetic friction of 0.25. What is its acceleration?

Solve It