Helmholtz Resonators

An interesting type of acoustic resonance - known as the Helmholtz resonance is present/operative in many musical instruments (*e.g*. the ocarina, the flute, the *f*-hole(s) of violin/viola/cello, the sound hole of an acoustic guitar, human whistling…) as well as *e.g*. in bass reflex and/or ported/vented speaker enclosures. Bass traps are sometimes used in listening room environments to ameliorate or suppress problematic acoustic resonances of the room.

The underlying theory of a Helmholtz resonator is quite simple – it is related to the 1-D mass-spring system, which has a natural resonance frequency $f_o \equiv \frac{1}{2\pi}\sqrt{k_s/M}$ where k_s is the spring constant (*N*/*m*) of the spring and *M* is the mass (*kg*). In a Helmholz resonator – an arbitrarily-shaped container of {inner} volume $V\left(m^3\right)$ with an {assumed circular} opening of cross-sectional area $A = \pi R^2 \left(m^2\right)$, the mass of air *M* contained within the opening/“neck” of the container oscillates back and forth due to the effective spring constant k_s associated with the air contained within the {inner} volume *V* of the whole container - alternatingly being compressed/rarified above/below atmospheric pressure once each cycle of oscillation.

The Helmholtz resonance frequency (derived using first principles in the Appendix) is $f_H = \frac{v_{air}}{2\pi}\sqrt{A/\left(V\cdot\ell_{neck}^{eff}\right)}\ (Hz)$, where $v_{air} \simeq 344\ m/s$ is the longitudinal propagation speed of sound in air (@ NTP), $\ell_{neck}^{eff} = \ell_{neck} + \delta_{end}^{tot} \simeq \ell_{neck} + 1.7R\,(m)$ is the e*__ffective__* length of the “neck” of the Helmholtz resonator (or the wall thickness of the resonator, if it has no neck); ℓ_{neck} is the physical length/thickness of the “neck” of the opening, and $\delta_{end}^{tot} = \delta_{end}^{inner} + \delta_{end}^{outer} \simeq R + 0.7R = 1.7R$ is the so-called {total} end correction† – adding to the physical length of the neck on the inside (outside) of the container on the order of 1.0 (0.7) of a radius R of the mouth/opening of the resonator, and the cross sectional area of the opening $A\left(m^2\right)$ of the Helmholtz resonator.

Since the mass of air oscillating back and forth in the neck of the Helmholtz resonator at frequency f_H is $M = \rho_{air}V_{neck} = \rho_{air}A\cdot\ell_{neck}$, a little algebra gives us the effective spring constant of the Helmholtz resonator, $k_s \simeq \rho_{air}v_{air}^2A^2/V$. The spring constant k_s of the Helmholtz resonator is linearly proportional to the mass density of air, $\rho_{air} = 1.204\,kg/m^3\ (@\ NTP)$, linearly proportional to the square of both the speed of sound in air v_{air} and the cross sectional area of the opening *A* in the resonator, and is also inversely proportional to the {inner} volume *V* of the resonator.

A familiar example of a Helmholtz resonator is a simple bottle – with an opening in it. The Helmholtz resonance of an open bottle can be excited by placing the mouth of the bottle against one’s lips and simply exhaling (or gently blowing) across the mouth of the bottle. The pix below shows various sized and shaped bottles that we have in the POM lab for investigation/measurement of their associated Helmholz resonances.

† For a circular opening… The end corrections are different *e.g*. for a rectangular opening…

Experimental Determination of the Resonance Frequency f_H of a Bottle Helmholtz Resonator:

Various experimental methods can be used to measure the Helmholtz resonance frequency f_H of an arbitrarily-shaped bottle/container. First, it is useful to obtain a rough, *qualitative* idea of f_H simply by exhaling or gently breathing across the mouth/opening of the bottle/container and listening to the pitch – is it high or low? Then, *e.g*. placing the mic of guitar tuner in proximity to the mouth/opening of the bottle/container while the Helmholz resonance of the bottle/container is being excited, the tuner will indicate what note of the piano is closest to it, and how many cents high/low from this piano note the Helmholtz resonance is. Another method that can be used to excite the Helmholtz resonance is to use the palm of your hand to impulsively strike the resonator. If you are able to get a ~ reasonably stable reading off of the guitar tuner, you can then *e.g*. refer to the table of frequencies of the 88 notes of the piano in the UIUC POM lecture notes on musical intervals/musical scales to obtain a better estimate of f_H. A third, more accurate method is to connect up one of the UIUC POM labs pressure microphones to a Tek 3012 'Scope, excite the Helmholz resonator of the bottle using e.g. the one of the Q-Electronics ear buds inserted into the bottle, driven by a ~ 100 mV amplitude sine wave output from an Agilent 33220A FG, with the pressure microphone either in proximity to the neck/opening of the bottle or actually inserted part way into the neck/opening of the bottle. Then, vary the frequency of the FG in small increments above/below your previous estimate(s) of f_H to find the f_H where the sine-wave signal output from the pressure microphone is maximized. Note that you can equivalently use the cursors on the 'Scope to obtain an accurate measurement of the period τ of this sine wave, then take the reciprocal of the period to obtain the frequency of the Helmholtz resonance, *i.e*. $f_H = 1/\tau$, which should be in good agreement with the FG frequency. A fourth method is excite the Q-Electronics ear bud inside the bottle with white noise (*i.e*. all frequencies simultaneous) and use one of the UIUC POM Lab's Dynamic Signal Analyzers to analyze the so-called power spectrum (*aka* auto-correlation) of the white-noise signal output from the pressure microphone. A lab handout on how to setup and use one of the UIUC POM Lab's Dynamic Signal Analyzers (DSA) is posted/available on both the Physics 193/406 POM websites. Don't hesitate to ask one of the POM Lab TA's for help in carrying out such measurements. It's not <u>*that*</u> hard to set up this equipment and use it! We can also read out the spectral contents of the DSA into one of the DAQ PC's via the GPIB interface, so that it can be plotted up, *e.g.* using a Matlab script – don't hesitate to ask one of the POM TA's to show you how to do this!

Once you have obtained measurement(s) of the frequency of the Helmholtz resonance of one of the bottles, it is useful to compare it to the theory prediction: $f_H = \frac{v_{air}}{2\pi}\sqrt{A/\left(V \cdot \ell_{neck}^{eff}\right)}\ (Hz)$. We have already measured $v_{air} = 345.8 \pm 0.1\ m/s$ here in the POM Lab/6105 ESB ($T \simeq$ 20 ^oC, 760 *ft* elevation). You need to measure $A = \pi R^2 \left(m^2\right)$, $V\left(m^3\right)$, $\ell_{neck}\left(m\right)$ and then compute $\ell_{neck}^{eff} = \ell_{neck} + \delta_{end}^{tot} \simeq \ell_{neck} + 1.7R\left(m\right)$ and then finally calculate $f_H = \frac{v_{air}}{2\pi}\sqrt{A/\left(V \cdot \ell_{neck}^{eff}\right)}\ (Hz)$. In practice, it is difficult for arbitrarily-shaped bottles to accurately measure/determine ℓ_{neck}. Even the determination of *A* can be problematic if *e*.g. if the bottle has a long neck and tapers from where the neck joins the body of the bottle to where the opening/mouth of bottle. In this situation, *e.g.* using the simple geometrical mean for *A* would be better. Hence, one should not be disappointed if your experimentally determined f_H disagrees with the theory prediction, even by up to ~ 20-30%, depending on the complexity of the shape of the bottle.

<u>Experimental Determination of the Width of the Resonance of a Bottle Helmholtz Resonator</u>:

Another important physical property of a resonance is the width of the resonance, Δf . A narrow (broad) resonance indicates that relatively little (lots) of dissipation/losses present, respectively. By convention, one is usually interested in determining the Full Width at Half-Maximium (FWHM). If you used the fourth (*i.e.* DSA-based) method described above to determine f_H, the frequencies f_{Hi} and f_{Lo} can be determined from your spectral data where the microphone's signal amplitude^2 is a factor of ½ that at the peak of the resonance. Then $\Delta f \equiv f_{Hi} - f_{Lo} = FWHM$. The so-called *Q*uality-factor (*Q*-factor, for short) is defined as the ratio of the resonance frequency f_o to *FWHM, i.e.* $Q \equiv f_o/\Delta f$. Physically, the *Q*-factor is large for narrow/low-loss resonances and small for wide/lossy resonances. The *Q*-factor is also defined as $Q \equiv \frac{\text{Energy Stored}}{\text{Energy Dissipated per Cycle}} = \frac{\text{Energy Stored}}{\text{Power Loss}/\omega} = \omega\left(\frac{\text{Energy Stored}}{\text{Power Loss}}\right)$. Note that *Q* is a dimensionless quantity.

If you used only the third method for determining f_H (*i.e.* the 'Scope method), the frequencies f_{Hi} and f_{Lo} can be determined from the microphone's signal amplitude on the 'Scope when it is reduced by a factor of $1/\sqrt{2} \simeq 0.707$ of that at the peak of the resonance.

The *Q*-factor for a Helmholz resonator is theoretically predicted to be $Q_H^{Thy} = 2\pi\sqrt{V\left(\ell_{neck}^{eff}/A\right)^3}$. Hence, additionally compare your experimental determination of the *Q*-factor, $Q_H^{Expt} \equiv f_H/\Delta f_H$ with its theory prediction. How well do they agree with each other?

Appendix: Derivation of the Resonance Frequency of a Helmholtz Resonator