Experiment # 2
Simple Harmonic Oscillation part II: Helmholtz Resonators and the Bottle Band
PHYS/MUS 102—Spring 2016

Background

In the previous lab, we studied the general principle of simple harmonic motion and saw its connection to sound production in musical instruments. Now let's check out another class of musical instrument that relies on a clever (and not so obvious) system akin to mass-on-a-spring oscillations—the ***Helmholtz Resonator***. You are no doubt familiar with Helmholtz Resonators in the form a glass bottle. Blowing over the edge, causes a steady tone to be produced. The basic principle of operation of sound generation is brilliantly explained in the first 2 minutes of following video By Rimstar on youtube. Please watch the first 2 min of this video before continuing on with this lab: `https://www.youtube.com/watch?v=PZVeJ2rh6ts`.

Figure 1: Example of pre-Colombian "whistle jar" that operates on the principle of a Helmholtz resonator. Image credit: `http://www.whyyouhearwhatyouhear.com/subpages/chapter13.html`

Historically, many pre-Colombian instruments of current day Central and South America have been constructed that work according to the same principle (Figure 1). These instruments had significant social importance in times of celebration, feast, famine, war, and mourning. For example, please see the wonderfully informative 1.5 min clip about Mayan instruments here: `https://www.youtube.com/watch?v=hXv7SBy7y7M`. There is also a very nice demo of other pre-Colombian instruments here: `https://www.youtube.com/watch?v=zMKQWGTKuEA`. Please see a few minutes of the video for historical and musical context before venturing on with this lab.

Theory

The relatively simple physics model explaining Helmholtz Resonance is summarized on the University of New South Wales Musical Acoustics website: `http://newt.phys.unsw.edu.au/jw/Helmholtz.html`. Be sure to check out the physclip animation: If a picture is worth a 1000 words, a good animation is worth a million. In brief, blowing air across the opening causes a small plug of to oscillate on the large volume of air in the main chamber (see Figure 2).

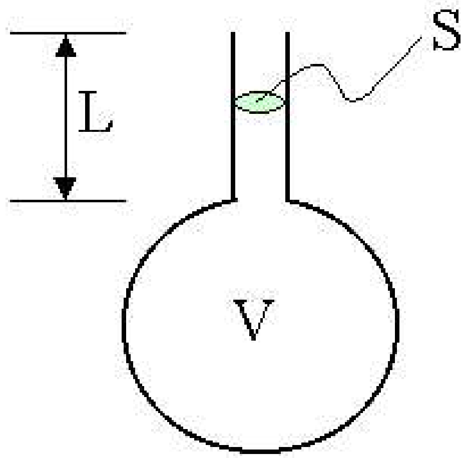

Figure 2: Simplified physics model of Helmholtz Resonator. The plug of air in the neck with length L and cross-sectional area S pushes against the volume of air in the main chamber V . Image credit: `www.diracdelta.co.uk/science/source/h/e/helmholtz%20resonator/source.html`

This is similar to a buoy bobbing in the lake or ocean. The resonance frequency f_o of the cork depends on its mass and the springiness of water. So, too, does the resonant frequency—aka the pitch—of the Helmholtz resonator. Specifically, the resonance frequency is *approximated* by the following equation:

$$f_o = \frac{v_{sound}}{2\pi}\sqrt{\frac{S}{VL}} \tag{1}$$

where v_{sound} is the velocity of sound in air (about 343 m/s), S and L are the the cross-sectional area and the length of the bottle neck, and V is the volume in the main chamber. This equation says that larger volumes and longer necks produce lower pitches, while larger cross-sectional areas lead to higher pitches. You likely already know that changing the level of fluid in the bottle changes the pitch produced. Now you can see why! Changing the level of liquid changes V and possibly also L.

Carefully note that this formula is *approximate*. Why? Well, just look at the geometry of the bottles in front of you relative to the simple physics model in Fig 2. They are very likely to be different in several aspects—observe carefully which ones. **One of the big themes of today's experiment is to critically analyze the accuracy of the assumptions of a simple theoretical model.** One short-coming of this model you should think about straight out of the gate is this: The plug of air bobbing in the neck eventually bobs outside of the bottle. In doing so, it radiates energy outside of the bottle, pushing a bunch of air around it, effectively lengthening the neck of the bottle. A more sophisticated model takes into account this *radiation mass elongation*. The L in Eqn 1 can be better approximated by:

$$L_e \approx L + 1.5r \tag{2}$$

where L_e is the *effective length* , and r is the radius at the opening of the bottle neck.

All righty, with all this info in mind, it is time to make a bottle band!

Experiment

Figure 3: Coke bottles. Time to tune up your resonators!

Obtain about 6 bottles. The bottles can (should) be of various shapes and sizes. Tune each one to a desired note so that you can play a desired song. For instance, you can use a 12 oz coke bottle, a wine bottle, etc. For each bottle:

1. Tune each to a desired pitch. State what desired pitch/musical note you are trying to achieve. You will likely need to add water to the bottle to achieve this.
2. Carefully observe and document the bottle geometry. A nice hand-sketch will do; perhaps a photo is even better. Make measurements to obtain the geometric parameters L, S, and V. This may involve some approximation—aka your best judgment call. Be sure to describe in words how your rationale for any approximations made. Carefully annotate your pictures to indicate where you defined the neck and main chamber volume to be.
3. Using Eqn 1, compute the theoretical resonance frequency f_o. Note that it likely makes good sense to use the effective length (see Eqn 2) plugged into Eqn 1.
4. Record the tone actually generated by your instrument using a microphone plus Audacity software. Determine the experimental (actual) resonance frequency. Quantify how similar or different they were by computing the % difference.