

## Skill Drill 12

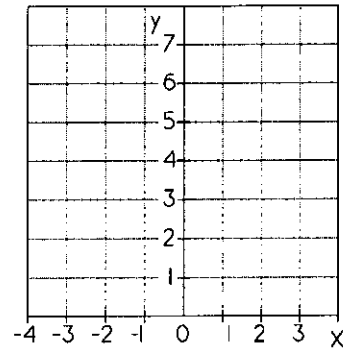
The exercises in this drill emphasize the algebra of quadratic equations, with a variety of applications to word problems. Quadratic equations will be revisited in the next review, which deals with the intelligent use of formulas in physics.

1. Review of major points. Consider the quadratic expression

$$y = ax^2 + bx + c.$$

(a) Suppose  $b = c = 0$ . If  $y = 4$  for a certain value of  $x$ , what is  $y$  when  $x$  is doubled?

(b) Again assuming  $b = c = 0$  plot  $y$  versus  $x$  on the accompanying graph, for  $a = 0.5$ . From your plot find  $x$  when  $y = 5$ .



(c) Now take  $a = 0.5$ ,  $b = 1$ , and  $c = 2$ . Before plotting the equation, determine the value of  $y$  at which the curve will cross the vertical axis. Now plot the equation on the same graph which was used in question (b).

(d) Again assume the values of the constants given in part (c) Use the "quadratic formula" to find  $x$  when  $y = 14$ .

(e) For the values of the constants given in part (c) find  $x$  when the simultaneous equation  $y = 2x + 4$  is also satisfied.

2. Solve each of the following equations and check your results by substituting back into the original expression:

(a)  $x^2 + 3x + 2 = 0$

(b)  $x(5x - 1) = (x + 1)(5x + 2)$

(c)  $x^2 - 6x + 9 = 0$

(d)  $16x^2 - 48x = 0$

(e)  $8x^2 + 26x = -21$

(f)  $x^2 - 1 = (5/6)x$

3. Solve each of the following pairs of simultaneous equations:

(a)  $5x^2 + 15x - 6 = y$   
 $-y = 6x + 10$

(c)  $y = x^2$   
 $y = 45 - 12x$

4. The product of two consecutive even numbers is 224. What are the numbers?

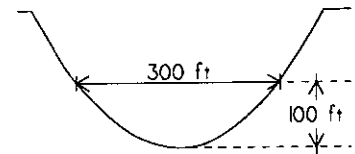
5. The sum of two numbers is 9, and the difference of their squares is 45. What are the numbers?

6. The combined areas of two squares is  $424 \text{ in}^2$ . If the side of one square is 8 inches greater than the side of the other square, what are the dimensions of both figures?

7. Ralph made a trip of 200 miles at an average rate of speed  $v$ . Had he decreased his average speed by 10 mi/hr the same trip would have taken 1 hour more. Find  $v$ .

8. Farmer Smith tethers her goat on a rope attached to a post so that the goat can graze over a circular area of meadow. If the length of the tether rope is increased by 100 feet the goat can graze over a circle with 9 times the area. What is the length of the original rope?

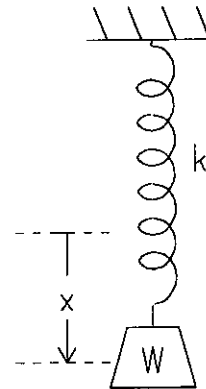
9. A certain ravine has a parabolic cross-section, as illustrated. If the ravine is 300 feet across at a height 100 feet above the bottom, what is the width at the top of the ravine 200 feet above the bottom?



10. The so-called "potential" energy  $E$  of a system consisting of a spring with a weight  $W$  suspended from it is given by

$$E = \frac{1}{2}kx^2 - Wx$$

where  $x$  is the difference between the length of the spring and its length before the weight is attached. If the spring constant  $k = 10$  lb/ft and  $W = 30$  lb, what values of  $x$  correspond to an energy  $E = 2.5$  ft-lb?



11. A billiard ball collides head-on with a stationary marble. Using laws of mechanics it is possible to write equations which relate the speeds of the billiard ball and marble after the collision ( $v_B$  and  $v_M$ , respectively) to the incoming speed of the billiard ball. Suppose for a certain case the equations are as follows:

$$10 v_B + v_M = 150 \text{ cm/s} \quad \text{and} \quad 10 v_B^2 + v_M^2 = 10 (15 \text{ cm/s})^2 .$$

Solve this pair of simultaneous equations to determine  $v_M$ .