1. What is 23° in radians?

Solve It

2. What is $\pi / 16$ in degrees?

Solve It

When describing the way things go in circles, you don't just use radians; you also can specify the time it takes. The time it takes for an object to complete an orbit is referred to as its *period*. For example, if the object is traveling at speed v, then the time it takes to go around the circle — the distance it travels in the circle's circumference, $2\pi r$ — will be

$$T = \frac{2\pi r}{v}$$

Note the symbol of the radius of a circle: r. That's half the circle's diameter, which is d. So $r = d / 2$. Note also the symbol for the period: T. With this equation, given an orbiting object's speed and the radius of the circle, you can calculate the object's period.

Another time measurement you'll see in physics problems is *frequency*. Whereas the period is the time an object takes to go around in a circle, the frequency is the number of circles the object makes per second. The frequency, f, is connected to the period like this:

$$f = \frac{1}{T}$$

Q. The moon's orbital radius is 3.85×10^8 m, and its period is about 27.3 days. What is its speed as it goes around the Earth?

A. The correct answer is 1024 m/sec.

1. Convert 27.3 days to seconds:

$$27.3 \text{ days} \frac{24 \cancel{\text{hours}}}{\cancel{\text{day}}} \frac{60 \cancel{\text{minutes}}}{\cancel{\text{hour}}} \frac{60 \text{ seconds}}{\cancel{\text{minute}}} = 2.36 \times 10^6 \text{ sec}$$

2. Use the equation for the period to solve for speed:

$$v = \frac{2\pi r}{T}$$

3. Plug in the numbers:

$$v = \frac{2\pi r}{T} = \frac{2 \cdot \pi \cdot 3.85 \times 10^8}{2.36 \times 10^6} = 1024 \text{ m/sec}$$

3. You have a ball on a string, and you're whipping it around in a circle. If the radius of its circle is 1.0 m and its period is 1.0 sec, what is its speed?

Solve It

4. You have a toy plane on a wire, and it's traveling around in a circle. If the radius of its circle is 10.0 m and its period is 0.75 sec, what is its speed?

Solve It

There are analogs of every linear motion quantity (such as distance, velocity, and acceleration) in angular motion, and that's one of the things that makes angular motion easier to work with. The velocity of an object in linear motion is shown in the following equation (this is actually a vector equation, of course, but I don't get into the vector nature of angular motion until Chapter 10, so I look at this equation in scalar terms):

$$v = \frac{\Delta s}{\Delta t}$$

What's the analog of this equation in angular terms? That's easy; you just substitute angle θ for the distance, so the angular velocity is θ / t. That means that angular velocity ω is the angle (in radians) that an object sweeps through per second.

$$\omega = \frac{\Delta \theta}{\Delta t}$$

Figure 6-2 shows a line sweeping around in a circle. At a particular moment, it's at angle θ, and if it took time t to get there, its angular velocity is θ / t.

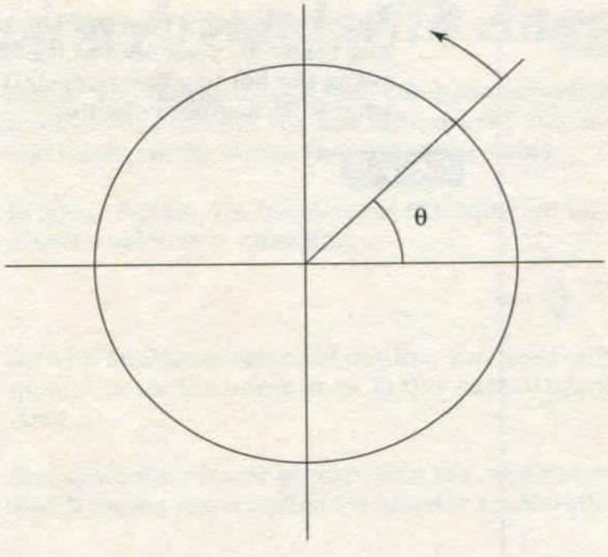

Figure 6-2: Angular velocity in a circle.

So if the line in Figure 6-2 completes a full circle in 1.0 sec, its angular velocity is $2\pi/1.0$ sec = 2π radians/sec (because there are 2π radians in a complete circle). Technically speaking, radian isn't a physical unit of measure (it's a ratio), so the angular velocity can also be written $2\pi\ sec^{-1}$.

The symbol for angular velocity is ω, so you can write the equation for angular velocity this way:

$$\omega = \frac{\Delta \theta}{\Delta t}$$

Given the angular velocity, you also can find the angle swept through in a number of seconds:

$$\Delta \theta = \omega \cdot \Delta t$$

Q. The moon goes around the Earth in about 27.3 days. What is its angular velocity?

A. The correct answer is 2.66×10^{-6} radians/sec.

1. Convert 27.3 days to seconds:

$$27.3 \ \frac{24 \text{ hours}}{\text{day}} \ \frac{60 \text{ minutes}}{\text{hour}} \ \frac{60 \text{ seconds}}{\text{minute}} = 2.36 \times 10^{6} \text{ sec}$$

2. Use the equation for angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

3. Plug in the numbers:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{2.36 \times 10^{6} \text{ sec}} = 2.66 \times 10^{-6} \text{ radians/sec}$$

5. You have a toy plane on a string that goes around three complete circles in 9.0 sec. What is its angular velocity?

Solve It

6. You're swinging a baseball bat around, getting ready for your shot at the ball. If you swing the bat in a half circle in 1.0 sec, what is its angular velocity?

Solve It

7. A satellite is orbiting the Earth at 8.7×10^{-4} radians/sec. How long will it take to circle the entire world?

Solve It

8. A merry-go-round is spinning around at 2.1 radians/sec. How long will it take to go in a complete circle?

Solve It

Just as with linear motion, you can have acceleration when you're dealing with angular motion. For example, the line in Figure 6-2 may be sweeping around the circle faster and faster, which means that it's accelerating.

In linear motion, the following is the equation for *acceleration*, the rate at which the object's velocity is changing:

$$a = \frac{\Delta v}{\Delta t}$$

As with all the equations of motion, you need only to substitute the correct angular quantities for the linear ones. In this case, v becomes ω. So the angular acceleration is $\Delta\omega / \Delta t$.

The symbol for linear acceleration is a, and the symbol for angular acceleration is α, which makes the equation for angular acceleration:

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

The unit for angular acceleration is radians/sec^2 (or, technically, just sec^{-2}).

Q. Your toy plane on a string accelerates from ω = 2.1 radians/sec to 3.1 radians/sec in 1.0 sec. What is its angular acceleration?

A. The correct answer is 1.0 radians/sec^2.

1. Use the equation for angular acceleration:

$$\alpha = \frac{\Delta \omega}{\Delta t}$$

2. Plug in the numbers:

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{(3.1 - 2.1)}{1.0} = 1.0 \text{ radians/sec}^2$$

9. Your model globe is turning at 2.0 radians/sec, which you decide isn't fast enough. So you give it a push, accelerating it in 10^{-1} sec to 5.0 radians/sec. What is its angular acceleration?

Solve It

10. You have a toy plane on a wire, and it's traveling around in a circle at 3.5 radians/sec. You speed it up to 5.4 radians/sec in 3.0 sec. What was its angular acceleration?

Solve It

11. You're square dancing, turning your partner around at 1.0 radians/sec. Then you speed up for 0.50 sec at an angular acceleration of 10.0 radians/sec^2. What is your partner's final angular speed?

Solve It

12. You're trying a new yoga move, and starting your arm at rest, you accelerate it at 15 radians/sec^2 over 1.0 sec. What's your arm's final angular velocity?

Solve It

You can connect the distance traveled to the original velocity and linear acceleration like this:

$$s = v_0(t_f - t_o) + \frac{1}{2}a(t_f - t_o)^2$$

And you can make the substitution from linear to angular motion by putting in the appropriate symbols:

$$\theta = \omega_0(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

Using this equation, you can connect angular velocity, angular acceleration, and time to the angle.

A marble is rolling around a circular track at 6.0 radians/sec and then accelerates at 1.0 radians/sec^2. How many radians has it gone through in 1 minute?

The correct answer is 2200 radians.

1. Use this equation:

$$\theta = \omega_0(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2$$

2. Plug in the numbers:

$$\theta = \omega_0(t_f - t_o) + \frac{1}{2}\alpha(t_f - t_o)^2 = (6.0)(60) + \frac{1}{2}(1.0)(60^2) = 360 + 1800 = 2160$$

13. Your model globe is spinning at 1.0 radians/sec when you give it a push. If you accelerate it at 5.0 radians/sec^2, how many radians has it turned through in 5.0 sec?

Solve It

14. Your toy plane on a wire is traveling around in a circle at 8.0 radians/sec. If you accelerate it at 1.0 radians/sec^2 for 20 sec, how many radians has it gone through during that time?

Solve It

15. You're whipping a ball on a string around in a circle. If it's going 7.0 radians/sec and at the end of 6.0 sec has gone through 60.0 radians, what was its angular acceleration?

Solve It

16. A roulette wheel is slowing down, starting at from 12.0 radians/sec and going through 40.0 radians in 5.0 sec. What was its angular acceleration?

Solve It