M1

Figure 10-1 shows a rotating ball at the end of a stick. You should remember that the force needed to keep the ball moving in a circle is given by $F = m \cdot a$, so the torque, which equals $F \cdot r$, is

$$F \cdot r = \tau = m \cdot r \cdot a$$

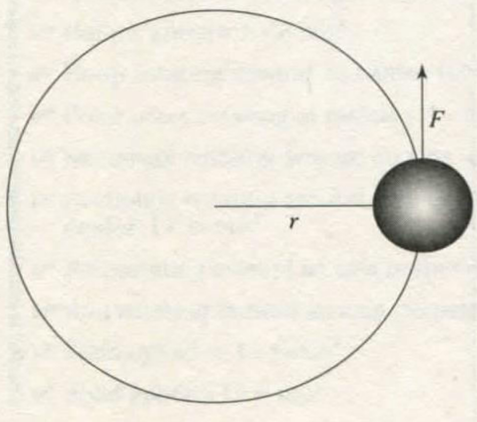

Figure 10-1: A rotating ball.

Look at this situation in terms of α, angular acceleration. Angular acceleration is one of those items you can multiply by the radius to get the linear equivalent, which in this case is equal to the tangential acceleration:

$$a = r \cdot \alpha$$

Substituting $a = r \cdot \alpha$ in the torque equation gives you:

$$\tau = m \cdot r^2 \cdot \alpha$$

This is an important result because it relates torque and angular acceleration. The quantity mr^2 is called the *moment of inertia*, I, and it represents the effort you need to get something to turn, so this equation is usually written as:

$$\tau = I \cdot \alpha$$

This equation is a general result, but the moment of inertia, I, differs depending on the situation. For example, I is different when you're spinning a solid cylinder versus a solid sphere. For a single small mass such as the ball on the end of a stick, $I = mr^2$.

The units of moment of inertia are kilogram meters^2 (kg-m^2) in the MKS system.

Q. You're whipping a cannonball around on the end of a 1.0 m iron rod. If the cannonball has a mass of 10.0 kg, what torque do you need to apply to get an angular acceleration of 0.50 radians/sec^2?

A. The correct answer is 5.0 N-m.

1. Use the equation $\tau = m \cdot r^2 \cdot \alpha$.
2. Plug in the numbers:

$$\tau = m \cdot r^2 \cdot \alpha = (10) \cdot (1.0^2) \cdot (0.5) = 5.0 \text{ N-m}$$

1. You're rotating a cannonball at the end of a 1.0 m rod in a circle and want an angular acceleration of 1.0 radians/sec^2. What torque do you need to supply?

Solve It

2. The cannonball on the end of a rod has an angular acceleration of 2.0 radians/sec^2. What torque are you applying?

Solve It

3. You're shoveling snow, holding the shovel handle in your right hand. Assuming that you can use the equation $I = mr^2$ to determine the moment of inertia of the shovel, if the 2.0 kg shovel has an angular acceleration of 10.0 radians/sec^2 and a length of 1.5 m, what torque are you applying?

Solve It

4. A 100.0 g clock pendulum on the end of a 1.0 m rod has an angular acceleration of 2.0 radians/sec^2. What torque is being applied?

Solve It

You know that the moment of inertia of a small mass on the end of a thin rod is $m \cdot r^2$. What are the moments of inertia for other configurations, such as a solid sphere? By treating each mass as a collection of small masses, the moment of inertia for a number of other shapes have been figured out; here are some of them:

- Disk rotating around its center: $I = \frac{1}{2} \cdot m \cdot r^2$
- Hollow cylinder rotating around its center (such as a tire): $I = m \cdot r^2$
- Hollow sphere: $I = \frac{2}{3}\, m \cdot r^2$
- Hoop rotating around its center (like a tire): $I = m \cdot r^2$
- Point mass rotating at radius r: $I = m \cdot r^2$
- Rectangle rotating around an axis along one edge: $I = \frac{1}{3}\, m \cdot r^2$
- Rectangle rotating around an axis parallel to one edge and passing through the center: $I = \frac{1}{12} \cdot m \cdot r^2$
- Rod rotating around an axis perpendicular to it and through its center: $I = \frac{1}{12} \cdot m \cdot r^2$
- Rod rotating around an axis perpendicular to it and through one end: $I = \frac{1}{3}\, m \cdot r^2$
- Solid cylinder: $I = \frac{1}{2} \cdot m \cdot r^2$
- Solid sphere: $I = \frac{2}{5}\, m \cdot r^2$

Q. A solid cylinder with a mass of 5.0 kg is rolling down a ramp. If it has a radius of 10 cm and an angular acceleration of 3.0 radians/sec^2, what torque is operating on it?

A. The correct answer is 0.075 N-m.

1. Use the equation $\tau = I \cdot \alpha$.
2. In this case, $I = \frac{1}{2}\, m \cdot r^2$.
3. Plug in the numbers:

$$\tau = \frac{1}{2} \cdot m \cdot r^2 \cdot \alpha = \frac{1}{2} \cdot (5.0) \cdot (0.1^2) \cdot (3.0) = 0.075 \text{ N-m}$$

5. You're spinning a 5.0 kg ball with a radius of 0.5 m. If it's accelerating at 4.0 radians/sec^2, what torque are you applying?

Solve It

6. A tire with a radius of 0.50 m and mass of 1.0 kg is rolling down a street. If it's accelerating with an angular acceleration of 10.0 radians/sec^2, what torque is operating on it?

Solve It

7. You're spinning a hollow sphere with a mass of 10.0 kg and radius of 1.0 m. If it has an angular acceleration of 15 radians/sec^2, what torque are you applying?

Solve It

8. You're throwing a 300.0 g flying disc with a radius of 10 cm, accelerating it with an angular acceleration of 20.0 radians/sec^2. What torque are you applying?

Solve It

9. If you're spinning a 2.0 kg solid ball with a radius of 0.5 m, starting from rest and applying a 6.0 N-m torque, what is its angular speed after 60.0 sec?

Solve It

10. If you're spinning a 2.0 kg hollow ball with a radius of 0.50 m, starting from rest and applying a 12.0 N-m torque, what is its angular speed after 10.0 sec?

Solve It

What if you apply some force to the edge of a tire to try to get a car moving, and you apply a force of 500 N, what work do you do over 1.0 m of travel? That work looks like this equation, where s is the distance the force was applied over:

$$W = F \cdot s$$

You can also apply force rotationally. In the case of you applying force to the edge of a tire to get a car moving, the distance s equals the radius multiplied by the angle through which the wheel turns, $s = r \cdot \theta$, so you get this equation:

$$W = Fs = F \cdot r \cdot \theta$$

But the torque, τ, equals $F \cdot r$ in this case. So you're left with this:

$$W = F \cdot s = F \cdot r \cdot \theta = \tau \cdot \theta$$

Talk about a cool result — work equals torque multiplied by the angle through which that torque is applied.

Q. If you apply a torque of 500.0 N-m to a tire and turn it through an angle of 2π radians, what work have you done?

A. The correct answer is 3140 J.

1. Use the equation $W = \tau \cdot \theta$.
2. Plug in the numbers:

$$W = \tau \cdot \theta = (500) \cdot (2\pi) = 3140 \text{ J}$$

11. How much work do you do if you apply a torque of 6.0 N-m over an angle of 200 radians?

Solve It

12. You've done 20.0 J of work turning a steering wheel. If you're applying 10.0 N-m of torque, what angle have you turned the steering wheel through?

Solve It

13. How much work do you do if you apply a torque of 75 N-m through an angle of 6π radians?

Solve It

14. You've done 350 J of work turning a bicycle tire. If you're applying 150 N-m of torque, what angle have you turned the wheel through?

Solve It

Kinetic Energy linear:

$$KE = \frac{1}{2} mv^2$$

Convert that equation to its angular analog:

$$KE = \frac{1}{2} I \cdot \omega^2$$

In other words, the first equation becomes the second when you're going rotational.

Q. You have a 100 kg solid sphere with a radius equal to 1.0 m. If it's rotating at ω = 10.0 radians/sec, what is its rotational kinetic energy?

A. The correct answer is 2000 J.

1. Use this equation:

$$KE = \frac{1}{2} I \cdot \omega^2$$

2. For a solid sphere, $I = \frac{2}{5} \cdot m \cdot r^2$.

3. Plug in the numbers:

$$KE = \frac{1}{2} I \cdot \omega^2 = \frac{1}{2}\frac{2}{5} \cdot m\ r^2\ \omega^2 =$$

$$(0.5) \cdot (0.4) \cdot (100) \cdot (1.0^2) \cdot (10^2) = 2000\ J$$

15. How much rotational kinetic energy does a spinning tire of mass 10.0 kg and radius 0.50 m have if it's spinning at 40.0 rotations/sec?

Solve It

16. How much rotational kinetic energy does a spinning tire of mass 12 kg and radius 0.80 m have if it's spinning at 200.0 radians/sec?

Solve It

17. How much work do you do to spin a tire, which has a mass of 5.0 kg and a radius of 0.40 m, from 0.0 radians/sec to 100.0 radians/sec?

Solve It

18. How much work do you do to spin a hollow sphere, which has a mass of 10.0 kg and a radius of 0.50 m, from 0.0 radians/sec to 200.0 radians/sec?

Solve It

Q. If a solid sphere is at the top of a 3.0 m-high ramp, what is its speed when it reaches the bottom of the ramp?

A. The correct answer is 6.5 m/sec.

1. Use this equation:

$$v = \sqrt{\frac{2mgh}{m + I/r^2}}$$

2. For a solid sphere, $I = \frac{2}{5} \cdot m \cdot r^2$. That means that v is equal to this:

$$v = \sqrt{\frac{2mgh}{m + (2/5)\,m}}$$

3. That breaks down to:

$$v = \sqrt{\frac{2gh}{1 + (2/5)}}$$

4. Plug in the numbers:

$$v = 6.5 \text{ m/sec}$$

19. If a hollow cylinder is at the top of a 4.0 m-high ramp, what is its speed when it reaches the bottom of the ramp?

Solve It

20. If a solid cylinder is at the top of a 2.0 m-high ramp, what is its speed when it reaches the bottom of the ramp?

Solve It

21. A tire is rolling down a ramp, starting at a height of 3.5 m. What is its speed when it reaches the bottom of the ramp?

Solve It

22. A basketball (that is, a hollow sphere) is rolling down a ramp, starting at a height of 4.8 m. What is its speed when it reaches the bottom of the ramp?

Solve It

In linear motion, momentum looks like this:

$$\mathbf{p} = m\mathbf{v}$$

Momentum is conserved in collisions. In addition to linear momentum, you can have angular momentum, which is represented by the symbol L. Following is the equation for angular momentum; note that this is a vector equation and that **L** points in the same direction as **ω**, the object's angular velocity:

$$\mathbf{L} = I \cdot \boldsymbol{\omega}$$

In physics, angular momentum is conserved. For example, if you have a skater spinning around and then she spreads her arms (giving her a different moment of inertia), because angular momentum is conserved, you get this:

$$I_1 \cdot \omega_1 = I_2 \cdot \omega_2$$

With this equation, if you know the skater's original and final moments of inertia and her original angular speed, you can calculate her final angular speed like this:

$$\omega_2 = \frac{I_1}{I_2} \cdot \omega_1$$

Q. If a 500.0 kg merry-go-round with a radius of 2.0 m is spinning at 2.0 radians/sec and a boy with a mass of 40.0 kg jumps on the outer rim, what is the new angular speed of the merry-go-round?

A. The correct answer is 1.7 radians/sec.

1. Use this equation:

$$I_1 \cdot \omega_1 = I_2 \cdot \omega_{2w}$$

2. For a solid disc like the merry-go-round, $I = ½\, m \cdot r^2$.

3. When the boy jumps on, he adds $m_b \cdot r^2$ to I, where m_b is the mass of the boy. This means that:

$$(½ \cdot m \cdot r^2) \cdot \omega_1 = (½ \cdot m \cdot r^2 + m_b \cdot r^2) \cdot \omega_2$$
$$(½ \cdot m \cdot r^2) \cdot \omega_1 = 2000 \text{ kg-m}^2$$

4. Solve for ω_2:

$$\omega_2 = \frac{\frac{1}{2} \cdot m \cdot r^2 \cdot \omega_1}{\left(\frac{1}{2} \cdot m \cdot r^2 + m_b \cdot r^2\right)}$$

where $½ \cdot m \cdot r^2 = 1000 \text{ kg} \cdot \text{m}^2$ and $m_b \cdot r^2 = 160 \text{ kg} \cdot \text{m}^2$.

5. Plug in the numbers:

$$\omega_2 = \frac{2000}{(1000 + 160)} = 1.7 \text{ radians/sec}$$

23. A merry-go-round with a mass of 500.0 kg and radius of 2.0 m is rotating at 3.0 radians/sec when two children with a combined mass of 70.0 kg jump on the outer rim. What is the new angular speed of the merry-go-round?

Solve It

24. A 2000.0 kg space station, which is a hollow cylinder with a radius of 2.0 m, is rotating at 1.0 radians/sec when an astronaut with a mass of 80.0 kg lands on the outside of the station. What is the station's new angular speed?

Solve It