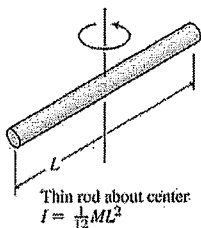
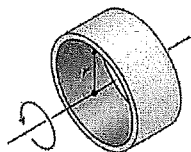


# M2 Rotational Dynamics

Point Mass :  $I = mr^2$

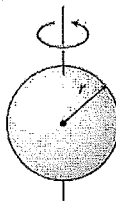


Thin rod about center  
 $I = \frac{1}{12}ML^2$

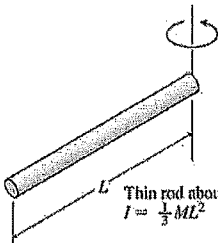
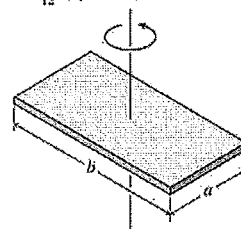


Thin ring or hollow cylinder about its axis  
 $I = MR^2$

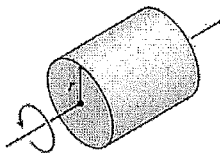
Solid sphere about diameter  
 $I = \frac{2}{5}MR^2$



Flat plate about perpendicular axis  
 $I = \frac{1}{12}M(a^2 + b^2)$

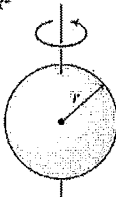


Thin rod about end  
 $I = \frac{1}{3}ML^2$

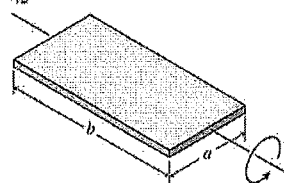


Disk or solid cylinder about its axis  
 $I = \frac{1}{2}MR^2$

Hollow spherical shell about diameter  
 $I = \frac{2}{3}MR^2$

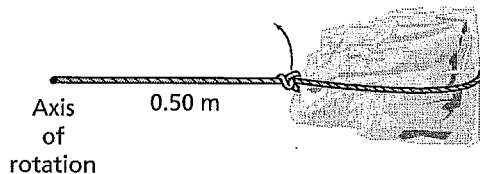


Flat plate about central axis  
 $I = \frac{1}{12}Ma^2$



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A 0.50-m length of string with a 0.20-kg weight at one end is rotated around the other end, as shown in the diagram.



Assuming the mass of the string is zero and the weight is a point mass, what is the moment of inertia of the weight?

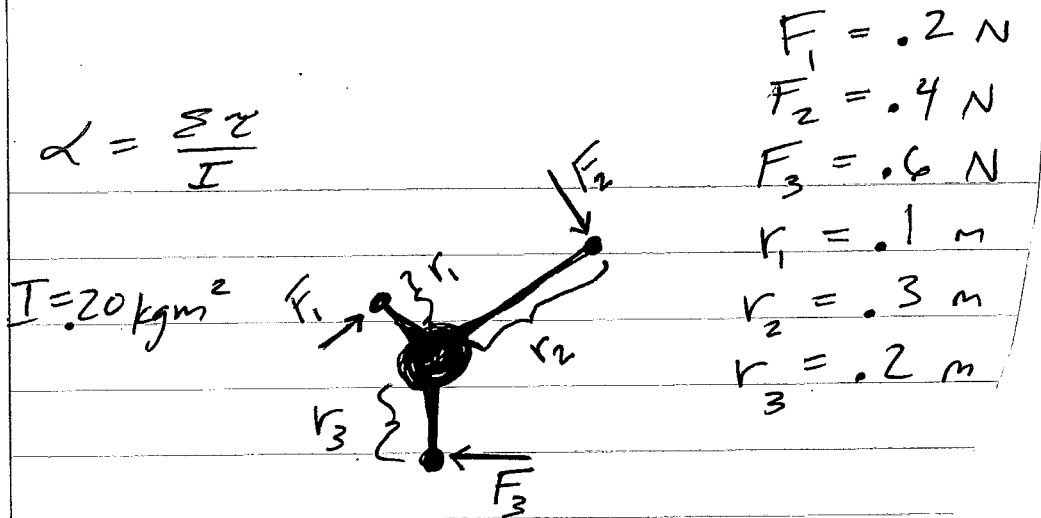
- A 0.020 kg·m<sup>2</sup>
- B 0.050 kg·m<sup>2</sup>
- C 0.10 kg·m<sup>2</sup>
- D 0.50 kg·m<sup>2</sup>

A solid sphere with a radius of 0.050 m and a mass of 3.0 kg has a rod through its center. A net torque of  $6.0 \times 10^{-4}$  kg·m<sup>2</sup> is used to spin the sphere about the rod. What is the sphere's angular acceleration? *Hint: The moment of inertia for the sphere is  $\frac{2}{5}mr^2$ .*

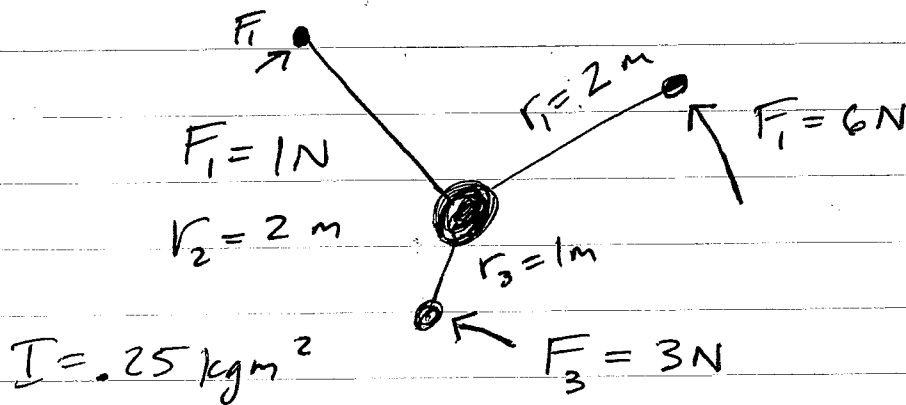
- A 0.20 rad/s<sup>2</sup>
- B 0.21 rad/s<sup>2</sup>
- C 2.5 rad/s<sup>2</sup>
- D 5.0 rad/s<sup>2</sup>

$$\sum \tau = I \alpha$$

$$\sum \tau = r_{\perp} \cdot F$$

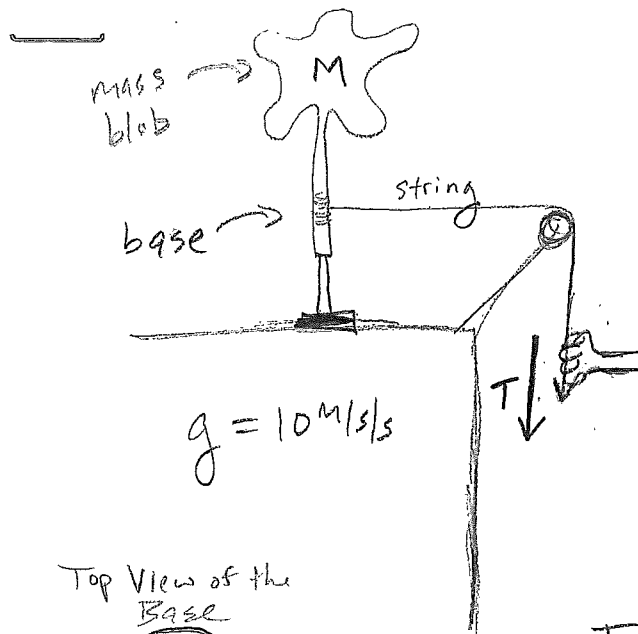


- A.) An object with moment of inertia is acted upon by three torques.
1. What is the  $\sum \tau$ ?
  2. What is  $\alpha$ ?



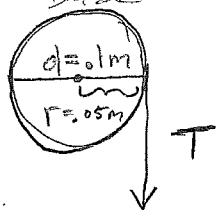
- B.)
1. What is the  $\sum \tau$ ?
  2. What is the  $\alpha$ ?

1. A 75 kg grinding disc ( $I = \frac{1}{2}mr^2$ ) has radius  $r = .28 \text{ m}$ . A force of 180 N is applied tangentially counter clockwise to the disk. What is the torque?
2. What is the resulting  $\alpha$ ?
3. If an additional friction force is applied tangentially at .015 m from the axis of rotation with magnitude 20 N at the same instant the 180 N force is applied, what is the new resulting  $\alpha$ ?

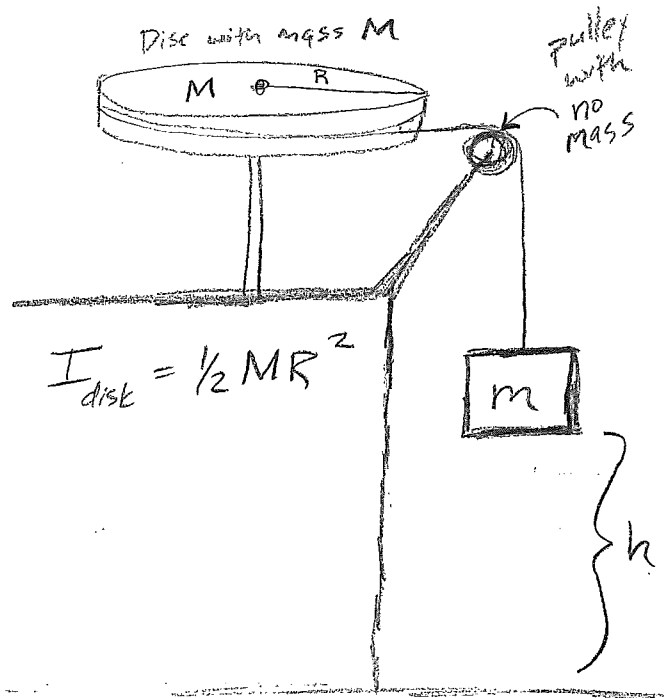


The mass blob has mass  $M$ .  
 The diameter of its base on which a string is wrapped is .1 meters.  
 The string is connected to a hand pulling with tension  $40 \text{ N}$  through a massless pulley system.

Top View of the Base



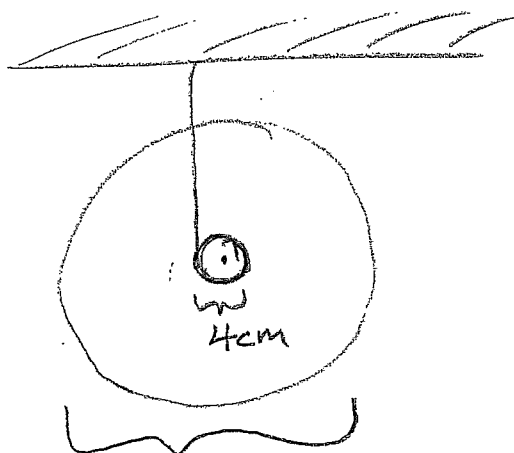
If the angular acceleration of the blob is  $3 \text{ rad/s}^2$ , what is the blob's moment of inertia?



1. Determine the literal equation for the translational acceleration of the hanging mass ' $m$ ' in terms of  $M$ ,  $m$  and  $g$ .

2. Assuming that  $h = 4$  meters,  $m = 7$  kg,  $M = 15$  kg,  $v_i = 0$  and  $g = 20$  m/s<sup>2</sup>, what is the time required for ' $m$ ' to hit the ground?

A large yoyo of mass 80 grams and moment of inertia  $5 \times 10^{-5} \text{ kgm}^2$



16 cm

$$g = 10 \text{ m/s}^2$$

$$a = r\alpha$$

1. Draw a free body diagram for the yoyo.

2. Write the literal net force equation for the vertical direction.  
Solve for tension.

3. Using the two torque equations:

$$\sum \tau = r_{\perp} F \quad \& \quad \sum \tau = I\alpha$$

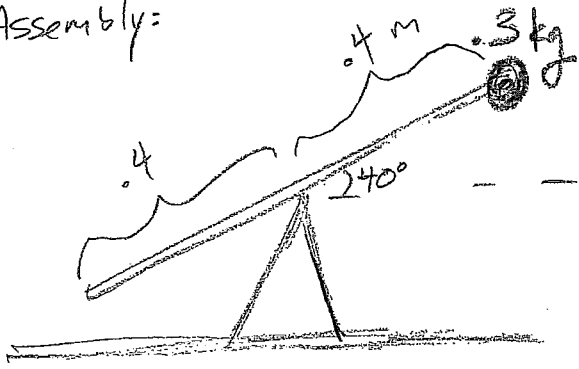
to create a literal equation for tension.

4. Combine the equations you created in #2 and #3 and solve for 'a'.

5. Solve for tension in the string.

6. Solve for  $\alpha$  of the yoyo.

Assembly:



A uniform rod .8 m long with mass of .5 kg is pivoted at its midpoint. A .3 kg point mass is placed on the right side as shown: and the rod is tilted  $40^\circ$  from the horizontal.

$$I_{rod} = \frac{1}{12} m L^2$$

$$I_{point\ mass} = m r^2 \quad g = 10 \text{ m/s}^2$$

1. Find  $\alpha$  of the assembly
2. Find  $a$  of the .3 kg mass.

- \* Hints:
1. Use the two net torque equations; set them equal to one another.  $\sum \tau = I \alpha \quad \sum \tau = r \cdot F \cdot \sin \theta$
  2. To find  $I_{total}$  sum  $I_{rod} + I_{point\ mass}$ .
  3. The torque from the rod only is at the pivot point; the torque from the .3 kg force/gravity is at the rod's end.

An external torque is applied to an object with a narrow base. Which location for the center of mass would make the object less likely to tip over?

- A just above the base
- B high above the base
- C just outside the base
- D far outside the base