Chapter Two

MOTION IN ONE DIMENSION

KEY IDEAS

Motion is the change of position in time. *Displacement* is the directed change of an object's position. *Velocity* is the time rate of change of displacement, and *acceleration* is the time rate of change of velocity.

Under conditions of constant acceleration (also known as uniform acceleration), the motion of an object is governed by a set of interrelated equations. Objects that fall freely near the surface of the Earth are uniformly accelerated by gravity.

The motion of an object can be described by a series of motion graphs. The object's position, velocity, and acceleration can be plotted as functions of time. These graphs can then be used to illustrate various aspects of the object's motion at every point in time.

KEY OBJECTIVES

At the conclusion of this chapter you will be able to:

- Define the terms *motion, distance, displacement, average velocity, speed, instantaneous velocity,* and *acceleration,* and state their SI units.
- Solve problems involving average velocity and constant velocity.
- Distinguish between average velocity and instantaneous velocity, and relate these terms to a position-time graph.
- Solve problems involving the equations of uniformly accelerated motion.
- Solve problems involving freely falling objects.
- Interpret the data provided by motion graphs and solve problems related to them.

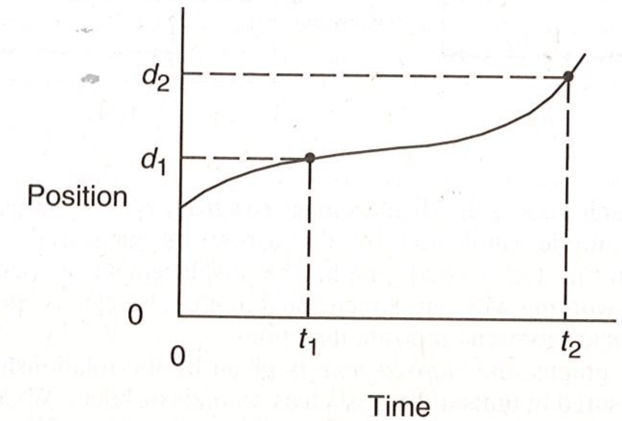
* 2.1 MOTION DEFINED

How do we know when an object is in motion? If we look at the hour hand of a watch, it does not appear to be moving, yet over a period of time we see a change in its position. Therefore, a reasonable definition of **motion** is the change of an object's position in time.

* indicates that material is part of the New York State core curriculum.

* 2.2 GRAPHING AN OBJECT'S MOTION

Graphs are especially useful for analyzing an object's motion. The position of the object is plotted along the *y*-axis, and the elapsed time along the *x*-axis. Here is a graph of very general motion in one dimension:



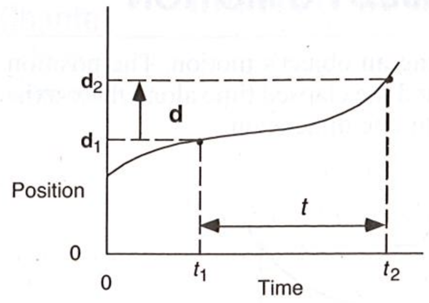
The origin of the graph (0, 0) marks the reference point for both position and time. When we say "zero time" we mean the time when we begin the event—the time when we "start the clock," so to speak. Similarly, "zero position" means the specific place where we begin measuring. It may be the ground or a table top or a spot on the wall. Generally, we use the letters d to represent position and t to represent time. If we know that we are measuring *horizontal* position, we may use the letter x, or d_x in place of d. If we are measuring a vertical position we can substitute the letter y or d_y in place of d.

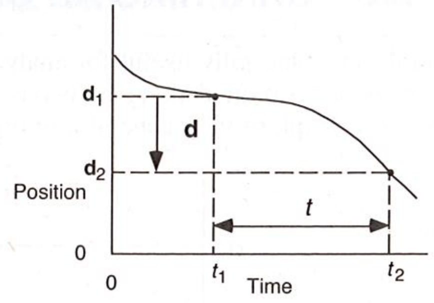
The dotted lines on the graph tell us where the object is at a given time. At time t_1, the object is at position d_1; at time t_2, the object is at position d_2.

* 2.3 DISPLACEMENT

The **displacement** of an object is the change in its position and is measured in units of length (such as meters or inches). In the graph in Section 2.2, the displacement of the object between times t_1 and t_2 is given by the relationship $\mathbf{d} = \mathbf{d}_2 - \mathbf{d}_1$. Since we are subtracting two coordinates ($\mathbf{d}_1$ from $\mathbf{d}_2$), we are not primarily concerned about the exact path taken by the object between these points. We assume that the magnitude of the displacement is given by the length of a straight line between $\mathbf{d}_1$ and $\mathbf{d}_2$ along the axis representing position.

Displacement is known as a *vector* quantity because, in addition to magnitude, it has *direction.* (Vector quantities, which are discussed in detail in Chapter 4, are set in boldface type.) The magnitude of displacement is known as *distance.*



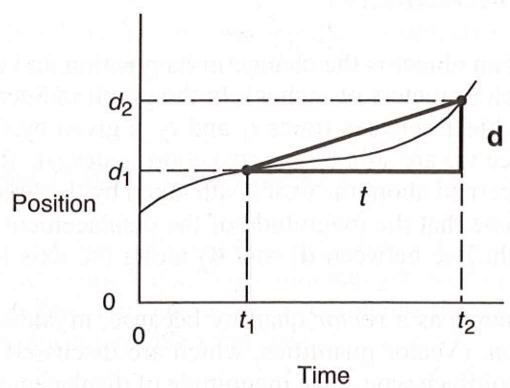


In the first graph above, the displacement (**d**) from t_1 to t_2 is positive because its magnitude (indicated by the arrow) is measured in the positive direction; in the second graph, the displacement is negative. Whenever we are working with one-dimensional motion, we can use positive and negative signs to represent opposite directions.

In each of the graphs, the *elapsed time* is given by the relationship $t = t_2 - t_1$ and is measured in units of time, such as seconds or hours. We always read the time axis from left to right since normally we do not travel backward in time.

✱ 2.4 VELOCITY

In the graph below, the slope of the straight line connecting the two points on the graph is given by the relationship **d**/*t*, and it indicates *how rapidly* the position of the object (**d**) has changed over the time interval (*t*). This quantity is known as the **average velocity** ($\bar{\mathbf{v}}$) of the object and is measured in units such as meters per second (m/s) or miles per hour (mph). Mathematically, the average velocity of the object is defined by the equation



PHYSICS CONCEPTS

$$\bar{\mathbf{v}} = \frac{\mathbf{d}}{t} = \frac{\mathbf{d}_2 - \mathbf{d}_1}{t}$$

Velocity, like displacement, is a vector quantity because it has both magnitude and direction. As with displacement, we can use positive and negative signs to represent motion in opposite directions. The magnitude of the velocity is known as **speed**.

PROBLEM
The position of an object is +35 meters at 2.0 seconds and is +87 meters at 15 seconds. Calculate the average velocity of the object.

SOLUTION

[illegible] − (+35 m) / [illegible] − 2.0 s = +52 m / 13 s = +4.0 m/s

This object is traveling in [illegible] rection with an average speed of 4.0 m/s.

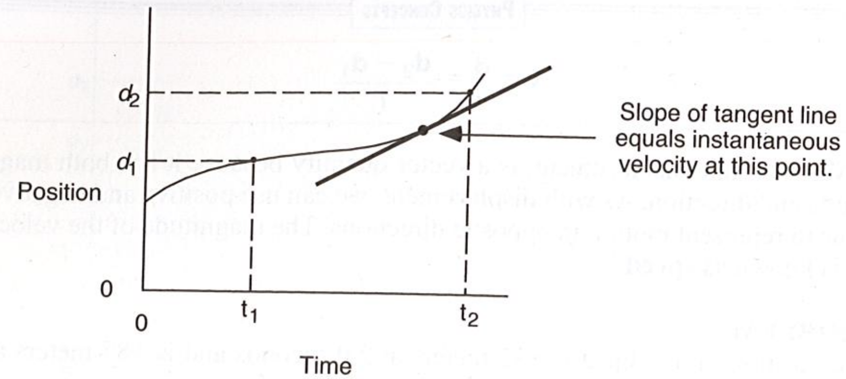
We use the term *average velocity* because we do not know exactly what is happening *between* the two points in question. For example, suppose we traveled by automobile due west 1000 miles and the trip took 20 hours. When we calculate our average velocity for the trip, we obtain

$$\bar{\mathbf{v}} = \frac{1000\ \text{mi [W]}}{20\ \text{h}}$$

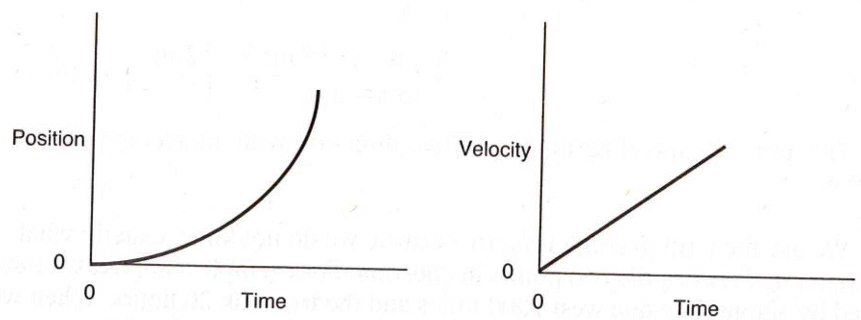
$$= 50\ \text{mph [W]}$$

Does this mean that we traveled the entire distance at a constant speed of 50 miles per hour? Not necessarily! We would probably have had to add fuel, eat, pay tolls, or engage in other activities on the trip. There might have been construction delays or reduced speed zones. All we can say with certainty is that our average speed was 50 miles per hour and our direction of travel was west.

How then could we measure our velocity at any *point* on our trip—our **instantaneous velocity**? (This is the value that we read on the speedometer of our car.) One way would be to measure our average velocity over *smaller and smaller* time intervals. To accomplish this, however, we would need to use mathematical techniques that are beyond the scope of this book. The other way is to use a position versus time graph. The instantaneous velocity at any point on the graph is the slope of a line drawn *tangent* to the graph at that point, as shown in the diagram.



2.5 ACCELERATION



Consider the two graphs shown above. The first graph represents the position of an automobile as a function of time. Note that the graph becomes steeper (curves upward) as time passes. This occurs because the automobile's instantaneous velocity is *increasing.*

The second graph represents the instantaneous velocity of the same automobile as a function of time. Note that the graph is a straight line directed upward. This graph also shows that the instantaneous velocity of the automobile is increasing with time.

Actually, the graphs represent the motion of the automobile from two different viewpoints: that of position (as measured by the automobile's *odometer*) and that of velocity (as measured by the automobile's *speedometer*).

The *slope* of the velocity–time graph is given by the relationship $\Delta \mathbf{v}/t$ and indicates the rate at which the velocity of the object ($\Delta \mathbf{v}$) has changed over the time interval (t). This quantity is known as the **acceleration (a)** of the object and is measured in units such as meters per second^2 (m/s^2). Since the graph is a straight line, the acceleration in this case is constant or *uniform.* Mathematically, the uniform acceleration of the object is defined by the equation

PHYSICS CONCEPTS

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{t} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

Acceleration is also a vector quantity because it has both magnitude and direction. A positive acceleration means that the velocity of an object is becoming more positive with time; a negative acceleration, that the velocity of the object is becoming more negative with time.

PROBLEM

The velocity of an object is +47 meters per second at 3.0 seconds and is +65 meters per second at 12.0 seconds. Calculate the acceleration of the object.

2.6 THE EQUATIONS OF UNIFORMLY ACCELERATED MOTION

In the problem we solved in Section 2.5, an object accelerates uniformly, at 2.0 meters per second^2, from 47 meters per second to 65 meters per second in 9.0 seconds. There is a great deal of additional information about the object we might wish to learn. For example:

1. What is the *average velocity* of the object over 9.0 seconds?
2. What is the *displacement* of the object at the end of 9.0 seconds?
3. What is the *instantaneous velocity* of the object at any given time (at 6.0 s, for example)?

To solve these problems, we use a set of five equations that describe the motion of an object undergoing uniform acceleration. In each of these equations, we use the subscripts *i* (for *initial* value), *f* (for *final* value), and $\bar{v}$ to indicate average velocity. Your textbook or teacher may use different subscripts or notation, but they all yield the same results. The equations for uniformly accelerated motion are as follows:

PHYSICS CONCEPTS

1. $\bar{\mathbf{v}} = \frac{\mathbf{d}}{t}$
2. $\bar{\mathbf{v}} = \frac{\mathbf{v}_i + \mathbf{v}_f}{2}$
3. $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a} \cdot t$
4. $\mathbf{d} = \mathbf{v}_i \cdot t + \frac{1}{2}\mathbf{a} \cdot t^2$
5. $\mathbf{v}_f^2 = \mathbf{v}_i^2 + 2 \cdot \mathbf{a} \cdot \mathbf{d}$

Equation 1 is the definition of average velocity. Equation 2 tells us that, under uniform acceleration, the average velocity lies midway between the initial and final velocities. It should be noted that although this equation is not included on the Reference Table, it is essential to solving problems on the exam. Equation 3 is just the definition of acceleration ($\mathbf{a} = \Delta\mathbf{v}/t$) rearranged in a more convenient form for solving problems. Equations 4 and 5 are relationships that have been derived from the first three equations.

Which equation should you use to solve a particular problem? The answer depends on the data you are given. In the problem we have been considering, an object with an initial velocity of 47.0 meters per second accelerates uniformly at 2.0 meters per second^2 for 9.0 seconds. Suppose we wish to calculate the *displacement* of this object at the end of 9.0 seconds. We list the variables that are part of the problem, along with their values:

$$\mathbf{v}_i = 47.0 \text{ m/s}$$
$$\mathbf{a} = 2.0 \text{ m/s}^2$$
$$t = 9.0 \text{ s}$$
$$\mathbf{d} = ???$$

If we examine the list of equations given above, we see that equation 4 contains the four variables that form the basis of our problem. We solve the problem by substituting the values and calculating the answer:

$$\mathbf{d} = \mathbf{v}_i \cdot t + \frac{1}{2}\mathbf{a} \cdot (t)^2$$
$$= (47.0 \text{ m/s})(9.0 \text{ s}) + \frac{1}{2}(2.0 \text{ m/s}^2)(9.0 \text{ s})^2$$
$$= 504 \text{ m} = 500 \text{ m}$$

2.7 FREELY FALLING OBJECTS

The following table represents the motion of an object falling from rest near the surface of the Earth when air resistance is ignored.

t (s)	0.00	1.00	2.00	3.00	4.00	5.00	6.00
v (m/s)	0.00	9.80	19.6	29.4	39.2	49.0	58.8
d (m)	0.00	4.90	19.6	44.1	78.4	122	176

If we analyze this motion, we see that the speed of the object increases uniformly by 9.8 meters per second for each second of travel. This suggests that the object is subject to a constant acceleration of 9.8 meters per second^2. The distance traveled by the object over time verifies that the object's acceleration is constant.

PROBLEM

How does the distance traveled by the object described above over time verify that the object's acceleration is constant?

SOLUTION

The distance traveled by an objec[illegible] uniform acceleration is given by the equation:

$$\mathbf{d} = [illegible] \frac{1}{2}\mathbf{a} \cdot (t)^2$$

Since the object starts from rest, [illegible]uation reduces to:

$$\mathbf{d} = [illegible] (t)^2$$

If we substitute each of the corre[illegible]g values of d and *t* given in the table, we find that the acceleration in [illegible]se is 9.8 m/s^2.

If we were to investigate fur[illegible] would find that *all* objects falling near the surface of the Earth exp[illegible] a constant acceleration of 9.8 meters per second^2 if air resistance is ig[illegible] This phenomenon is due to the presence of gravity, which affects e[illegible] every object. If we were to travel to the Moon, we would find that a[illegible]s also fall to its surface with a uniform acceleration. However, this [illegible]ation is only 1.6 meters per second^2 because of the Moon's weaker gravitational forces.

Since free fall involves uniform acceleration, the five equations we developed in Section 2.6 can be used to solve all free-fall problems. We need only remember that objects can move up as well as down in the presence of gravity. Therefore, we assign the up direction as positive and the down direction as negative. Since gravity *always* points downward (i.e., toward the Earth), its value is taken to be –9.8 meters per second^2. Gravitational acceleration is denoted by the lowercase letter *g*.

PROBLEM

An object is dropped from rest from a height of 49 meters.

(a) How long does the object take to hit the ground?

(b) What is its speed as it hits the ground?

SOLUTION

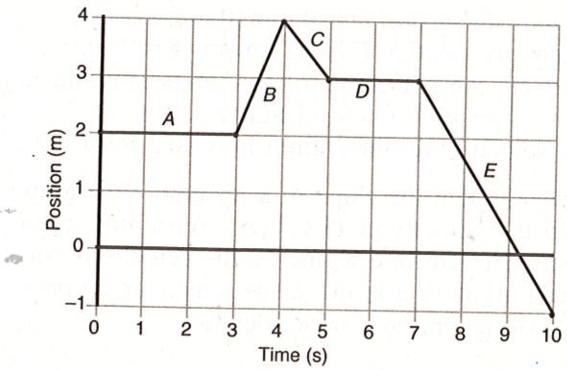
the i

2.8 MOTION GRAPHS REVISITED

Throughout this chapter we have used motion graphs as aids to understanding the concept of motion. In this section, we take a more detailed look at these graphs and the information they can provide. We shall examine three types of graphs: position–time, velocity–time, and acceleration–time.

Position–Time Graphs

The following graph illustrates the position of an object as a function of time.



We recall from Section 2.2 that "zero time" represents the start of an event and that "zero position" represents some arbitrary reference point. We have divided the position–time graph into five sections: *A, B, C, D,* and *E*. Since each section is a straight-line segment, the velocity within each section is *constant* and the acceleration over each section is *zero*. We will learn how to interpret this graph by considering the following problem.

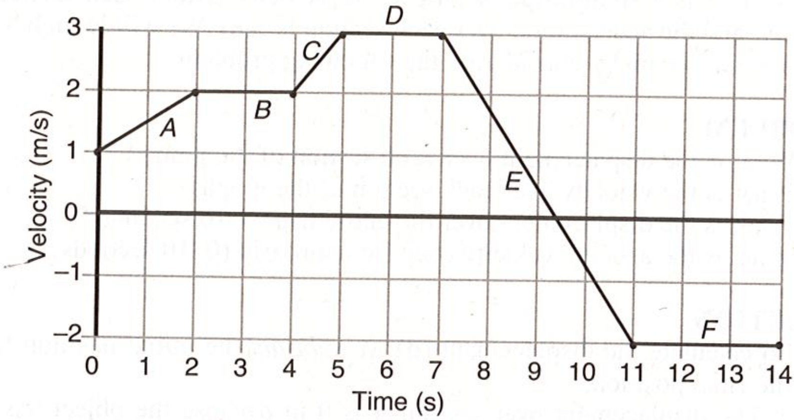
PROBLEM

1. What is the displacement over each section of the graph?
2. What is the velocity over each section of the graph?
3. What is the displacement over the entire trip (0–10 seconds)?
4. What is the average velocity over the entire trip (0–10 seconds)?

SOLUTION

Velocity–Time and Acceleration–Time Graphs

The graph below illustrates the velocity of an object as a function of time.



The values on the *y*-axis represent the *instantaneous* velocities of the object at the times marked on the *x*-axis. It is as though we were looking at a car's speedometer at various times. We have divided the graph into six sections: *A, B, C, D, E,* and *F*. Since each section is a straight-line segment, the object's acceleration within each section is *constant.* We will learn how to interpret this graph by considering the following problem.

PROBLEM

1. What is the average velocity within each section of the graph?
2. What is the acceleration within each section of the graph?
3. When does the object come to rest?
4. When does the object reverse the direction of its motion?
5. What is the displacement within each section of the graph?
6. What is the displacement over the entire trip (0–14 seconds)?
7. What is the average velocity over the entire trip (0–14 seconds)?
8. What is the shape of the corresponding *acceleration versus time* graph?