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Pressure vs. depth/Blood pressure

Air pressure: We will take the pressure of the atmosphere at sea level to be $1 \text{ atm} = 100,000 \text{ N/m}^2$. This is about 15 psi.

What is the source of atmospheric pressure?

Tire pressure. A tire gauge reads a pressure of 35 psi in a tire. What is the real air pressure in the tire?

A tire gauge reads a pressure of 0 psi in a flat tire. What is the real air pressure in the flat tire?

Underwater pressure vs. depth. Divers know that pressure increases rapidly as one goes deeper below the surface of the water. The change in pressure is given by $P_2 - P_1 = \rho g(y_2 - y_1)$.

Density of water: many people know that the density of water is 1 gram/cm^3 . Show that this is the same as 1000 kg/m^3 .

1. The pressure at the surface of the water is $100,000 \text{ N/m}^2$ or 1 atm. What is the pressure at a depth of 10 m? Give answer in N/m^2 and in atm.

2. What is the pressure at a depth of 20 m? Give answer in N/m^2 and in atm.

3a) Without using a calculator, what is the pressure at a depth of 40 m?

b) At a depth of 40 m, what is the external force of the water on a submarine window that measures $25 \text{ cm} \times 25 \text{ cm}$?

What is the net force on the window (assuming a pressure of 1 atm inside the submarine)?

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Blood pressure: Blood pressure (like tire pressure) is relative to the external atmospheric pressure.

a) If a person's blood pressure at the heart is $1.3 \times 10^4 \text{ N/m}^2$, what is the true pressure of the blood?

b) Convert this pressure to mmHg. ($1 \text{ atm} = 1 \times 10^5 \text{ N/m}^2 = 760 \text{ mmHg}$)

c) Why is blood pressure measured on the upper arm?

1. The density of blood is 1050 kg/m^3 .

a) What is the relative pressure of the blood (relative to the atmosphere) at the brain, 35 cm above the heart?

b) Convert this pressure to mmHg.

2. What is the relative pressure of the blood at the feet, 1.5 m below the heart? Convert this pressure to mmHg.

PHY 111 Density measurements

Find the density of the following materials by measuring a given mass and volume.

1. Water. mass = volume =

$\rho_{\text{water}} = \text{_____} = \text{g/cm}^3 = \text{kg/m}^3$

2. Brass mass = volume =

$\rho_{\text{brass}} = \text{_____} = \text{g/cm}^3 = \text{kg/m}^3$

3. air (in our classroom)

$\rho_{\text{air}} = 1.22 \text{ kg/m}^3$

V of room: V =

mass of air in room: m =

CHAPTER 13

Fluid Statics

Any form of matter that flows, such as a liquid or a gas, is called a **fluid**. In this chapter we use the ideas of statics to describe static fluids—fluids at rest. In the next chapter we consider fluid dynamics—fluids in motion.

Fluids are of paramount importance to living organisms. The human body is two-thirds water, and liquids in the body not only transport nutrients to its 10^{14} living cells but also carry waste products from those cells. The oxygen we need for combustion of foods flows in and out of the respiratory system along with other molecules in the air.

Fluids are also responsible for most of the climatic conditions on the earth. Air and ocean currents moderate the climates of different parts of the globe, while convection currents in the atmosphere carry pollutants away from large cities. Even the drift of the continents on the earth's mantle can be considered a $\frac{1}{2}$ m of flow.

Because fluids are not rigid, they can flow and change shape when exposed to external forces. If you walk into a wall, its rigid structure opposes your motion. If you walk into air, it simply flows past your body. Because fluids often lack definite size and shape, it is convenient to use the quantities pressure and density rather than force and mass when studying fluid statics and dynamics. We will use these quantities with Newton's laws to develop some of the important principles concerning static fluids.

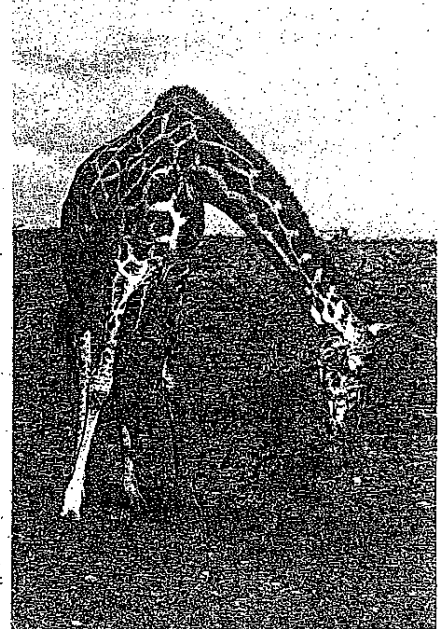
13.1 Pressure

When we study fluids, the quantity pressure assumes a role of primary importance. What force is to solids in the study of statics and dynamics, pressure is to fluids.

Pressure P was defined in Chapter 12 as the ratio of the magnitude of a force applied perpendicular to a surface and the area over which the force is exerted:

$$P = \frac{F}{A} \quad (12.1)$$

Pressure is a scalar quantity and has no direction. However, the force caused by the pressure of a fluid has a magnitude and a direction. The magnitude of the



The pressure in a fluid, such as blood, depends upon elevation. If a giraffe did not have special valves in its circulatory system, the pressure increase as it lowered its head could cause brain damage, and the pressure decrease as it raised its head could cause it to faint. (Tom Fix/Peter Arnold)

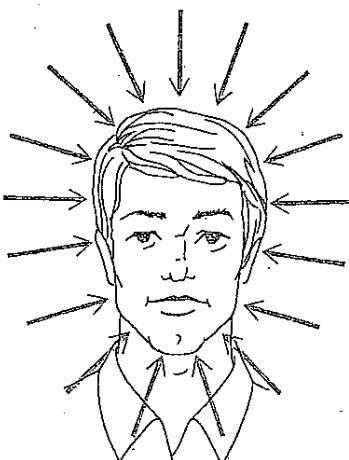


FIG. 13.1. The pressure of the earth's atmosphere causes a force on the skin that is always perpendicular to the skin.

force on a surface of area A is

$$F = PA.$$

The direction of the force caused by the pressure is perpendicular to and toward the surface. For example, the molecules of air colliding with a person's head cause a pressure of about 10 N/cm^2 , so the magnitude of the force on a 1-cm^2 area is 10 N . The direction of the force depends on the orientation of the skin, as depicted in Fig. 13.1. The air hitting a 1-cm^2 area at the top of the man's head causes a 10-N force directed downward, whereas a 1-cm^2 area under his chin experiences a 10-N force pointing up. A horizontal force acts on the sides of the face.

A static fluid at pressure P exerts a force F perpendicular to and toward a surface. The magnitude of the force against a surface of area A is

$$F = PA. \tag{12.1}$$

EXAMPLE 13.1 Estimate the force of the earth's atmosphere on the skin under your chin when your head is held upright.

SOLUTION The area of the skin under the chin is about $5 \text{ cm} \times 8 \text{ cm} = 40 \text{ cm}^2 = 40 \times 10^{-4} \text{ m}^2$. The pressure of the earth's atmosphere at sea level is $1.0 \times 10^5 \text{ N/m}^2$. Thus, the magnitude of the air's force on the skin is

$$F = PA \cong (1.0 \times 10^5 \text{ N/m}^2)(40 \times 10^{-4} \text{ m}^2) = \underline{400 \text{ N}}$$

(about 100 lb)! The direction of the force is perpendicular to and toward the surface. For this example the force points up (approximately). \square

13.3 Pressure Variation with Depth

The pressure exerted by a fluid varies with depth; the deeper an object is in a fluid, the greater the pressure acting on the object. You may have felt the increase in water pressure on your ears while swimming under water or the decrease in air pressure when driving or hiking from lower to higher elevations in the mountains, when taking off in an airplane, or while riding up in a skyscraper's elevator. When our ears "pop," air is released from the middle ear. This release of air causes a reduction in air pressure inside the middle ear that compensates at least partially for the reduced pressure outside the ear when at a higher elevation.

The pressure at a particular depth in a fluid is due to the weight of the fluid above. For instance, the blood pressure in the veins of a person's feet exceeds the pressure in the head because the blood in the feet supports the weight of the blood above it. This increased pressure may cause one's feet to swell, especially if one must stand while working.

The variation of pressure with depth is somewhat like stacking books on a table (Fig. 13.2). Imagine that each book is a layer of air. The only pressure on the top book is that due to the air pushing down from above. However, the second book from the top must support the weight of the top book plus the atmospheric air above it. The bottom book in the stack must support the weight of the five books above it plus the atmospheric air, and so forth.

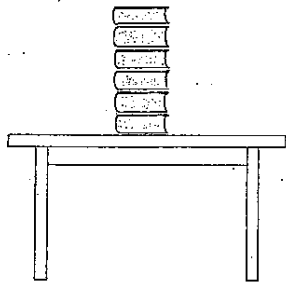


FIG. 13.2. The pressure is greatest on books deepest in the stack because they must support the weight of all the books above them.

We can derive an equation that allows us to calculate the difference in pressure at different depths in a fluid. Consider the shaded portion of the fluid shown in Fig. 13.3. The bottom surface is at vertical elevation y_1 and the top surface is at elevation y_2 . The difference in pressure between y_1 and y_2 should depend on the extra weight of fluid that must be supported at elevation y_1 compared to that at elevation y_2 .

The vertical forces acting on the shaded volume are shown in Fig. 13.4. The fluid above the shaded volume pushes down with a force F_2 . If the pressure at elevation y_2 is P_2 and the cross-sectional area of the cylinder is A , then the magnitude of the force from above is

$$F_2 = P_2 A.$$

Similarly, fluid from below the shaded section of fluid pushes up with a force F_1 equal in magnitude to

$$F_1 = P_1 A.$$

The third force acting on the shaded volume is its weight w , a downward-directed force.

Since the fluid is not moving and therefore is in equilibrium, these three vertically directed forces must add to zero. With the y axis pointing upward, this equilibrium condition is written as

$$\Sigma F_y = F_1 - F_2 - w = 0,$$

or

$$P_1 A - P_2 A - w = 0. \quad (13.2)$$

The weight of the shaded section of fluid depends on its density and volume. The volume is

$$V = A(y_2 - y_1),$$

where $(y_2 - y_1)$ is the height of the shaded section of fluid. The weight of this part of the fluid (assuming constant density) is

$$w = mg = (\rho V)g = \rho A(y_2 - y_1)g.$$

Substituting for w in Eq. (13.2), we find that

$$P_1 A - P_2 A - \rho g A(y_2 - y_1) = 0.$$

After dividing the equation by A , we have the desired result:

$$P_1 - P_2 = \rho g(y_2 - y_1)$$

or

$$P_1 = P_2 + \rho g(y_2 - y_1). \quad (13.3)$$

This equation allows us to calculate the pressure in a fluid at one elevation in terms of the pressure at a second elevation. The equation has been derived with the assumption that the y axis points upward and that the density of the fluid does not change with elevation.

EXAMPLE 13.4 Calculate the pressure on a skin diver who is 10 m below the surface of the water. The density of seawater is 1025 kg/m^3 , and the air pressure at the water's surface is $1.01 \times 10^5 \text{ N/m}^2$.

We can use $P = 1 \times 10^5 \text{ or } 100,000 \text{ N/m}^2$
or 100 kPa .

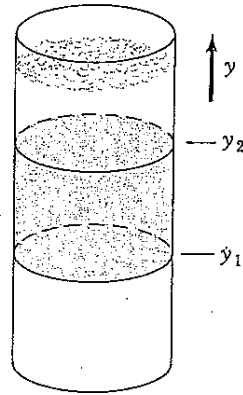


FIG. 13.3. A cylinder of fluid.

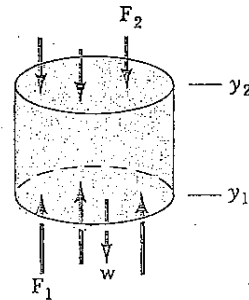


FIG. 13.4. The forces acting on the shaded section of fluid shown in Fig. 13.3.

EXAMPLE 13.5 Estimate the net force on your eardrum due to the air inside and out after you drive from Denver at an elevation of 1609 m to the top of Pikes Peak at 4301 m. Assume that the area of the eardrum is 0.20 cm^2 , that the pressure of the outside air and the air in the middle ear are balanced at Denver, and that no air leaves the middle ear during the trip. The average density of the outside air is about 0.80 kg/m^3 .

13.4 Externally Applied Pressure

We have seen that fluid pressure varies with depth. Another important property of fluids is the uniform increase in pressure throughout an enclosed volume of fluid that is caused by increased pressure at one location in the fluid. This consequence is predicted by Eq. (13.3):

$$P_1 = P_2 + \rho g(y_2 - y_1).$$

8

PHY 111 Pressure reduction: siphon

1. You would like to siphon water upward through 60 cm (2 feet) of hose. By what amount do you need to reduce the pressure in the hose?

2. You would like to siphon gasoline upward through 60 cm of hose. By what amount do you need to reduce the pressure in the hose? ($\rho_{\text{gasoline}} = .72\rho_{\text{water}}$)

If the pressure P_2 at elevation y_2 increases, there must be an equal increase in pressure P_1 at elevation y_1 . This property was first recognized in the seventeenth century by the French mathematician and philosopher Blaise Pascal (1623–1662). The principle, named in his honor, is stated as follows:

Pascal's principle: An external pressure exerted on a static, enclosed fluid is transmitted uniformly throughout the volume of the fluid.

If the pressure at the pumping station of a city water system is increased by 10 pounds per square inch (psi), then the pressure at all the homes connected by water lines to the pumping station also increases by 10 psi. The pressure varies in different homes, depending on their elevation relative to the pumping station, but an increase in pressure at the station causes an equal increase throughout the water system.

Pascal's principle has important consequences in biology and medicine. For example, glaucoma, an eye disease, involves Pascal's principle. A clear fluid called aqueous humor fills two chambers in the front of the eye (Fig. 13.5). In the normal eye, new fluid is continually secreted into these chambers while old fluid drains from the eye through small sinus canals. When a person has glaucoma, these drainage canals close, fluid accumulates in the front of the eye, and the pressure inside the eye increases. This pressure increase is transmitted to the retina at the back of the eye and causes degenerative changes in the retina that can eventually lead to blindness.

A practical application of Pascal's principle is the hydraulic press, a form of simple machine that converts small forces into larger forces, or vice versa. Hydraulic presses are used by automobile mechanics to lift cars and by dentists and barbers to raise and lower the chairs on which their clients sit. The hydraulic jacks of an automobile are also a form of hydraulic press. Most of these devices work on the simple principle illustrated in Fig. 13.6, although the actual devices are usually more complicated in construction.

A force F_1 is applied to a piston that has a cross-sectional area A_1 . The pressure in the fluid just under the piston is

$$P = \frac{F_1}{A_1}$$

Because the pressure is transmitted uniformly throughout the fluid, the pressure of the fluid under piston 2 is also $P = F_1/A_1$.^{*} But since the area of piston 2 is greater than that of piston 1, we find that the upward force of the fluid on piston 2 is greater than the force causing the pressure at piston 1. The upward force of the fluid on piston 2 is

$$F_2 = PA_2 = \left(\frac{F_1}{A_1}\right)A_2 = \left(\frac{A_2}{A_1}\right)F_1 \quad (13.4)$$

Since A_2 is greater than A_1 , F_2 is also greater than F_1 .

EXAMPLE 13.6 How large a force is needed on a small piston of area 2 cm^2 to support a 1000-N weight resting on a piston of area 20 cm^2 ?

^{*}We are assuming that piston 2 is at the same elevation as piston 1. If not, Eq. (13.3) will tell us the pressure differences under the two pistons because of their differences in elevation.

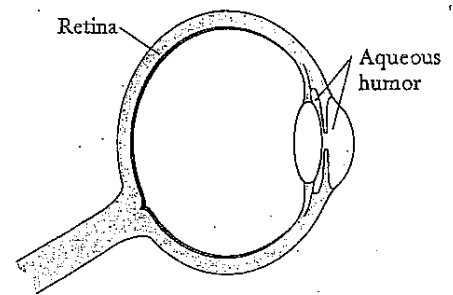


FIG. 13.5. When extra fluid cannot drain out of the eye's cornea, pressure increases throughout the eye, possibly causing blindness.

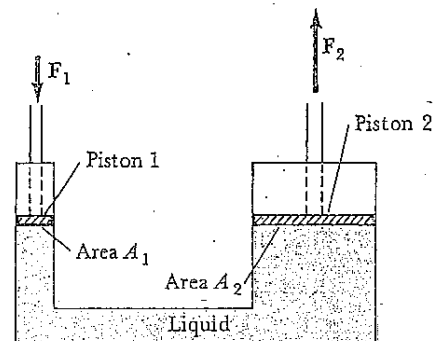


FIG. 13.6. A hydraulic press. A force on piston 1 causes an increase in pressure that is transmitted through the fluid. This increased pressure causes a large force on piston 2 because of its larger area.

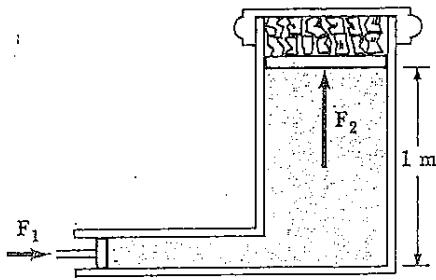


FIG. 13.7. A hydraulic can crusher.

SOLUTION We are given that $A_1 = 2 \text{ cm}^2$, $A_2 = 20 \text{ cm}^2$, $F_2 = 1000 \text{ N}$. We are asked to find F_1 . After rearranging Eq. (13.4), we find that

$$F_1 = \left(\frac{A_1}{A_2} \right) F_2 = \left(\frac{2 \text{ cm}^2}{20 \text{ cm}^2} \right) 1000 \text{ N} = \underline{100 \text{ N}}.$$

A 100-N force can support a 1000-N weight—like balancing a heavy weight on the short arm of a lever by a small force applied to the long arm. ■

EXAMPLE 13.7 A hydraulic can crusher is shown in Fig. 13.7. The large piston has an area of 8 m^2 and exerts a force F_2 of magnitude $2 \times 10^6 \text{ N}$ on the cans. Calculate the magnitude of the force F_1 exerted by the small piston (area 10 cm^2) on the fluid. **a)** Ignore the fact that the large piston is 1 m higher than the small piston.

b) Do not ignore the fact...

14.3 Bernoulli's Equation

In 1738, Daniel Bernoulli formulated an equation that could be used to determine the variation of pressure in fluids as a function of the speed of the fluid and the elevation of the vessel through which the fluid passed. This equation can be used to explain why airplane wings have lift, how to avoid having the roof of your house blown off during a tornado or hurricane, why plaque in arteries becomes dislodged (possibly causing a stroke), and many other phenomena involving moving fluids.

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Problems

13.1 Pressure

1. You hold your physics book in the air so that its back cover faces the ground. Approximately how large a force in newtons and in pounds does the atmosphere exert on the cover? Why does this force not support the book's weight?
2. Estimate the downward force of the earth's atmosphere on the state where you attend college.
3. Suppose that all the air inside your bedroom were removed (you would have to be in a pressurized space suit). *Estimate* the force in units of newtons, pounds, and tons with which you would have to push on the door to get out. * See note.
4. The atmosphere at a pressure of 1.0 atm exerts a downward force of $0.80 \times 10^5 \text{ N}$ on a circular table. Calculate the radius of the table.
5. Calculate the average pressure of the needle of a turntable on a record if the needle's radius is $10 \mu\text{m}$ and it supports a mass of 1.5 g. Compare this to the pressure of your shoes on the ground when standing.
6. The tires of a 980-kg car have absolute pressure of $3.0 \times 10^5 \text{ N/m}^2$. Determine the average area of contact of each tire with the road.

13.2 Density

7. A single-level home has a floor area of 200 m^2 with ceilings that are 2.6 m high. Determine the mass of the air in the house when at 20°C .
8. About two-thirds of your body mass consists of water. Calculate the volume of water in a 70-kg person.
9. Calculate the average mass density of the earth. The earth's mass is $5.98 \times 10^{24} \text{ kg}$, and its radius is $6.38 \times 10^6 \text{ m}$.
10. A 75.0-kg person's density is changed from 970 kg/m^3 to 990 kg/m^3 . Calculate the person's percent change in volume.
11. If the atmosphere had a uniform density of 1.3 kg/m^3 from the earth's surface up to the "top" of the atmosphere, how high would the atmosphere be? The pressure at the earth's surface is $1.0 \times 10^5 \text{ N/m}^2$. Explain why there is air even at an elevation of $2.5 \times 10^4 \text{ m}$ above the earth's surface.
12. A person's mass while dieting decreases by 5 percent. Exercise creates muscle and reduces fat, causing the person's density to increase by 2 percent. Calculate the percent change in the person's volume.

13.3 Pressure Variation with Depth

13. The pressure at the top of the water in a city's gravity-fed water reservoir is $1.0 \times 10^5 \text{ N/m}^2$. Calculate the pressure at the faucet of a home 42 m below the reservoir.
14. *Estimate* the gauge pressure of blood in your brain and in your feet when standing, relative to the average $1.3 \times 10^4 \text{ N/m}^2$ gauge pressure (100 mm Hg) in your heart.
15. A glucose solution of density 1050 kg/m^3 is transferred from a bottle exposed to the atmosphere through a tube and syringe into the vein of a person's arm. The pressure in the arm exceeds atmospheric pressure by 1400 N/m^2 . How high above the arm must the top of the liquid in the bottle be so that the pressure in the glucose solution at the needle exceeds the pressure of the blood in the arm? Ignore the pressure drop across the needle and tubing due to viscous forces.
16. Determine the change in air pressure as you climb from an elevation of 1650 m at the timberline of Mount Rainier to its 4392-m summit, assuming an average air density of 0.82 kg/m^3 .
17. A person climbing a vertical distance of 2000 m up a mountain carries a half-filled jar of water. At the start of the hike the pressure inside and outside the jar is $1.0 \times 10^5 \text{ N/m}^2$. Calculate the force on the lid (magnitude and direction) of area 75 cm^2 at the end of the hike. Assume that the air density is constant at 1.0 kg/m^3 .
18. The window of a deep-sea diving vessel has an area of 0.36 m^2 . The window can withstand a net force of $1.4 \times 10^6 \text{ N}$ before breaking. Calculate the maximum depth in the ocean that the vessel can go. The air pressure inside is the same as the air pressure at the ocean's surface.
19. Your car slides off an embankment into a pond. *Estimate* the force needed to open the door if the top of the door is 0.50 m below the water's surface. How might you escape without opening the door?
20. You can develop a gauge pressure of -30 mm Hg in your lungs while sucking on a straw. Up what length straw can you suck a fruit drink with a density of 1200 kg/m^3 ?
21. A device known as Hare's apparatus (Fig. 13.19) is used to measure the relative density of a fluid. Suction in a common, center tube causes water to rise in the tube on the right and a different type of fluid of unknown density to rise in the tube on the left. When the valve in the center tube is closed, the water has risen to a level of 10.5 cm and the fluid on the left has risen to a

3. * Door is $2 \text{ m} \times 1 \text{ m}$.

$1000 \text{ kg} = 1 \text{ ton} = 2000 \text{ lbs}$.

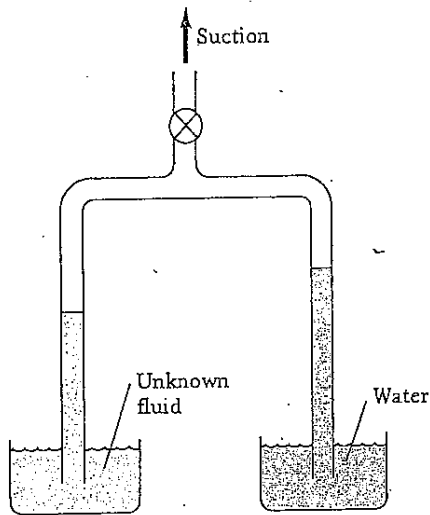


FIG. 13.19. Hare's apparatus for determining relative density in liquids.

level of 9.2 cm. Calculate the density of the fluid on the left. The density of water is 1000 kg/m^3 .

22. Olive oil is poured into a U-tube containing mercury with the result shown in Fig. 13.20. Both sides of the tube are open to the atmosphere. Calculate the density of the olive oil.

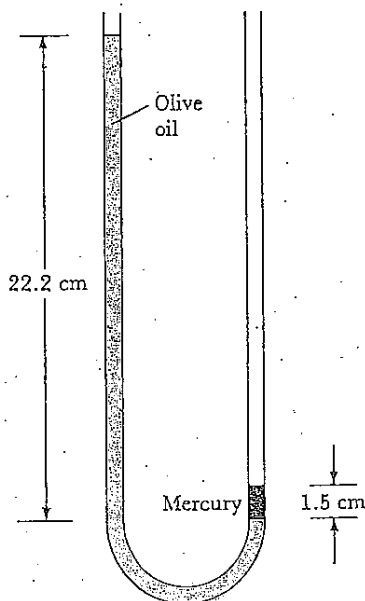


FIG. 13.20

13.4 Externally Applied Pressure

23. A hydraulic lift is used to raise a 900-kg car. Compressed air with a pressure of $4.0 \times 10^5 \text{ N/m}^2$ is applied to a small cylinder. Calculate the area of a large piston supporting the car's weight.

24. A 10-kg mass is placed on top of a piston of a hydraulic lift; the piston's area is 3.0 cm^2 . A 100-kg woman wrestler sits on a

piston of 60-cm^2 area. Will she cause the 10-kg mass to rise? Explain.

25. A person is poked in the eye with a finger that exerts 60-N force over an area of 0.3 cm^2 at the front of the eye. The increased pressure in the eye is transmitted to the back of the eye. (a) Calculate the force on the 0.05-cm^2 area from which the optic nerve leaves the eye. (b) Calculate the force at the back of the eye on a single rod whose diameter is about 10^{-6} m .

26. As shown in Fig. 13.21, a 100-N force pulling up on a lever causes an increase in pressure under a small piston of 8.0-cm^2 area. (a) Calculate the pressure created under the small piston. (b) Calculate the upward force F on the large, 160-cm^2 piston of the hydraulic press.

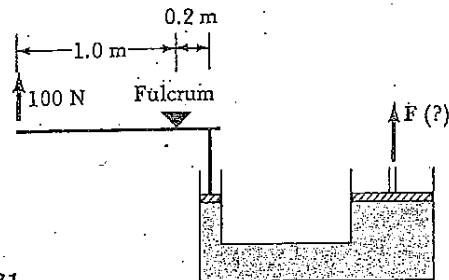


FIG. 13.21

27. The lever shown in Fig. 13.21 is applied to piston 1 of the hydraulic can crusher in Fig. 13.7. With what force must the end of the lever's handle be pulled to exert a $(2.4 \times 10^6)\text{-N}$ force F_2 on the cans? $A_1 = 24 \text{ cm}^2$ and $A_2 = 2400 \text{ cm}^2$.

Chapter 13

1. 5400 N, 1200 lb
3. $4 \times 10^4 \text{ lb}$, 20 tons, $2 \times 10^5 \text{ N}$
5. $4.7 \times 10^7 \text{ N/m}^2$
7. 630 kg
9. 5500 kg/m^3
11. $7.8 \times 10^3 \text{ m}$
13. $5.1 \times 10^5 \text{ N/m}^2$
15. 0.14 m
17. 150 N pushing out
21. 1140 kg/m^3
23. 0.022 m^2
25. (a) 10 N, (b) $1.6 \times 10^{-8} \text{ N}$
27. $4.8 \times 10^3 \text{ N}$
29. 110 N
31. (a) 37.9 N, (b) 38.4 N
33. 172 N
35. (a) 1.5 N, (b) 2.9 m/s^2
37. 1960 N

42.
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EXAMPLE 14.1 The heart pumps blood at a flow rate of $80 \text{ cm}^3/\text{s}$ into the aorta, the diameter of which is 1.5 cm . Calculate the average speed of blood in the aorta.

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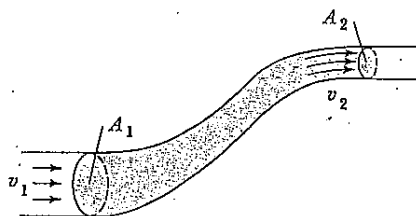


FIG. 14.4. A fluid flows through a vessel with changing cross-sectional area. For incompressible fluids, the flow rate across A_1 must equal that across A_2 .

The average speed of blood in the aorta is determined by rearranging Eq. (14.2):

$$\bar{v} = \frac{Q}{A} = \frac{80 \text{ cm}^3/\text{s}}{1.8 \text{ cm}^2} = \underline{44 \text{ cm/s.}}$$

Often, the cross-sectional area of a vessel through which fluid flows varies from one part of the vessel to another. If the fluid is incompressible—if its volume undergoes negligible change when the pressure acting on the fluid increases—then the volume of fluid that passes one cross section of the vessel equals that passing another cross section. During any period of time, the fluid entering the region between the cross sections (for example, A_1 and A_2 in Fig. 14.4) must equal that leaving the region. The flow rate in must equal the flow rate out. In terms of the cross sections A_1 and A_2 in Fig. 14.4, we can say that

$$Q = \bar{v}_1 A_1 = \bar{v}_2 A_2, \tag{14.3}$$

where \bar{v}_1 is the average speed of fluid passing cross section A_1 , and \bar{v}_2 is the average speed of fluid passing A_2 . In a narrow section of pipe where A is small, the speed of the fluid will have to be great in order for the flow rate to remain constant.

Equation (14.3) is called the **continuity equation** and is used to relate the cross-sectional areas and speeds of fluid flow in different parts of a vessel carrying the fluid. It will be an especially useful equation when we discuss applications of Bernoulli's equation later in this chapter.

- Example** 1. (a) Calculate the flow rate of water moving at an average speed of 32 cm/s through a garden hose of radius 1.2 cm . (b) Calculate the speed of the water in a second hose of radius 1.0 cm that is connected to the first hose.

PHY 111 Reduction in pressure due to fluid velocity: Bernoulli's Principle

When a fluid, such as air or water flows past a surface, there is a reduction in pressure due to the fact that the fluid is moving compared to what the pressure would be if the fluid was not moving.

$$\Delta P = \frac{1}{2} \rho v^2$$

EXAMPLE 14.6 A 45-m/s (100-mph) wind blows across the top of a flat-roofed house (Fig. 14.10). Calculate the net force on the roof due to the pressure differential from inside the house to the outside. Assume that the pressure in the house is atmospheric pressure and that the roof has an area of 200 m². The density of air equals 1.3 kg/m³.



FIG. 14.10. Air blowing across a roof causes the pressure to be reduced. If the wind blows fast enough, the internal air pressure can blow the roof off.

plaque such as shown in Fig. 14.11. If dislodged, the plaque moves downstream and may block a smaller vessel so that no blood flows through it. Should this happen to one of the arteries supplying blood to the heart muscles, a heart attack may occur. Bernoulli's principle helps us understand how a plaque becomes dislodged, as described in the simplified model of a plaque in the next example.

EXAMPLE 14.7 Blood of density 1030 kg/m^3 flows through the unobstructed part of the blood vessel shown in Fig. 14.12 at a speed of 0.50 m/s . The cross-sectional area through which blood flows past the plaque is one-sixth the normal area of the vessel. The bottom surface of the plaque facing down toward position 2 has an area of 0.60 cm^2 , as do the top surfaces in the channels facing position 1. Calculate the net downward force that tends to dislodge the plaque.

This is about the weight of one-fourth of a medium-sized apple pulling on the plaque.

In addition to the force calculated using Bernoulli's principle that tends to suck the plaque off the wall, there is also an "impact" pressure and force caused by blood hitting the plaque's upstream side.

14.5 Viscous Fluid Flow

In our previous discussion and examples we have ignored the role of friction forces. This is not appropriate in many practical cases. In fluids, just as in solids, friction is often responsible for the generation of thermal energy, but the way in

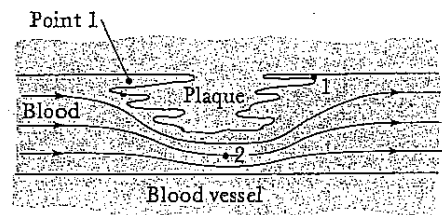


FIG. 14.11. Blood must accelerate as it moves past a plaque. The reduced blood pressure of this fast-moving blood may cause the plaque to become dislodged.

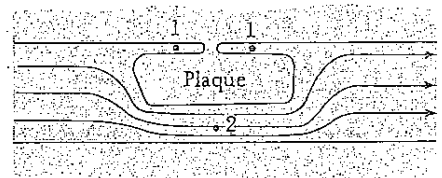


FIG. 14.12. A simplified version of a plaque.

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(y_2 - y_1). \quad (14.4)$$

This is one form of Bernoulli's equation. It relates the pressures, speeds, and elevations of a fluid at any two points in the fluid (our positions 1 and 2 in Fig. 14.5 were arbitrary). Equation 14.4 applies to the nonturbulent flow of a frictionless, incompressible fluid. The term on the left side of Eq. (14.4) is a consequence of the work done because of the difference in fluid pressure pushing

forward and that pushing back. The two terms on the right result from the changes in kinetic and gravitational potential energies of the fluid.

Bernoulli's equation is often rewritten in a form that is easier to use and happens easier to remember. A rearrangement of Eq. (14.4) yields

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2. \quad (14.5)$$

EXAMPLE 14.3 Water is pumped at a rate of $24 \text{ cm}^3/\text{s}$ through a 0.50-cm -radius pipe on the main floor of a house to a 0.35-cm -radius pipe in a solar hot water collector 4.0 m higher on the roof. If the pressure in the pipe on the roof is $1.20 \times 10^5 \text{ N/m}^2$, what is the pressure in the larger pipe on the main floor?

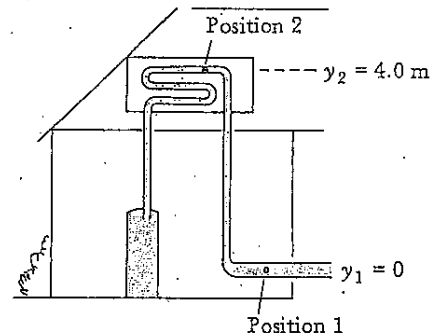


FIG. 14.6. A water line for a solar collector.

EXAMPLE 14.4 (a) Calculate the speed with which water flows from a hole in the dam of a large irrigation canal. The hole is 0.80 m below the surface of the water (Fig. 14.8). (b) If the hole has a radius of 2.0 cm, what is the flow rate of water from the hole?

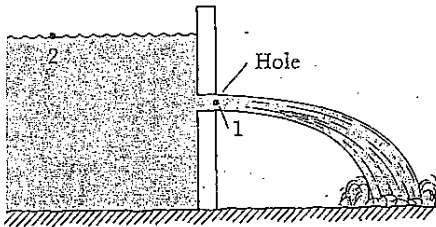


FIG. 14.8

EXAMPLE 14.6 A 45-m/s (100-mph) wind blows across the top of a flat-roofed house (Fig. 14.10). Calculate the net force on the roof due to the pressure differential from inside the house to the outside. Assume that the pressure in the house is atmospheric pressure and that the roof has an area of 200 m^2 . The density of air equals 1.3 kg/m^3 .

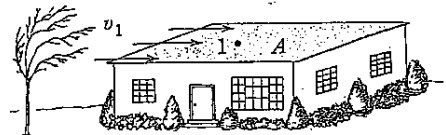


FIG. 14.10. Air blowing across a roof causes the pressure to be reduced. If the wind blows fast enough, the internal air pressure can blow the roof off.

EXAMPLE 14.5 Calculate the pressure needed for the pump of a 3.0-cm-radius firehose to pump water through its nozzle of radius 2.0 cm at an average speed of 4.0 m/s when the nozzle is 15 m above the fire truck.

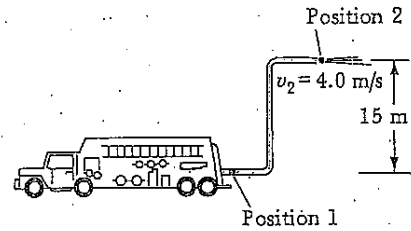


FIG. 14.9. What is the pressure in the hose at position 1 in order for an adequate flow of water from the nozzle at position 2?

Problems

Unless stated otherwise, assume in these problems that atmospheric pressure is $1.01 \times 10^5 \text{ N/m}^2$ and that the densities of water and air are 1000 kg/m^3 and 1.3 kg/m^3 , respectively.

14.2 Flow Rate and Equation of Continuity

2. (a) An irrigation canal has a rectangular cross section of 5.0-m width and 1.2-m depth. If water flows at a speed of 0.80 m/s, what is its flow rate? (b) If the width of the stream is reduced to 3.0 m and the depth to 1.0 m as the water passes a flow-control gate, what is the speed of the water past the gate?

3. Each second, 0.070 m^3 of water flows through a firehose. If the water is to leave the nozzle at a speed of 25 m/s, what should its diameter be?

4. A main waterline for a housing project must deliver water at a maximum flow rate of $0.010 \text{ m}^3/\text{s}$. If the speed of the water at this flow rate is 0.30 m/s, calculate the diameter of the pipe carrying the water.

5. The flow rate of blood in the aorta is $80 \text{ cm}^3/\text{s}$. Beyond the aorta, this blood travels through about 6×10^9 capillaries. If the radii of these capillaries is $8 \times 10^{-4} \text{ cm}$, what is the speed of the flow of blood through the capillaries?

6. A farmer's 4.0-cm-diameter pipe from an irrigation canal takes 20 h to flood a small field. How much time is required using a 6.0-cm pipe, assuming the water flows at the same average speed in both pipes?

14.3 and 14.4 Bernoulli's Equation and Applications

7. The pressure of water flowing through a pipe of radius 0.060 m at a speed of 1.8 m/s is $2.2 \times 10^5 \text{ N/m}^2$. Calculate (a) the flow rate of the water and (b) the pressure in the water after it goes up a 5.0-m-high hill and flows in a pipe of radius 0.050 m.

8. A garden hose of 0.80-cm radius is connected to one of 1.0-cm radius. The smaller hose is held on the roof of a house 4.0 m above the larger hose. Water leaves the smaller hose at a speed of 6.0 m/s and at atmospheric pressure. Calculate the pressure in the larger hose on the ground.

9. The large pipe of a waterline has radius 0.060 m and feeds ten smaller pipes of radius 0.020 m that carry water to homes. The flow rate of water in each of the smaller pipes is to be $6.0 \times 10^{-3} \text{ m}^3/\text{s}$, and the pressure is $4.00 \times 10^5 \text{ N/m}^2$. The homes are 10.0 m above the main pipes. Calculate the average speed of the water in (a) a smaller pipe and (b) in the main pipe. (c) Calculate the pressure in the main pipe.

10. Blood flows at an average speed of 0.40 m/s in a horizontal artery of radius 1.0 cm. The average gauge pressure is $1.4 \times 10^4 \text{ N/m}^2$. Calculate (a) the average speed of the blood past a constriction where the radius of the opening is 0.30 cm and (b) the gauge pressure of the blood as it moves past the constriction.

11. Air of density 1.3 kg/m^3 blows past a $1.2 \text{ m} \times 2.2\text{-m}$ window of a skyscraper at a speed of 25 m/s. The air inside the

building is at atmospheric pressure. (a) Calculate the difference in pressure between the inside of the window and the outside. (b) Calculate the net force of the air on the window (magnitude and direction).

12. Calculate the lift caused by the Bernoulli effect on an airplane wing of area 30 m^2 . Air of density 1.1 kg/m^3 moves across the top surface at a speed of 180 m/s and across the bottom at 165 m/s.

13. At what speed must air of density 1.2 kg/m^3 move across the flat roof of a house of area 160 m^2 to be able to lift the weight of the $2.1 \times 10^4\text{-kg}$ roof. Air inside the house is at atmospheric pressure and at rest.

14. A straw extends 3.0 cm out of a glass of water. How fast must air blow across the top of the straw to draw water to the top of the straw?

15. A U-shaped tube, open at both ends, contains water. If air blows across the top of one end at a speed of 10 m/s and not across the other, how much higher will the water be in the side below the moving air compared with that below the air at rest?

16. A large, open barrel is filled with water to a height of 1.0 m above a cork. Calculate the initial flow rate of water from the 1.2-cm-radius hole when the cork is removed.

17. A large, closed barrel of water has a 0.01-m-diameter opening at the bottom through which water leaves at a speed of 2.0 m/s. Calculate the pressure at the top of the water level in the barrel 0.50 m above the hole.

18. Water sits at rest behind an irrigation dam. The water is 1.2 m above the bottom of a gate that, when lifted, allows water to flow under the gate. To what height h from the bottom of the dam should the gate be lifted to allow a flow rate of $1.0 \times 10^{-2} \text{ m}^3/\text{s}$? The gate is 0.50 m wide.

19. The gauge pressure inside a 1.0-cm radius vessel carrying blood at a speed of 0.50 m/s is 5200 N/m^2 , and the gauge pressure outside the vessel is 3200 N/m^2 . To what radius must the vessel be reduced by a constriction so that the outside pressure is greater than that inside, thus causing the vessel to close at the constriction? (The vessel will flutter open and closed like a vibrating reed.)

Chapter 14

1. (a) $140 \text{ cm}^3/\text{s}$, (b) 46 cm/s
3. 0.060 m
5. $6.6 \times 10^{-3} \text{ cm/s}$
7. (a) $0.020 \text{ m}^3/\text{s}$,
(b) $1.7 \times 10^5 \text{ N/m}^2$
9. (a) 4.8 m/s , (b) 5.3 m/s ,
(c) $4.95 \times 10^5 \text{ N/m}^2$
11. (a) 406 N/m^2 , (b) 1070 N pushing out
13. 46 m/s
15. $6.6 \times 10^{-3} \text{ m}$
17. $9.81 \times 10^4 \text{ N/m}^2$
19. 0.50 cm

