Gauss's Law Applied to Capacitor

- Dotted line shows surface around +Q

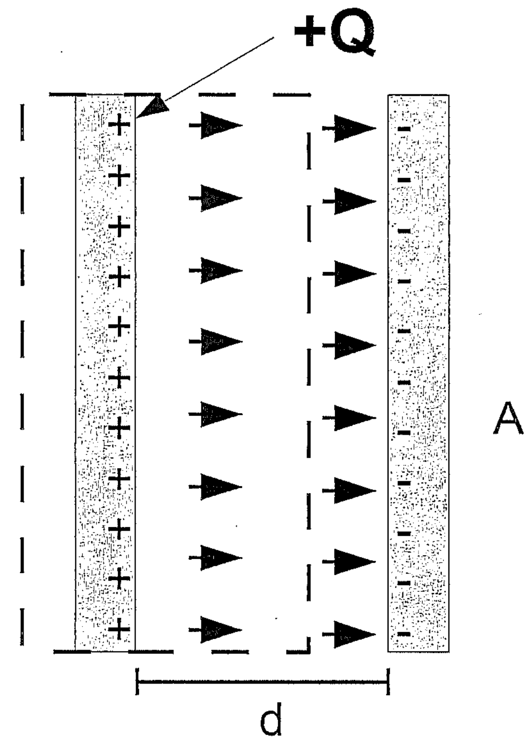
Gauss's Law:

$$A * \vec{E} = 4\pi k Q$$

$$Q = \frac{1}{4\pi k} A * E$$

$$Q = \frac{1}{4\pi k} A * \frac{V}{d}$$

- Geometrical factors A and d determine C



A = area
d = plate separation

Capacitor Equation

$$Q = C * V = \boxed{\frac{A}{4\pi k d}} * V$$

C

- Unit of capacitance is the Farad

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Charge stored in capacitor depends on Area, separation, and Voltage

Increasing capacitance density : Dielectric Constant

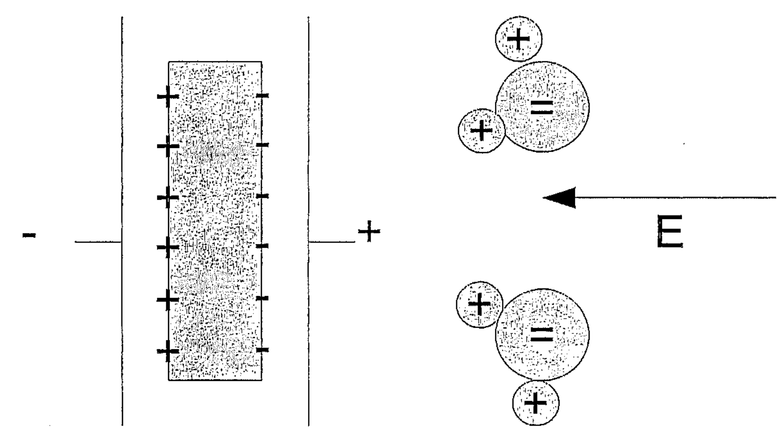
- It is difficult to store as much charge as we need in a parallel plate capacitor of reasonable size, so dielectrics are used to multiply the capacitance many times what they would be if filled only with air.

- Examples: dielectric constant of http://www.clippercontrols.com/info/dielectric_constants.html

Material	κ
Air	1
Water	80
Wax	up to 6.5
Tantalum Oxide	11.6
Aluminum Oxide	10

- In a liquid dielectric, the E field causes the liquid molecular dipoles to line up with the field. (Why?) This increases the amount of charge stored for a smaller electric field E, hence C = Q/V is increased.

$$C = \kappa \frac{A}{4\pi k d}$$



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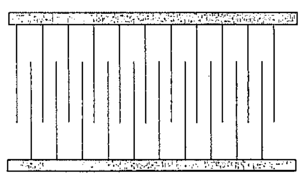
High dielectric constant makes same area into a bigger capacitance

Questions about capacitor size

- In a 1 farad capacitor, if the separation between the plates is 1 cm, what is the area of the plates?

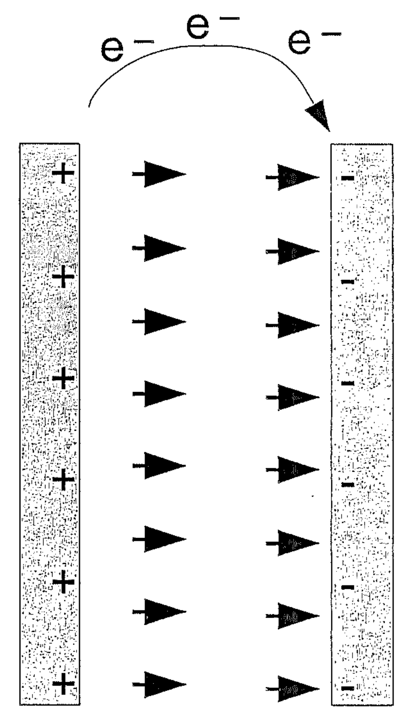
- A radio frequency tuning capacitor consists of a comb of 21 plates with area 4 cm^2. The separation between the plates is 1.5 mm. What is the capacitance of the tuning capacitor?



- If the capacitor dielectric consists of wax paper that is 10^{-4} m thick, what area is needed to create a capacitance of 1 uF (assume the wax paper has a dielectric constant ~4)?

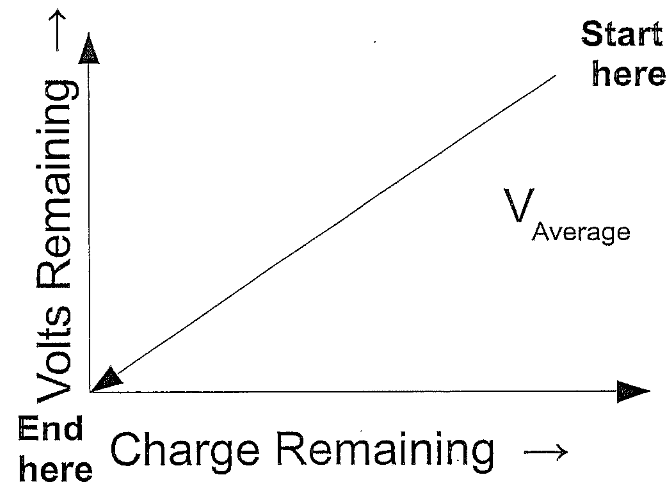
Energy is stored in a capacitor

- Move some electrons over to the negative plate:
  - This takes force and energy is added to the capacitor
  - Move charge $-q$ through voltage $-V$ takes energy qV
- Because the voltage builds up along with charge added, the average energy is ½ QV



Energy discharging from a Capacitor

- If a capacitor is charged at voltage V and charge Q, the stored energy is less than QV
  - Because when you release the energy a little at a time by transferring electrons slowly from negative to positive plate
  - As Q decreases $V = Q/C$ decreases too.
  - So energy is $Q*V_{average}$



$$Energy = Q*V_{average} = Q\frac{V}{2} = \frac{1}{2}CV^2$$

The energy in a capacitor is proportional to voltage squared

Energy storage and dielectric problems

- A radio frequency tuning capacitor consists of 21 pairs of plates with area 4 cm^2. The separation between the plates is 1.5 mm and the gaps are filled with polyethylene (dielectric constant = 2.3) . What is the maximum capacitance of the tuning capacitor?

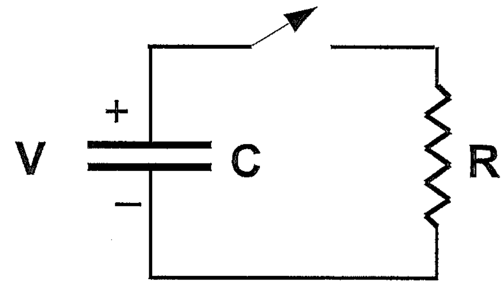
- A 0.22 F capacitor is charged to a voltage of 6V. How much energy is stored?

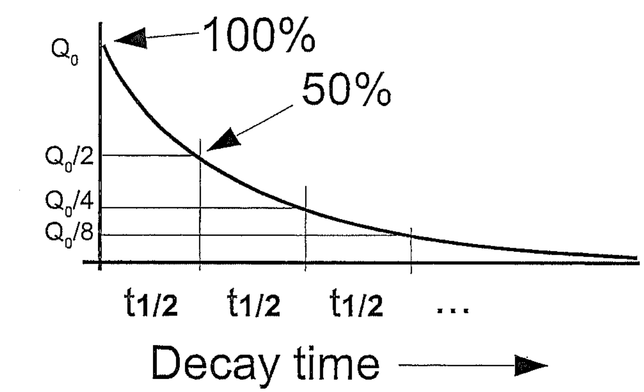
  - For how many seconds can this amount of energy keep a 6V, 20 Ω light bulb lit?

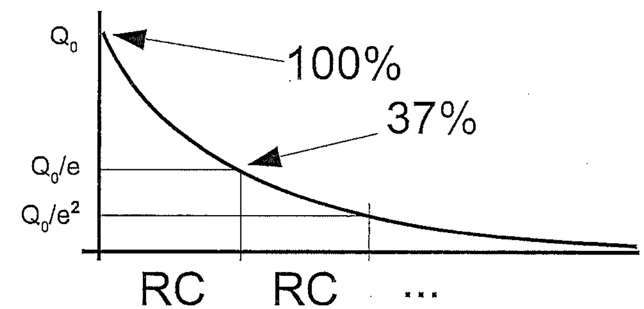
- A 0.07 F capacitor is charged to a voltage of 6V. How much energy is stored?

Time constant of resistor-capacitor

- A capacitor is charged up to voltage V. Then the switch is closed
  - Allowing the charge to circulate through load resistor R
- The charge on the capacitor approaches zero exponentially
- RC 'time constant' is 1/e-life, similar to ½-life but a little longer







τ = RC is time it takes charge to decay to 1/e = 37% initial charge

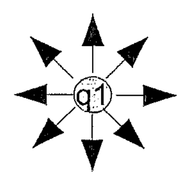
Review of Electric Field

- The Electrostatic force / field E
  - Between two charges
  - inverse square law.
  - Gauss's Law helps explain uniform Electric fields, e.g., *parallel plate capacitor*:

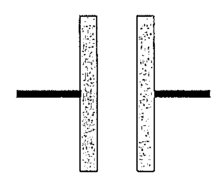
$$E = 4\pi k \frac{q}{A}$$

$$\vec{F} = k\,\frac{q_1 * q_2}{R^2}$$

$$\vec{F} = q * \vec{E}$$



$$\vec{E} = k\,\frac{q_1}{R^2}$$



$$Q = CV$$

$$C = \frac{A}{4\pi k d}$$

force

field

energy storage

E-field is essentially the story of the inverse square law

Energy stored in the E-field

- Start with the energy expression for the capacitor

$$Energy = \frac{1}{2} C V^2$$

- Change voltage to electric field

$$Energy = \frac{1}{2} C (E d)^2$$

- Change C to geometrical quantities

$$Energy = \frac{1}{2} \frac{A}{4\pi k d} (E d)^2$$

- Simplify:

$$Energy = \boxed{\frac{1}{8\pi k} (E)^2} A * d$$

energy density ↗

The energy per unit volume in the E-field is $E^2/(8\pi k)$