Physical Optics

- Index of Refraction, n
- Snell's Law (fish tank)
- Total Internal Reflection
- Diffraction & Interference
- Single & Double Slits
- Thin Lens & Polarization

Refraction: Light rays bend when they encounter a change in media.

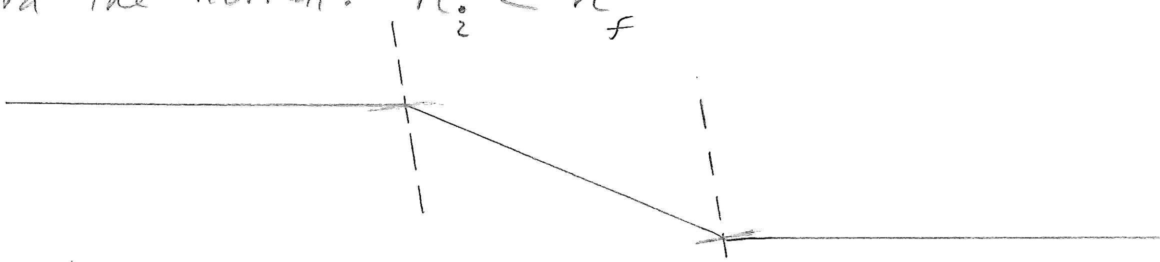
Index of Refraction, n: a unit-less measure of how much light bends when it enters a different medium.

$$n = \frac{c}{V_{medium}}$$

c = speed of light in a vacuum
$c \approx 3 \times 10^{8}$ m/s

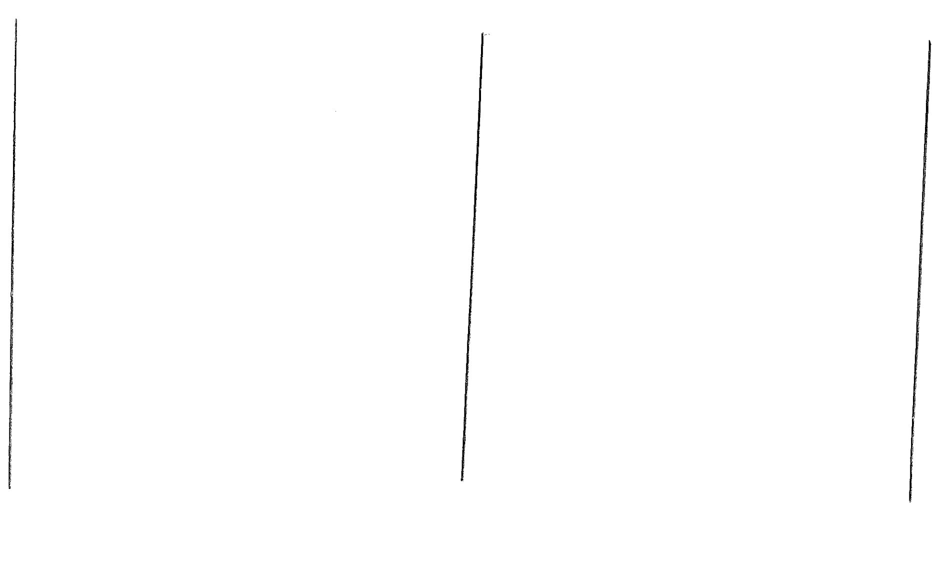
n will always be greater than 1

Toward the normal: $n_i < n_f$

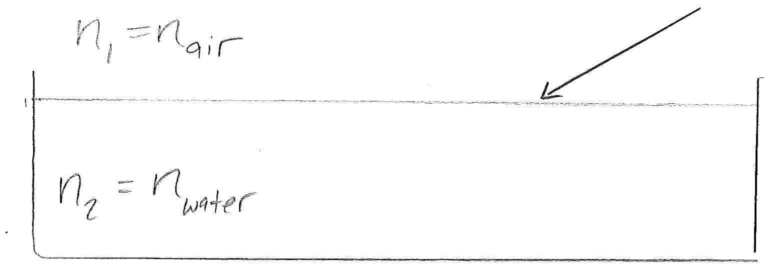


Away from normal: $n_i > n_f$

Car transitions from concrete to mud then back to concrete:



Laser into water from Air. Calculate n_{water}.



$n_{air} \approx 1.00$

$n_{water} =$

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Critical Angle occurs if $n_2 > n_1$ $\qquad \sin\theta_1 = \frac{n_2}{n_1}$

Index of Refraction and Law of Refraction

$c = 3 \times 10^8$ m/s = speed of light in a vacuum

Index of Refraction of a medium $n = c/v$

v = speed of light within the medium.

$n_{vacuum} = 1$ $\quad n_{air} = 1.000293 \approx 1$

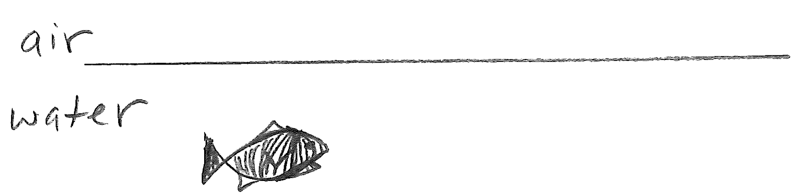
$n_{water} = 1.33$ $\quad V_{water} = ?$

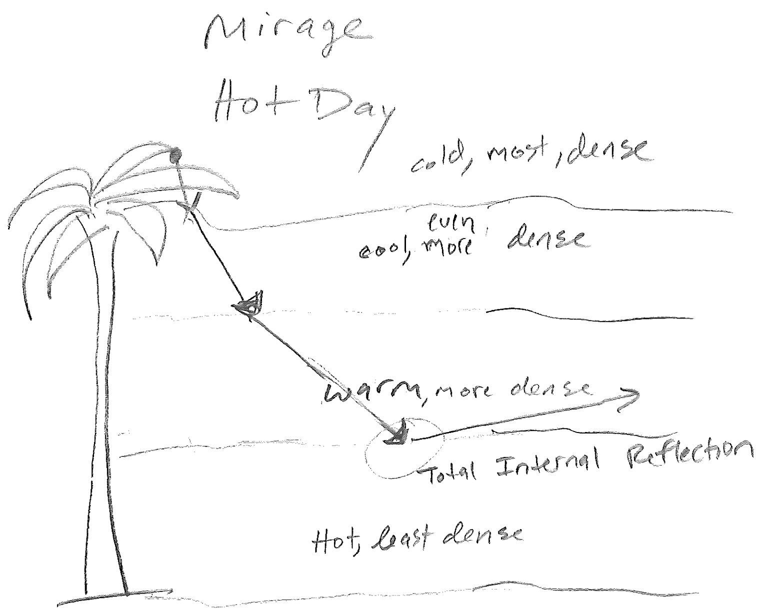
Snell's Law

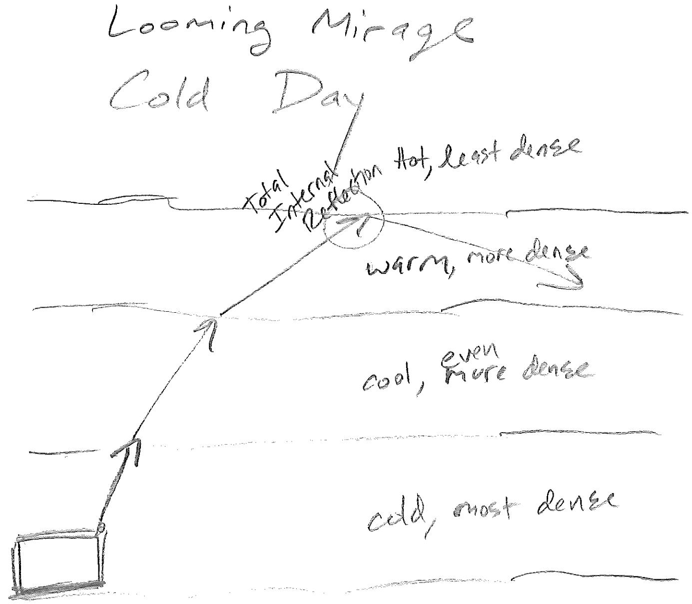
Law of Refraction: $\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{V_1}{V_2} = \dfrac{c/n_1}{c/n_2} = \dfrac{n_1}{n_2}$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \quad \text{or} \quad \boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Smaller θ for slower v, **bigger** n





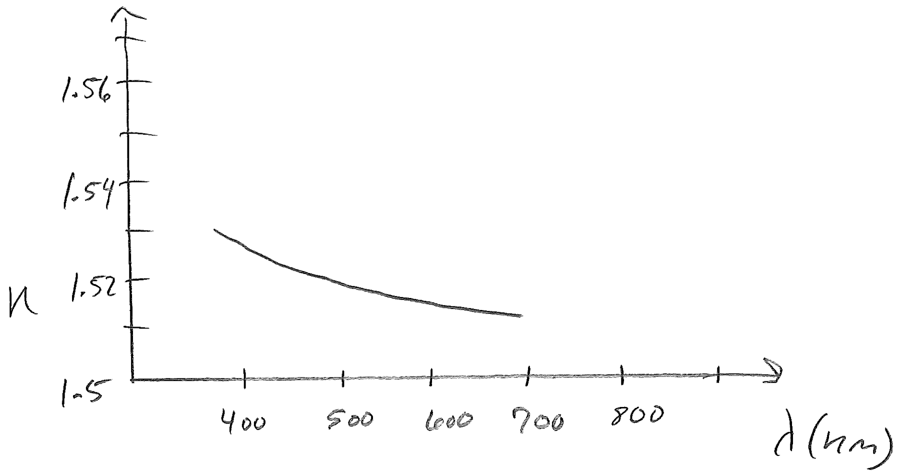


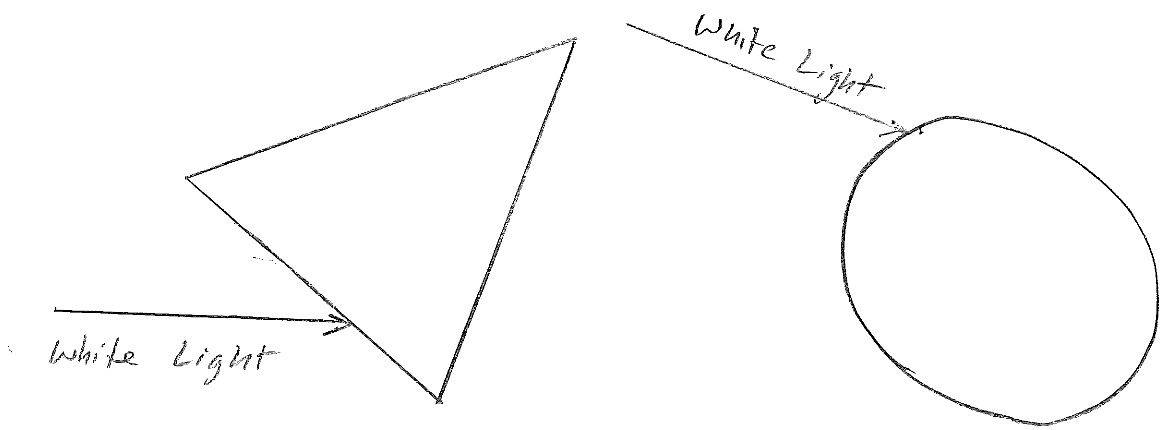
Snell's Law

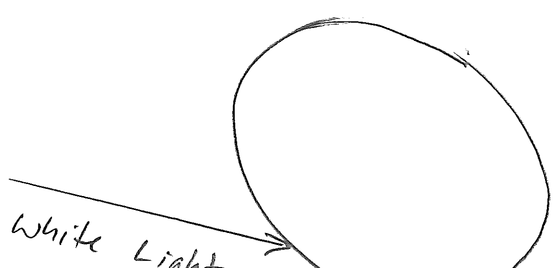
Less Dense to More Dense Medium → Bends toward normal

More Dense to Less Dense Medium → Bends <u>away</u> from normal

Dispersion Prisms and Rainbows

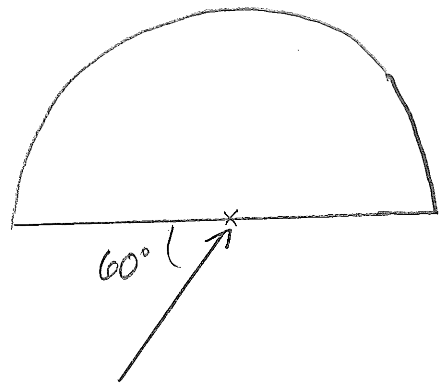






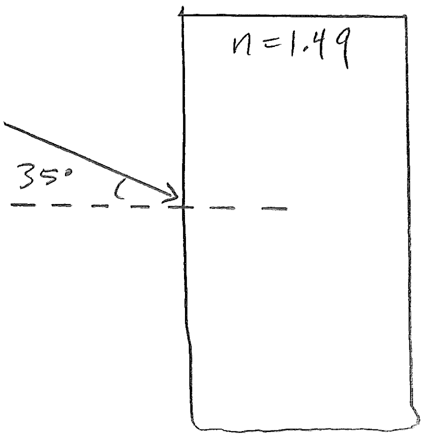
A semicircular dish is filled with glycerin.

a.) Draw the ray as it enters and then leaves.



b.) If the angle of the ray after it enters the dish is 20° from normal find 'n' for glycerin.

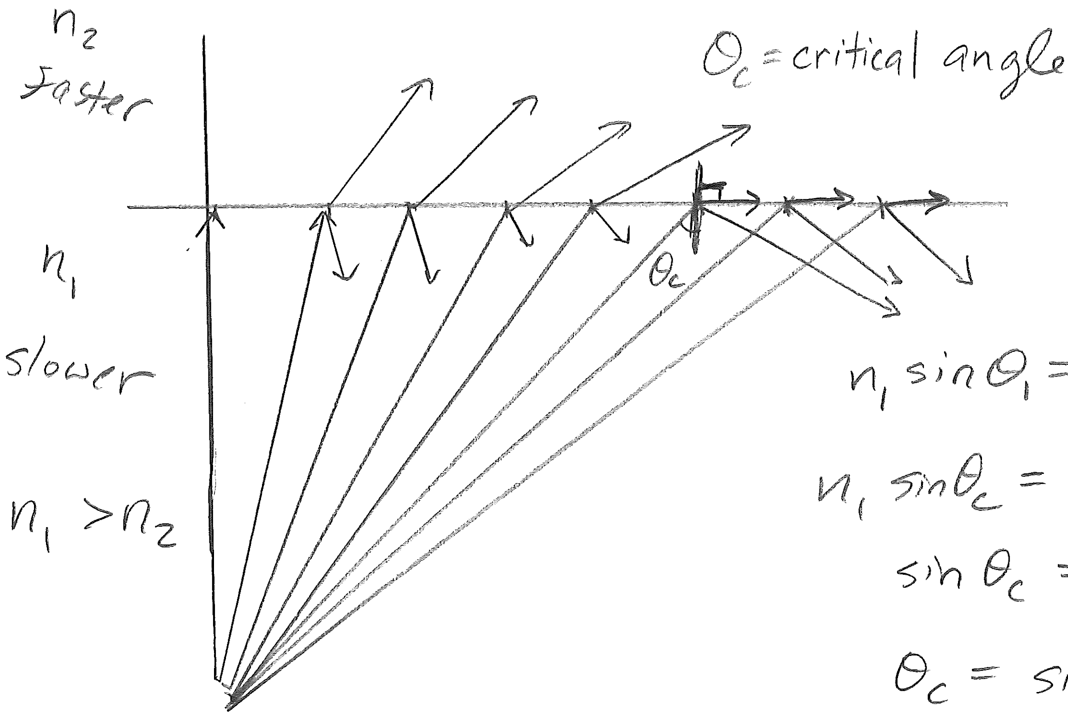
Rectangular Acrylic Rectangle



a.) Draw the refracted ray as it passes through and exits.

b.) Find the angles of the ray after it enters and after it exists.

Critical Angle and Total Internal Reflection



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1 \sin\theta_c = n_2 \sin 90^\circ$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

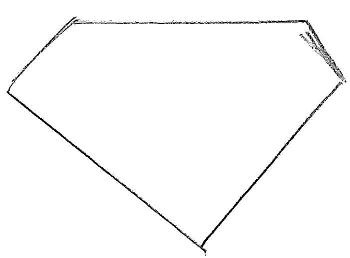
$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$\theta_i < \theta_c$: partial refraction, partial reflection

$\theta_i \geq \theta_c$: total internal reflection

Find θ_c for water-to-air interface ($n_{water} = 1.33$)

Find θ_c for diamond-to-air interface ($n_{diamond} = 2.42$)



Interfernce and Diffraction of Light Waves

Interference : when two (or more) waves pass through each other they cause a change in amplitude where they over lap.

- Constructive interference : waves interact to form a larger amplitude
- Destructive interference: waves interact to form a smaller amplitude.

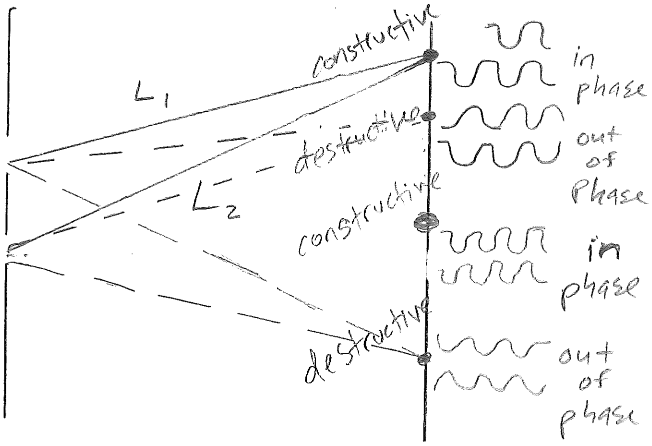
Combining interference and reflection.

Standing wave: when a series of waves reflect and intefere to create position of maximum amplitude.

Diffraction: when waves pass through an opening (that is close to the same size as the wavelength) the waves <u>curve</u>.

Two Coherent Source Interference

Coherent source : emits waves at a constant f & λ.



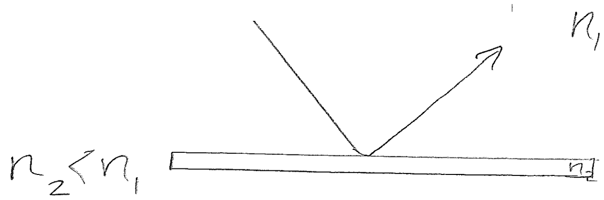
$\Delta\phi = 2\pi \Leftrightarrow \Delta x = \lambda$
$\Delta\phi = \pi \Leftrightarrow \Delta x = \lambda/2$
$\Delta\phi = 0 \Leftrightarrow \Delta x = 0$
$\Delta\phi = -\pi \Leftrightarrow \Delta x = -\lambda/2$

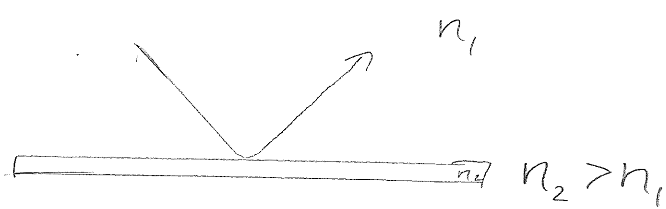
$\Delta x = L_2 - L_1$ or the path length difference

f = frequency

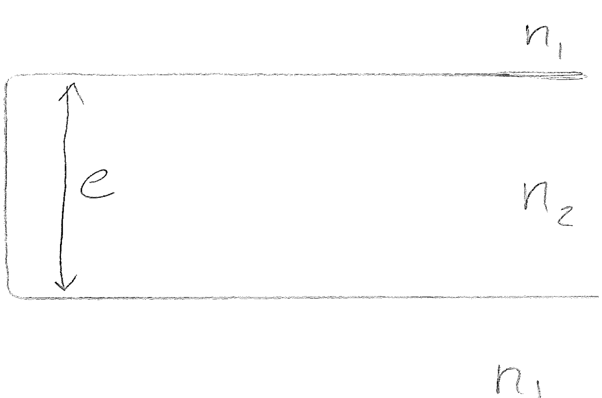
$\Delta\phi$ = phase shift

L_1 = length of path 1 L_2 = length of path 2



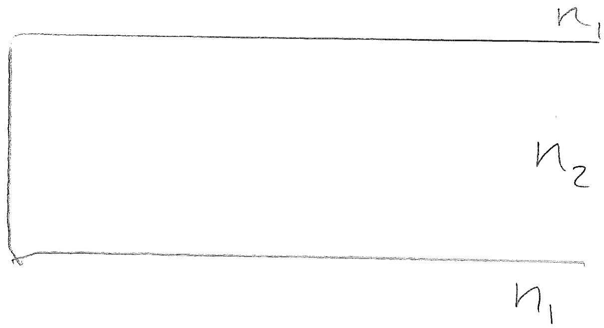


Thin Film Interference ($n_2 > n_1$)

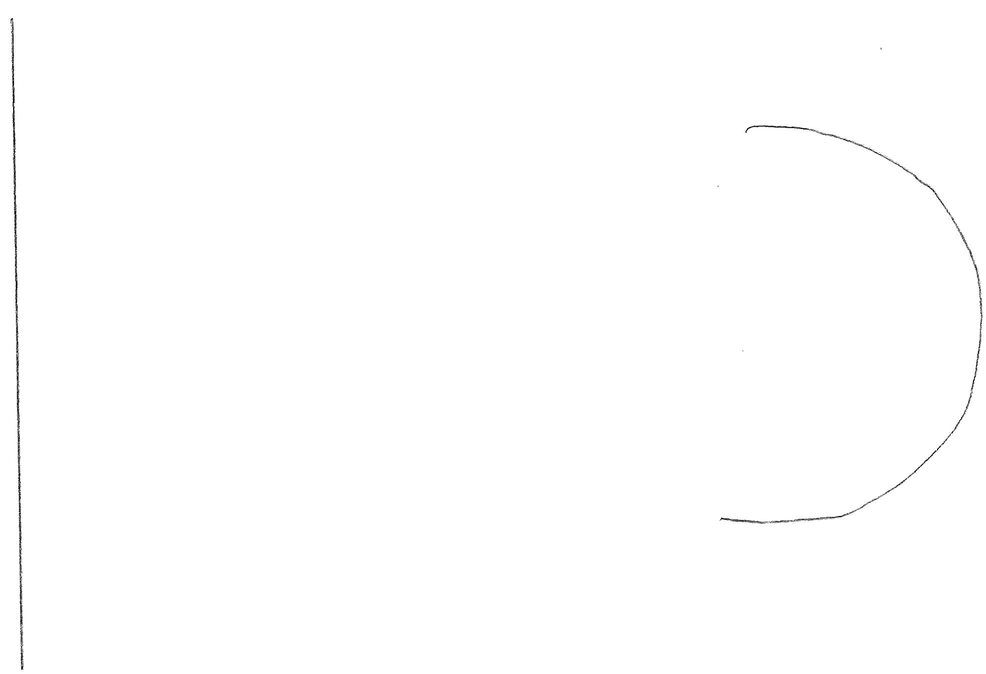


* Phase shifts result in a $+\lambda/2$ additional travel length.

Thin film Interference ($n_2 < n_1$)

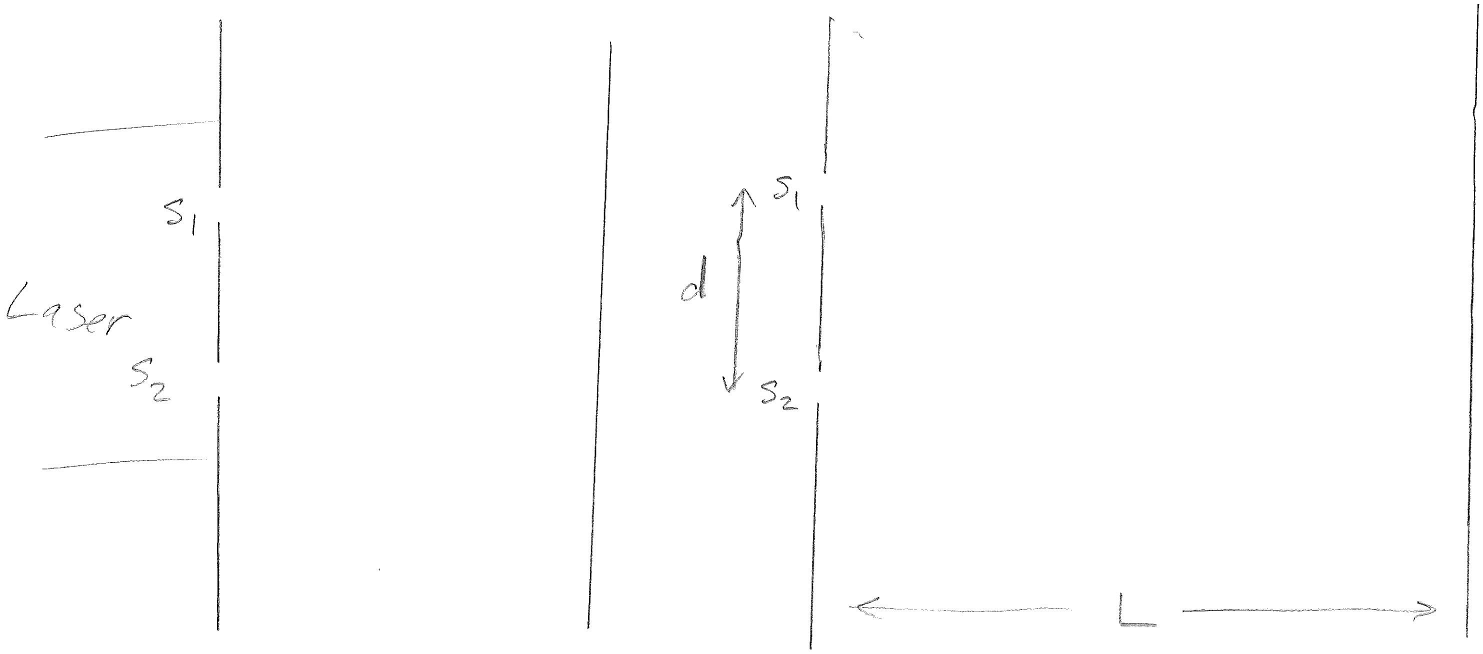


Huygen's Principle - all points of a wave front act as point sources of circular waves that expand to create a new wave front. After a Δt, the new wave front is the unique surface tangent to all forward propagating wave lets.



Double Slit Diffraction

$d << L$ so we can assume rays are $\approx$ parallel.



Young's Double Slit Experiment

$$x_{(m)} = \frac{m\lambda L}{d}$$

$\Delta x = 0, \lambda, 2\lambda, 3\lambda, \ldots$ constructive

$\Delta x = \lambda/2, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$ destructive

$x_{(m)}$: distance to bright spot number (m) away from the central bright spot

m : the number assigned to the bright spots starting with zero for the central bright spot $m = 0, 1, 2, 3, \ldots$

L : distance from double slit to the screen

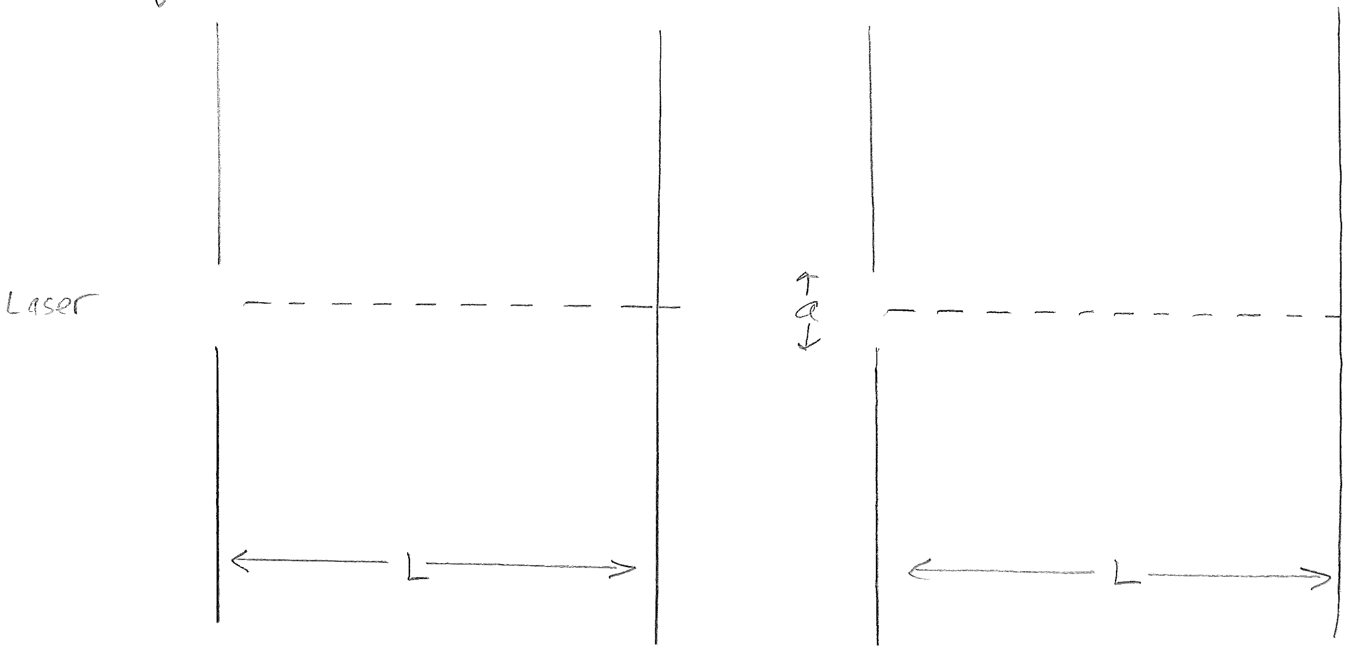
d : separation distance between slits

$$x_{(m)} = \frac{m\lambda L}{d} \Rightarrow \lambda = \frac{x_{(m)} d}{mL}$$

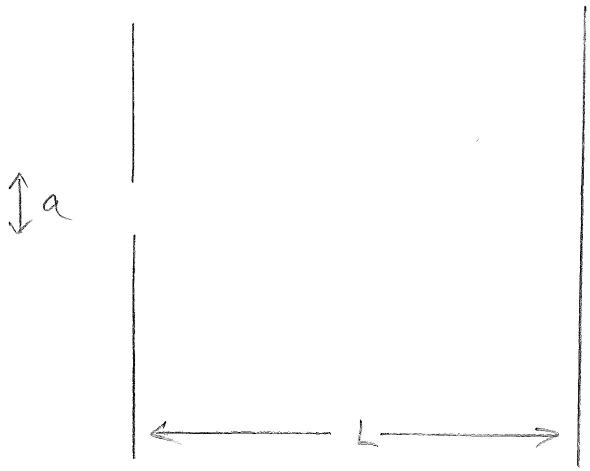
Implications of the Double Slit Experiment

- Light has wave properties
- Electrons also diffract and interfere through certain crystals to create interference patterns
- Electrons exhibit wave properties even though electrons are classically thought of as particles.
- Could this imply that particles have wave properties.

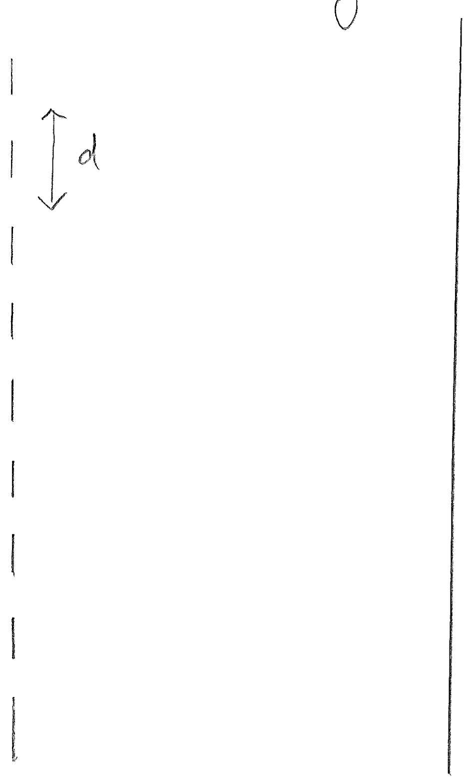
Single Slit Diffraction $(a << L)$



White Light Diffraction



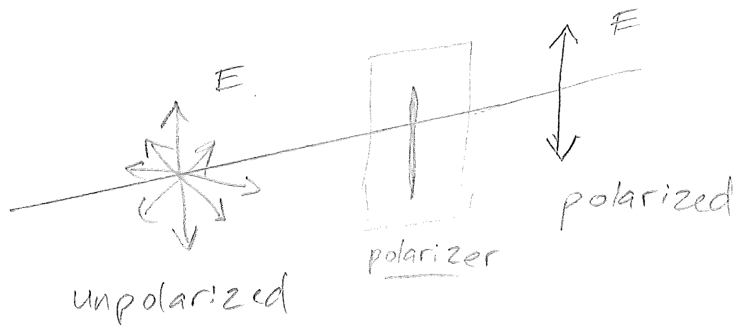
Diffraction Grating



Polarization is a property of transverse waves.

For EM waves the direction of the electric field is the direction of polarization.

A beam of unpolarized has an electric field in every direction parallel to its direction of motion. When passed through a polarizer the electric field will only oscillate in the direction parallel to the polarization slits.



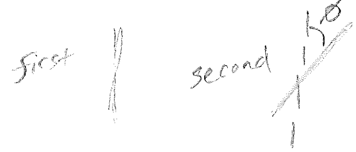
Unpolarized intensity is I_0

After being polarized, it becomes $\frac{1}{2} I_0$.

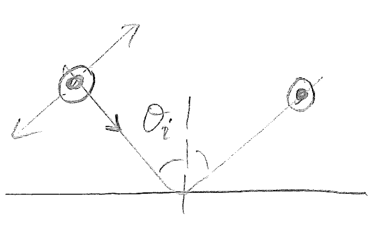
If you take two identical polarizers on top of one another and then you rotate one of them 90°, what will happen?

Intensity is proportional to the E^2

When a polarized beam travels through another polarizer at a different angle the intensity changes by $\frac{1}{2} I_0 (\cos^2 \phi)$.

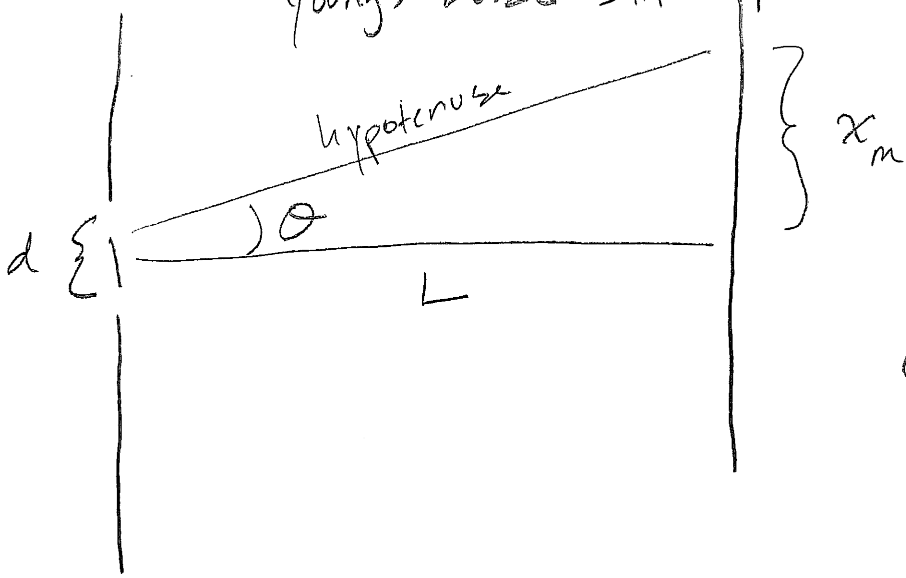


Polarization can occur by reflecting off of a non-conducting surface.



If θ_i = Brewster's angle, no parallel-to-page E gets reflected and the beam is completely polarized.

Young's Double Slit Experiment



$d \sin \theta = m\lambda$

$m = 0, 1, 2, 3, \text{etc....}$

θ in radians.

Light must be monochromatic

Newton believed light was a particle, but Young demonstrated it behaved as a wave. Newton did not realize light waves were so small??

The double slit experiment only works with slits an order of magnitude ($\times 10$ or $\div 10$) from the wavelength of light. for example 550nm light diffracts for slits of width 55 nm to 5500 nm.

hypotenuse $\approx L$ if θ is small, so...

$$d \sin \theta = m\lambda$$

$$d\left(\frac{x}{L}\right) = m\lambda \quad \text{or} \quad x = \frac{m\lambda L}{d}$$

↑ Spacing of fringes.

A double slit experiment set up has a slit spacing of 1.5 mm. A screen is set up 3.5 meters from the slits. Monochromatic 565 nm light is incident on the slits. Find the angle θ for the first order bright fringe. Radians & Degrees.

"First order" means one away from the brightest one.

$$d \sin\theta = m\lambda \qquad m = 0, 1, 2, 3, \text{etc...}$$

<u>radian mode</u>

$$x = \frac{m \lambda L}{d}$$

Red light ($\lambda = 664$ nm) shines through slits separated by a distance of 1.2×10^{-4} m. A screen is 2.75 m away. Find the distance on the screen between the central bright fringe and the third order ($m=3$) bright fringe.

What happens to the angle of diffraction for light of higer wave length?

What happens to the spacing of the bright dots if the slits are closer together?

Thin Films

n_1 Air
glass n_2

When light crosses a boundary between two materials, some reflects and some crosses the boundary. The reflected light may undergo a phase change. If n_2 is <u>greater</u> than n_1, there will be a 180° shift. That means when light in air bounces off glass, it undergoes a phase change. $n_{glass} = n_2 = 1.5$ $\quad n_{air} = n_1 = 1$

If n_2 is <u>less</u> than n_1, there is <u>no</u> phase change. But lets start with $n_2 > n_1$:

Assume the wave enters perpendicular to the film.

Several things happen to the incident light:

- (*) Some reflects off the top of the film and changes phase 180°.
- * Some of the light refracts into the film
- * Some of that light refracts out the other side.
- (*) Some of the light reflects off the back of the film and undergoes no phase change. This light then refracts out the other side. The total distance the light travels is twice the film thickness while it is in the film.

For the light that reflects off the top of the film which changed phase 180° and the light reflected off the back of the film then out the other side, there is both constructive and destructive interference.

Some reflected waves add up constructively, making those wavelengths more intense.

Some reflected waves add up destructively making those wavelengths absent from the reflection.

t = thickness

The total distance traveled by the light that is refracted in through the top, reflected off the back, then refracted out the top must be a whole wavelength for destructive interference to occur. It must be an odd multiple of half wavelength for constructive interference.

Destructive : $t_{min} = \frac{1}{2} \lambda_{film}$ Constructive : $t_{min} = \frac{1}{4} \lambda_{film}$

λ in the film is different from λ in the air.

$$n_1 \lambda_1 = n_2 \lambda_2$$

$$\lambda_{film} = \frac{\lambda_{air}}{n_{film}}$$

555 nm light is incident on a soap bubble with $n = 1.3$. Calculate the minimum thickness for destructive interference. $n_{air} = 1$

Destructive interference also happens in a film if the thickness is $\lambda, 1.5\lambda, 2\lambda, 2.5\lambda, \ldots$

Constructive interference also happens in a film if the thickness is $.75\lambda, 1.25\lambda, 1.75\lambda, 2.25\lambda, \ldots$

film } t = thickness

destructive : $t_{min.} = \lambda/2$

constructive : $t_{min.} = \lambda/4$

Thin Coatings

A thin coating is frequently used on glass lenses.

Another example of an oil slick in which a thin layer of oil rests on water.

Air	$n_1 = 1$
Oil	$n_2 = 1.6$
water	$n_3 = 1.33$

Same Rules as thin films.

$n_2 > n_3$

Air	$n_1 = 1$
Thin Coating	$n_2 = 1.4$
glass	$n_3 = 1.57$

Rules are opposite to thin films.

$n_2 < n_3$

Thin Coating Opposite Rules

Destructive : $t_{min} = \frac{1}{4} \lambda_f$

Constructive : $t_{min} = \frac{1}{2} \lambda_f$

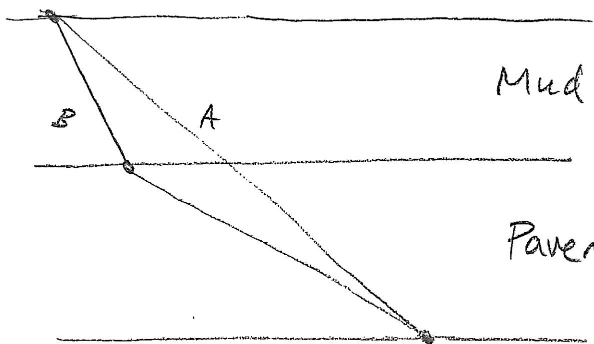
Constructive interference also happens in a film if the thickness is λ, 1.5λ, 2λ, 2.5λ, ...

Destructive interference also happens in a film if the thickness is $.75\lambda$, 1.25λ, 1.75λ, 2.25λ, ...

A very thin oil with $n_{oil} = 1.15$ floats on water $n_{water} = 1.33$.
Calculate the minimum oil thickness for constructive interference.

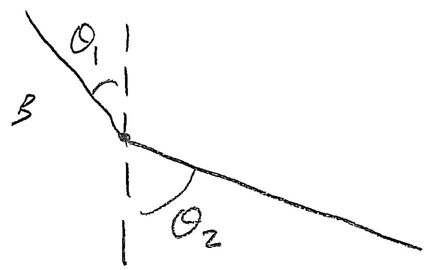
air	$n_1 = 1$
oil	$n_2 = 1.2$
water	$n_3 = 1.33$

Finding the path of least time (rather than distance) is called a brachistochrone problem. Analogy: Path A is the "least distance" path.



Assume you can travel faster on pavement compared to mud.

Path B is the "least time" path because you can travel faster on pavement. It gives you the optimal combination of less distance and greater speed.

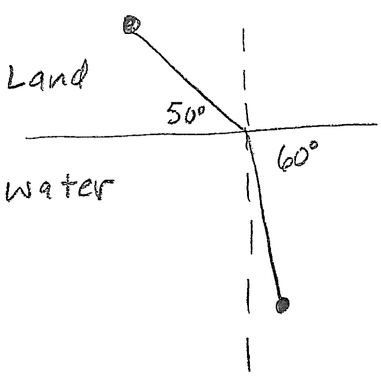


$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{\text{speed in mud}}{\text{speed on pavement}}$$

Snell's Law

Light always follows the path that is the fastest to take or the "path of least time".

An animal swims at 5 m/s. If the fastest path from a position on land to its home in water is shown below, how fast is the animal on land in m/s?



A screen is 1.5 meters from a set of double slits. The distance between the 2 slits is .07 mm. When a monochromatic light shines on the slits, the bright fringes on the screen are 1.2 cm apart. Find:

a.) λ of the light

b.) How far from center of screen is 1st dark fringe?

$L = 1.5\,m \quad d = .07\,mm \quad y = 1.2\,cm \quad \lambda = ?$

A 470-nm blue light is shone on a single slit to produce a central max. on a screen .8 m away. If the central max. is 30° wide: a.) What is the width of the slit?
b.) What is the width of the central max. on the screen?

A monochromatic light shines on a 400 line per mm grating. 1st-order line on screen is 20 cm from the 0th order line. If the screen is .8 m away: a.) $\lambda = ?$

b.) How far from the center of the screen is the 2nd order line?

An unpolarized light passes through 2 polarized sheets. If the intensity of the transmitted light is 20% that of the original light, what is the angle between transmission axes of the two polarizer sheets?

A glass lens is coated with a thin film of MgF_2 ($n = 1.38$) to reduce reflection. Find the minimum thickness of the coating to eliminate the reflection at 550 nm, the middle of the visible light spectrum at normal incidence.

A rhinestone is made of glass ($n = 1.5$). To make it more reflective, it is coated with a thin film of SiO ($n = 2$). Find the minimum thickness of SiO coating to provide strong reflection for 570 nm light at normal incidence.