

N1

Hooke's law is represented by this equation:

$$\mathbf{F} = -k \cdot \Delta \mathbf{x}$$

This deceptively simple equation is at the heart of explaining the motion of objects on springs. It says that the force on an object in simple harmonic motion is proportional to the displacement (that's  $\Delta \mathbf{x}$ ) from rest.

The constant of proportionality is  $k$  (Hooke's constant, also called a spring constant), which must be measured for every situation (because no two springs are exactly identical, for example). The negative sign in the equation indicates that  $k$  is a restoring force — that is, that the force points toward the equilibrium position of the object.

What are the units of spring constants? Just check out the equation: Because  $\mathbf{F} = -k \cdot \Delta \mathbf{x}$ ,  $k$  must have the unit N/m.

You'll commonly see Hooke's law applied to springs. Don't get confused by the minus sign in this equation; it's just there to indicate that the force opposes the displacement, which you know is true about springs.

If it's easier for you to understand, put in the minus sign after you've done the rest of the problem. You can easily figure out which way the force is going, and if it's in a positive direction as defined by the axes in the problem, the force is positive.

**Q.** You're stretching a spring with spring constant 5.0 N/m. If you stretch it 2.0 m, what pull do you feel from the spring?

**A.** The correct answer is  $-10.0$  N.

1. Use the equation  $F = -k \cdot \Delta x$ .
2. Plug in the numbers:

$$F = -k \cdot \Delta x = -(5.0) \cdot (2.0) = -10.0 \text{ N}$$

**1.** You have a spring whose spring constant is 200 N/m, and you want to stretch it by 6.0 m. What force do you need to apply?

*Solve It*

**2.** You have a spring of spring constant 50.0 N/m. What force do you need to stretch it by 5.3 m?

*Solve It*

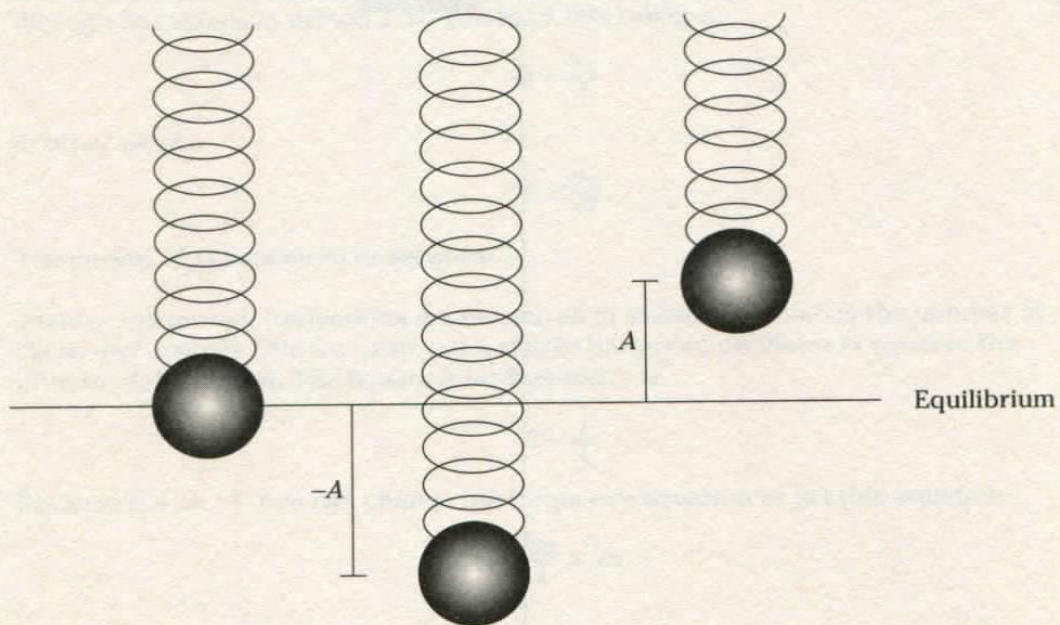
3. You have a spring of spring constant  $73 \text{ N/m}$ . What force do you need to stretch it by  $15.0 \text{ m}$ ?

**Solve It**

4. It takes  $200.0 \text{ N}$  to stretch a spring  $2.0 \text{ m}$ . What is its spring constant?

**Solve It**

Take a look at the spring in Figure 11-1. It starts at rest, drops down to distance  $A$ , and then moves back up to distance  $A$ .



**Figure 11-1:**  
A weight on  
a spring.



How do you describe this motion? In terms of the distance  $A$ , called the *amplitude* of the spring's motion, of course:

$$y = A \cos (\omega \cdot t)$$

In this equation for displacement,  $t$  is time, and  $\omega$  is the *angular speed*. (It's called angular speed here for a variety of reasons; it turns out that simple harmonic motion is only one component — hence the cosine — of full circular motion.)

That equation assumes that you start at full extension,  $t = 0$ . If you start at a different time,  $t_0$ , you can adjust the equation to match the spring's motion like this:

$$y = A \cos [\omega(t - t_0)]$$

- Q. A weight on a spring is making that spring oscillate up and down. If the amplitude of the motion is 1.0 m and the angular speed is 1.0 radians/sec, where will the oscillating mass be after 10.0 sec?

- A. The correct answer is  $-0.84$  m.  
 1. Use the equation  $y = A \cos (\omega \cdot t)$ .  
 2. Plug in the numbers:

$$y = A \cos (\omega \cdot t) = (1.0) \cos [(1.0) \cdot (10)] = -0.84 \text{ m}$$

5. A spring with a weight on it has an amplitude of motion of 2.5 m and an angular speed of 2.0 radians/sec. Where will the weight be after 60.0 sec?

**Solve It**

6. A spring with a weight pulling it down has an amplitude of motion of 5.0 m and an angular speed of 16.0 radians/sec. Where will the weight be after 60.0 sec?

**Solve It**

7. A spring is at  $-3.0$  m at time 60.0 sec and has an angular speed of 6.0 radians/sec. What is its amplitude?

**Solve It**

8. A spring is at  $-4.5$  m at time 10.0 sec and has an angular speed of 16.0 radians/sec. What is its amplitude?

**Solve It**

A weight on the end of a spring bounces up and down periodically. The time it takes to bounce up and down, completing a full cycle and coming back to where it started, is called its *period*, represented by symbol  $T$ . Because  $y = A \cos(\omega \cdot t)$ , the object goes through  $2\pi$  radians in period  $T$ , so you have this relation:

$$\omega = \frac{2\pi}{T}$$

In other words:

$$T = \frac{2\pi}{\omega}$$

The period,  $T$ , is measured in seconds.

Besides the period, oscillations are measured in *frequency*, which is the number of cycles per second. The frequency of a simple harmonic oscillator is equal to the inverse of the period. The equation for frequency is

$$f = \frac{1}{T}$$

Because  $\omega = 2\pi / T$ , you can change the frequency equation to get this equation:

$$\omega = \frac{2\pi}{T} = 2\pi \cdot f$$

Frequency is measured in Hertz, abbreviation Hz, which is one cycle per second.

It's worth noting that you can call  $\omega$  angular speed, but when you're working with simple harmonic motion, it's usually called angular frequency.

**Q.** A weight on a spring is bouncing up and down with an angular frequency of 12.0 radians/sec. How long does it take to complete each cycle?

**A.** The correct answer is 0.52 sec.

1. Use this equation:

$$T = \frac{2\pi}{\omega}$$

2. Plug in the numbers:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{12.0} = 0.52$$

**9.** A spring has a weight with an angular frequency of 4.5 radians/sec. What is the weight's period?

**Solve It**

**10.** A spring has a weight pulling it down, and the angular frequency of oscillation is 1.5 radians/sec. What is the weight's period?

**Solve It**



- 11.** A spring with a weight on it has an angular frequency of 0.70 radians/sec. What is its frequency?

**Solve It**

- 12.** A spring with a weight on it is bouncing up and down with an angular frequency of 1.3 radians/sec. What is its frequency?

**Solve It**

As I explain in the earlier section "Simply Simple Harmonic Motion," the displacement of an object on a spring looks like this:

$$y = A \cos (\omega \cdot t)$$

It turns out that you also can express the object's velocity; here's that equation:

$$v_y = -A \cdot \omega \sin \theta = -A \cdot \omega \sin (\omega \cdot t)$$

This equation assumes that you start at full extension,  $t = 0$ , which means that the initial velocity of the object is zero. If you want to adjust this equation to start at some other time,  $t_0$ , you alter the equation like so:

$$v_y = -A \cdot \omega \sin [\omega (t - t_0)]$$

When the object is at its fullest extension, the velocity is zero, and when it's swooping back through the equilibrium point, its velocity is at its maximum.

- Q.** A weight on a spring is bouncing up and down with an angular frequency of 3.4 radians/sec and an amplitude of 1.4 m. What is its speed at  $t = 5.0$  sec?

- A.** The correct answer is 4.6 m/sec.

1. Use this equation:

$$v_y = -A \cdot \omega \sin (\omega \cdot t)$$

2. Plug in the numbers:

$$\begin{aligned} v_y &= -A \cdot \omega \sin (\omega \cdot t) = \\ &= -(1.4) \cdot (3.4) \cdot \sin [(3.4) \cdot (5.0) \cdot (180 / \pi)] = \\ &= 4.6 \text{ m/sec} \end{aligned}$$

- 13.** If you have a spring with a weight on it, and the weight's angular frequency is 1.7 radians/sec and amplitude is 5.6 m, what is the weight's velocity at 30.0 sec?

*Solve It*

- 14.** A spring with a weight on it is bouncing up and down with an angular frequency of 2.7 radians/sec and amplitude of 1.7 m. What is its velocity at 10.0 sec?

*Solve It*

In addition to calculating the displacement and velocity of an object on a spring, you can calculate its acceleration.

Displacement goes from  $-A$  to  $A$ , where  $A$  is the amplitude. And the velocity of an object on a spring goes from  $-A\omega$  to  $A\omega$ ; the velocity is at its minimum (zero) at either end of its maximum extension, and it's at a maximum at the equilibrium point (where it would be at rest). The acceleration varies from  $-A\omega^2$  to  $A\omega^2$ , and it's at its maximum when the object is speeding up (or down!) the most, which is at the ends of its oscillations (when the velocity is zero). The acceleration of the object is zero when the object is passing through the equilibrium position because the net force on the object is zero at the equilibrium position.

Here's the equation for the acceleration of an object on a spring:

$$a = -A\omega^2 \cdot \cos \theta = -A\omega^2 \cdot \cos (\omega \cdot t)$$

- Q.** A weight on a spring is bouncing up and down with an angular frequency of 3.4 radians/sec and an amplitude of 1.4 m. What is its acceleration at 5.0 sec?

- A.** The correct answer is 4.6 m/sec.

1. Use this equation:

$$a = -A\omega^2 \cdot \cos (\omega \cdot t)$$

2. Plug in the numbers:

$$\begin{aligned} a &= -A\omega^2 \cdot \cos (\omega \cdot t) = \\ &= -(1.4) \cdot (3.4^2) \cdot \cos ((3.4) \cdot (5.0) \cdot (180 / \pi)) = \\ &= 4.3 \text{ m/sec}^2 \end{aligned}$$

15. A spring has a weight on it. If the weight's angular frequency is 1.3 radians/sec and its amplitude is 1.0 m, what is the weight's acceleration at  $t = 4.9$  sec?

*Solve It*

16. You have a spring with a weight pulling it down; the angular frequency is 1.7 radians/sec and the amplitude is 6.0 m. What is the weight's acceleration at 60 sec?

*Solve It*

17. A spring with a weight on it has an angular frequency of 3.7 radians/sec and an amplitude of 1.4 m. What is the weight's acceleration at 15 sec?

*Solve It*

18. A spring with a weight on it is moving up and down with an angular frequency of 1.3 radians/sec and an amplitude of 2.9 m. What is its acceleration at 9.0 sec?

*Solve It*



What is the period of a spring in terms of its spring constant,  $k$ ? You'll often come across that question in physics problems. You know that:

$$F = -k \cdot \Delta x$$

You also know that  $F = m a$ , so:

$$F = m \cdot a = -k \cdot \Delta x$$

For simple harmonic motion, you know that:

$$x = A \cos(\omega \cdot t)$$

$$a = -A \omega^2 \cdot \cos(\omega \cdot t)$$

Putting this all together gives you:

$$m \cdot a = -m \cdot A \omega^2 \cdot \cos(\omega \cdot t) = -k \cdot \Delta x = -k \cdot A \cos(\omega \cdot t)$$

That lengthy equation becomes

$$m \omega^2 = k$$

Which, if you solve for  $\omega$ , gives you the angular frequency of an object on a spring:

$$\omega = \sqrt{\frac{k}{m}}$$

**Q.** A weight on a spring is bouncing up and down. The spring constant is 1.6 N/m, the mass is 1.0 kg, and the amplitude is 3.0 m. What equation describes the spring's motion?

**A.** The correct answer is  $y = A \cos(\omega \cdot t) = 3.0 \cos(1.3 \cdot t)$ .

1. Use this equation:

$$\omega = \sqrt{\frac{k}{m}}$$

2. So this equation describes the motion:

$$y = A \cos(\omega \cdot t) = 3.0 \cos(1.3 \cdot t)$$



- 19.** If you have a spring with a mass of 1.0 kg on it and a spring constant of 12.0, what is the spring's period of oscillation?

*Solve It*

- 20.** What is the period of oscillation of a spring with a mass of 300 g on it and a spring constant of 7.0?

*Solve It*

- 21.** A weight on a spring is moving up and down. The spring constant is 1.9 N/m, the mass is 1.0 kg, and the amplitude is 2.4 m. What equation describes the spring's motion?

*Solve It*

- 22.** A weight on a spring is oscillating up and down. The spring constant is 2.3 N/m, the mass is 1.3 kg, and the amplitude is 5.6 m. What equation describes the spring's motion?

*Solve It*

When you stretch a spring, you create potential energy (just like when you lift a weight against the force of gravity). For a spring of spring constant  $k$  stretched a distance  $x$  from equilibrium, the potential energy in the spring is

$$PE = \frac{1}{2} k x^2$$

**Q.** What is the potential energy in a spring of spring constant 40 N/m that's stretched 5.0 m from equilibrium?

**A.** The correct answer is 500 J.

1. Use this equation:

$$PE = \frac{1}{2} k x^2$$

2. Plug in the numbers:

$$PE = \frac{1}{2} k x^2 = 500 \text{ J}$$

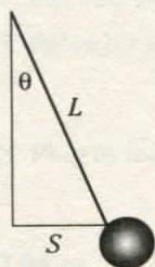
**23.** If you stretch a spring of spring constant 100 N/m by 5.0 m, what potential energy is in the spring?

**Solve It**

**24.** If you stretch a spring of spring constant 250 N/m by 6.5 m, what potential energy is in the spring?

**Solve It**

In addition to springs, physics problems about simple harmonic motion may ask you to deal with pendulums like the one in Figure 11-2.



**Figure 11-2:**  
A pendulum.

You can calculate the angular frequency of a pendulum of length  $L$  using the following equation, where  $g$  is the acceleration due to gravity:

$$\omega = \sqrt{\frac{g}{L}}$$

**Q.** What's the angular frequency of a pendulum of length 1.0 m?

**A.** The correct answer is 4.6 m/sec.

Use this equation and plug in the numbers:

$$\omega = \sqrt{\frac{g}{L}} = 3.1 \text{ radians/sec}$$



25. What's the period of a pendulum with a length of 1.5 m?

**Solve It**

26. What's the period of a pendulum with a length of 3.0 m?

**Solve It**

27. If you have a pendulum with a length of 1.0 m with a weight on the end of it, what equation describes the  $x$  position of that weight if the amplitude of motion is 0.5 m?

**Solve It**

28. You have a pendulum with a length of 1.5 m with a weight on the end of it. What equation describes the  $x$  position of that weight if the amplitude of motion is 0.75 m?

**Solve It**

