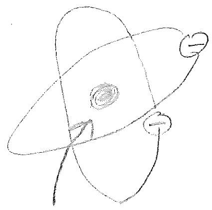
Nuclear Structure



Nucleus : Protons + Nuetrons (Nucleons)

$$^{A}_{Z}X$$

Q : net charge in Coulombs

A : mass # : # of nucleons

Z : atomic # : # of protons

X : Element symbol

$N = A - Z$: # of nuetrons

q : # of elementary charges in the net charge and can be + or −.

When $Q = 0$ there are Z electrons.

When $Q = qe$ there are $Z - q$ electrons.

Hydrogen isotopes : $^{1}_{1}H^{+}$ $^{2}_{1}H^{+}$ $^{3}_{1}H^{+}$

Isotopes have the same Z but a different A.

Nuclear Mass, Unified Atomic Mass Unit

$$\text{Mass}_{proton} = 1.67262171 \times 10^{-27} \text{ kg} = 1.00727646688 \text{ u}$$

$$\text{mass}_{nuetron} = 1.67492728 \times 10^{-27} \text{ kg} = 1.00866491560 \text{ u}$$

$$\text{mass}_{electron} = 9.10938356 \times 10^{-31} \text{ kg} = 0.000548799 0945 \text{ u}$$

$$1 \text{ u} = 1.66053886 \times 10^{-27} \text{ kg}$$

Electrons do not contribute much to the mass of an atom

Mass Defect, Binding Energy, & $E = (\Delta m)c^2$

The mass of a nucleus is less than the sum of the individual masses of the constituent protons and neutrons. This difference in the mass is known as the mass defect. This mass defect is equal to the energy that was released when the nucleus is put together.

The amount of energy released is known as the nuclear binding energy.

4 Fundamental Forces of Nature

- Gravitational

- Electromagnetic

- Strong Nuclear

- Weak Nuclear

Binding energy or bond energy : energy required to break a bond

which side has more mass, the whole atom or the separate composit particles?

$\underbrace{{}^{4}_{2}He \text{ atom}}$ + nuclear binding energy $\rightarrow \underbrace{2n^0 + 2p^+ + 2e^-}$

The mass difference or mass defect $= \Delta m$

The total binding energy $= \Delta m \cdot c^2$

$$\frac{\Delta m \cdot c^2}{A} = \text{average binding energy per nucleon}$$

The higher the binding energy per nucleon, the more energy required per nucleon to break the nucleus apart.

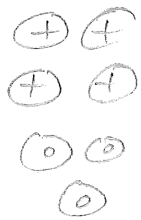
The higher the binding energy the more stable the atom.

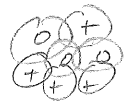
$E = mc^2$ $\boxed{1u = 931 \frac{MeV}{c^2}}$ $\frac{E}{c^2} = \frac{mc^2}{c^2}$

$m = \frac{\text{energy}}{\text{light speed}^2}$ $m = \frac{E}{c^2}$

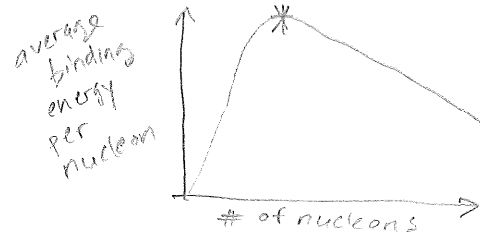
Mass Defect Binding Energy

$M_{before} > M_{after}$





Fe^{56} has the greatest average binding energy per nucleon.



Determine the mass defect and average binding energy per nucleon for an iron-56 nucleus.

$^{56}_{26}Fe$ Carry decimals to the 5th decimal place!

Determine the mass defect and average binding energy per nucleon for beryllium-7.

$^{7}_{4}Be$

Find the binding energy and the binding energy per nucleon for a $^{14}_{7}N$ nucleus.

$$^{14}_{7}N \text{ atom} + (\Delta m)c^2 \longrightarrow 7n^{\circ} + 7p^{+} + 7e^{-}$$

Radioactivity

The decay of an unstable nucleus.
It occurs spontaneously without external stimulus.

Nuclear Reaction

Not spontaneous. Occurs when a nucleus is struck by another nucleus or by a simpler particle such as γ-rays or n^0.

Radioactivity and nuclear reactions follow the conservation laws of momentum, electric charge, mass-energy, angular momentum, and number of nucleons.

Nuclear Reactions include Fission and Fusion

Fission was first observed in 1938 by two German Scientists.
Large nuclei are unstable and split, like large water droplets split into two droplets as it falls in the air.
Small nuclei are unstable just like large nuclei.
Fusion combines small nuclei to form a more stable nucleus.
Fusion releases more energy per nucleon compared to fission.
Our sun and the stars use fusion to produce energy.

Both fission and fusion release energy.

Fission and Fusion

- Nuclear reactions involving protons and nuetons.
- Both reactions results in less overall mass.

Fission

- Nucleus of an atom splits into smaller nuclei
- Often produces free neutrons
- Exothermic process
- Nuclear power and weapons

Fusion

- nuclei join together to form a heavier nucleus
- release or absorption of energy
- requires lots of energy
- occurs naturally is stars.
- hydrogen bombs

Fission Example:

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n$$

Fusion Example:

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H$$

Strong Nuclear Force becomes relevant at distances less than 1×10^{-15} meters.

In a nuclear fission reaction uranium-235 can be split into krypton-92 and barium-141. Using the average binding energy per nucleon determine:

a) the energy released per reaction

b) the number of joules released from 1 gram of uranium-235

	binding energy per nucleon		
^{235}U	7.59 MeV		
^{141}Ba	8.33 MeV		
^{92}Kr	8.51 MeV		

Nuclear Fusion

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{1}H + ^{1}_{1}H$$

Find the mass defect in u and Kg.

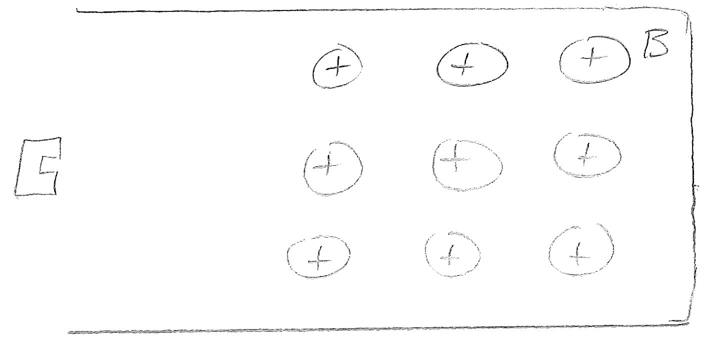
Find the binding energy in eV and J.

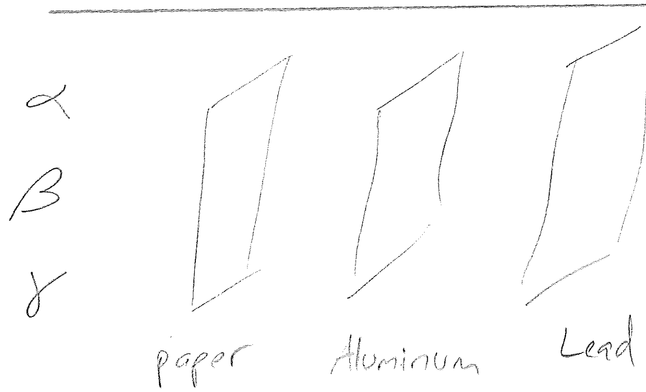
Chemical Reaction V. Nuclear Reaction:

Adding one electron to a hydrogen requires 13.6eV.

How does this energy compare to the binding energy?

Radioactivity





Alpha (α) Decay
- emits alpha particle
- Helium nucleus ${}^{4}_{2}He$ emitted
- positive charge (2+)
- mass numbers goes down by 4
- atomic number goes down by 2
- Transmutation means changing from one element to another.

Example: ${}^{238}_{92}U \rightarrow {}^{4}_{2}He + {}^{234}_{90}Th$

Complete the following: ${}^{212}_{84}Po \rightarrow {}^{4}_{2}He + \quad Pb$

I.) Beta (β) Decay : β^- [Atomic # increases by 1; mass # constant]
- Nucleus has too many neutrons
- Nuetron transforms into a proton emitting an electron e^- (β particle) and an anti neutrino ($\bar{\nu}_e$)

$$n \rightarrow p + {}^{0}_{-1}e^- + \bar{\nu}_e$$

β^- Decay Example: ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + e^- + \bar{\nu}_e$

Complete the following: $Pb \rightarrow {}^{209}_{83}Bi + e^- + \bar{\nu}_e$

II.) Beta (β) Decay : β^+ [Atomic # decreases by 1; mass # constant]
- Nucleus has too many protons
- Proton transforms into a neutron emitting a positron e^+ (β particle) and a neutrino (ν_e) $\quad p \rightarrow n + {}^{0}_{1}e^+ + \nu_e$

β^+ Decay Example: ${}^{19}_{10}Ne \rightarrow {}^{19}_{9}F + {}^{0}_{1}e^+ + \nu_e$

Complete the following: $Mg \rightarrow {}^{23}_{11}Na + {}^{0}_{1}e^+ + \nu_e$

III.) Beta (β) Decay : Electron Capture $\quad {}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}Y$

Gamma Decay (γ) Not a particle!

- Emits high energy photons
- No charge
- No change in the element.

Example: $^{60}_{28}Ni \rightarrow ^{60}_{28}Ni + \gamma$

- Two Gamma rays are emitted at once in order to conserve momentum.

Complete the following and indicate the type of decay.

$^{210}_{84}Po \rightarrow He + Pb$

$^{229}Th \rightarrow He + _{88}Ra$

$^{14}C \rightarrow N + ^{0}_{-1}e^{-} + \bar{\nu}_e$

$Ac \rightarrow ^{228}_{90}Th + ^{0}_{-1}e^{-} + \bar{\nu}_e$

$^{19}_{10}Ne \rightarrow F + ^{0}_{1}e^{+} + \nu_e$

$Mg \rightarrow ^{23}_{11}Na + ^{0}_{1}e^{+} + \nu_e$

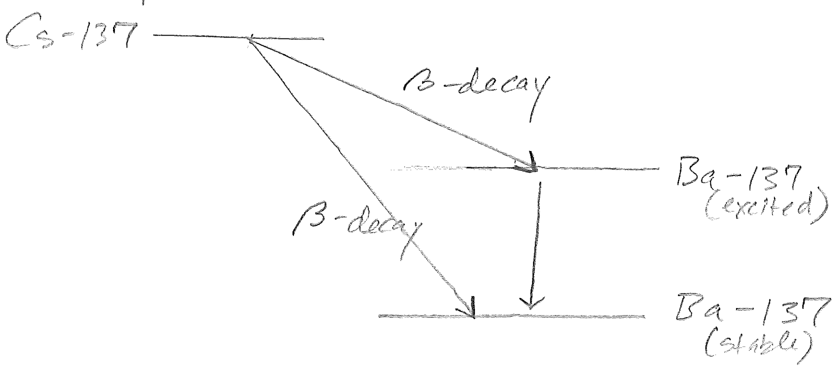
$^{60}_{28}Ni \rightarrow Ni + \gamma$

$Co \rightarrow ^{60}_{27}Co + \gamma$

$^{234}_{91}Pa \rightarrow U + \quad +$

$229 \quad \rightarrow \quad + ^{225}Ra$

Decay Scheme Cesium - 137



Cs-137 136.90709 u

Ba-137 136.90584 u

Find the mass defect.

Find the binding energy in eV.

Radioactive Decay Activity

Unit: 1 Bq = 1 decay / sec.

The Becquerel (Bq) is the unit for decay activity.

Highly radioactive → high activity

Activity depends on:

(1) Amount of the substance

Higher mass (number of nuclei), higher activity

Mass and activity are directly proportional

(2) Half-life

Longer the half-life, lower activity

Half-life and activity are inversely proportional

Calculating Activity (N = # of radioactive atoms)

$$\text{Activity} = \frac{\#\text{ of decays}}{\text{second}} = \frac{\Delta N}{\Delta t}$$

$$\lambda = \frac{\ln 2}{\text{Half-life in Seconds}}$$

or

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

$A(t)$ = Activity at time 't' = $\lambda \cdot N(t)$

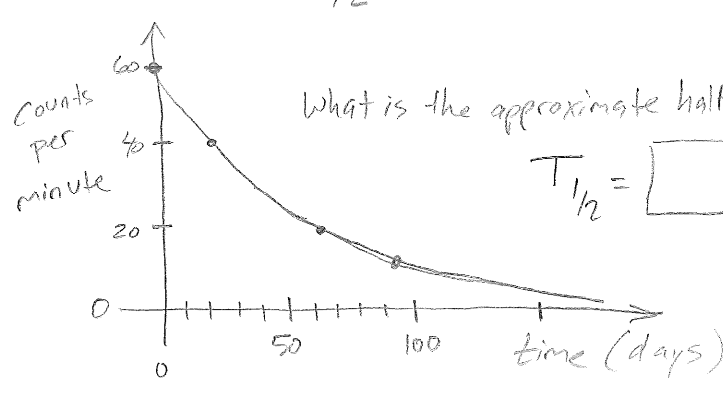
($N(t)$: # of radioactive atoms at time 't'.)

$T_{1/2}$ = Half-life

$$A(t) = \frac{\ln 2 \cdot N(t)}{T_{1/2}} \quad \text{or} \quad A(t) = \frac{.693 \cdot N(t)}{T_{1/2}}$$

$$A(t) = A_0 \cdot e^{-\lambda t}$$

(A_0: original activity; e: Euler's Number)



What is the approximate half-life?

$T_{1/2}$ = ☐ days

What is the activity of carbon-14 in a 1 gram sample of carbon from living tissue?

$$A(t) = \lambda \cdot N(t)$$

^{12}C 98.93% stable

^{13}C 1.07% stable

^{14}C Trace Radioactive
1 part per trillion
1 for every 1,000,000,000,000

$T_{1/2}$ = 5730 years

1. A sample of material contains one milligram of iodine-131. Iodine-131 has a half-life ($T_{1/2}$) of 8.02 days.

a.) Determine the # of atoms of iodine-131 initially.

b.) Determine the activity of the sample in Bqs.

molar mass of ^{131}I = 130.91 g $\qquad$ 1 mol = 6.02×10^{23} atoms.

2. What is the half-life ($T_{1/2}$) of Potassium-40 if 1.7×10^{19} nuclei have an activity of 300 Bq?

3. The activity of a At-211 sample at time equals zero is 400 Bq. Two hours later the sample's activity is 330 Bq. What is the half-life of At-211?

Radioactive Decay Law

The number of parent nuclei in a radioactive sample decreases exponentially over time. $N(t) = N_0 \cdot e^{-\lambda t}$

N_0 - number of radioactive nuclei at $t = 0$

$N(t)$ - number of radioactive nuclei remaining at time t.

e - Euler's number ≈ 2.718

λ = decay constant

t = time

Molar mass of $^{131}I = 130.91\ g$

A sample of material contains one milligram of iodine-131. This isotope is used in small doses in thyroid cancer treatments. It is also a dangerous product of nuclear fission if released into the environment. Iodine-131 has a half-life of 8.02 days.

1.) Determine the number of atoms of iodine-131 initially.

2.) Determine the number of iodine-131 atoms after 50 days.

1. In a sample there are originally 2.88×10^{20} radioactive nuclei. The decay constant is $\lambda = .1$ minute^{-1}

a.) Determine the number of radioactive nuclei remaining after 1 hour.

b.) Determine the half-life of the sample.

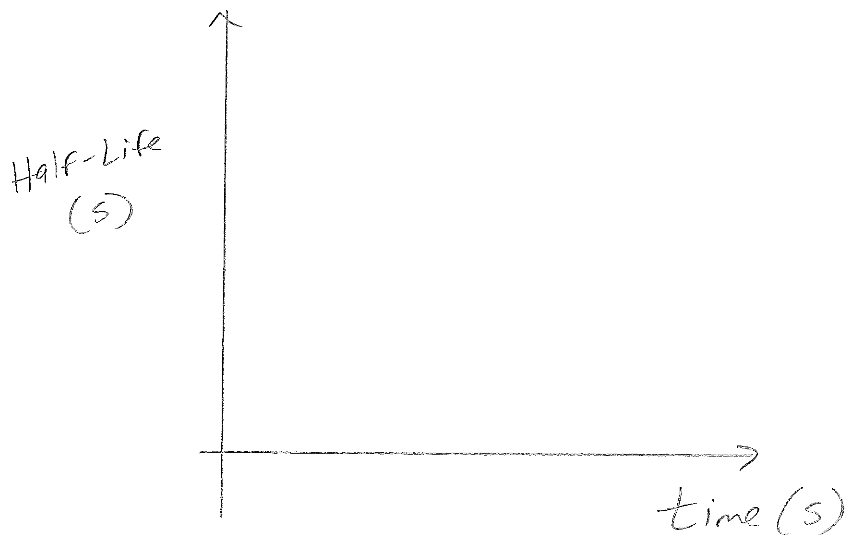
2. Tantalum - 184 undergoes beta decay with a half-life of 8.7 hours. Five hours and 18 minutes after the beginning of data collection the sample is found to contain 6×10^{19} nuclei.

a.) Determine the decay constant for Ta - 184

b.) Determine the number of radioactive nuclei that were present at $t = 0$.

3.) A radioactive sample is found to have 7.8×10^{13} nuclei at time equals zero hours. Twenty four hours later the number of radioactive nuclei has decreased to 2.7×10^{10}. Determine the half-life of the sample.

4. Sketch the qualitative graph of the radioactive half-life of a substance over time:



At an ancient burial site several researchers found a clay pot made with wooden beads. Back at the lab it was determined that 78% of the original carbon-14 remains in the wooden beads. How old are the wooden beads?

Phosphorus-32 undergoes beta minus decay with a half-life of 14.3 days. The molar mass of phosphorus-32 is 31.97 grams.

a.) Write the decay equation for phosphorus-32.

b.) Determine the decay constant for phosphorus-32

c.) Determine the initial activity of 1 gram of phosphorus-32.