# Review 13 – Problems, Formulas, and Physics

The art of successful problem solving in physics cannot be mastered simply by memorizing lists of useful formulas. Nevertheless, formulas are required to find most numerical answers. Using an example problem whose solution involves a quadratic equation, this review shows how formulas are used to solve problems in a discriminating and error-free way.

## A QUADRATIC EQUATION FROM PHYSICS

**Uniform acceleration.** Early in your physics course you will encounter a quadratic expression which describes the motion of objects whose speed along a straight path changes at a constant rate. Such an object is said to be undergoing *uniform acceleration*. An example of such behavior is the motion of a weight which is freely falling under the influence of gravity.

**The equation and an example problem.** The following quadratic equation describes how the position y of an object undergoing uniform acceleration depends on the time t:

$$y = \tfrac{1}{2}at^2 + v_o t + y_o$$

The constants $v_o$ and $y_o$ are, respectively, the speed and position of the object when the time $t = 0$ (when, for instance, a timing clock is started). The constant a, called *acceleration*, is a measure of the rate at which speed changes. A positive value of acceleration means that speed increases during motion in the +y direction, whereas a negative value means that speed decreases during such motion. Our concern at this point, however, is less on interpretation than on the successful use of the expression.

The following is a typical problem having to do with uniform acceleration:

> A ball is thrown straight downwards from the top of a 30 m high building with an initial speed of 5.0 m/s. Knowing that such a freely falling body has a constant acceleration a = 9.8 meters per second per second (9.8 $m/s^2$), determine how many seconds it takes for the ball to reach the ground.

Broadly speaking, the solution to a problem like this depends on correctly identifying both the given and unknown quantities and relating them through a valid equation (or equations) of physics. The answer is then obtained by solving and inserting numerical values in these equations. How this process can be carried out efficiently and correctly will be illustrated by reference to this example problem in the next section.

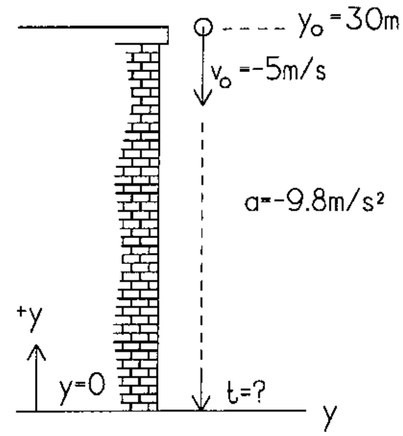
## INTELLIGENT USE OF FORMULAS

Several features commonly characterize the successful application of equations in the solution of physics problems. These characteristics are discussed below.

**(1) Making and labeling of a sketch.** In many cases even the most rudimentary drawing will help in visualizing and understanding a problem. This is often the best first step in organizing the information to be used in the solution.

As an example, the problem given above is illustrated by a sketch given below. Notice particularly that

- *factual information is very conveniently summarized in a sketch.*




In this drawing, the symbol for the initial speed $v_o$ is used as a label on a down-pointing arrow at the top of the building. Also, the main objective of the problem is summarized by "t=?" written at the bottom of the building. Notice also that in this case, as in other problems in which directions in space are important,

- *the choice of coordinate axes is explicitly indicated in the drawing.*

In the example diagrammed here, the y-axis has been chosen to point upwards; an arrow labelled "+y" indicates this. Likewise, the choice of the origin (y = 0) at the base of the building has been clearly indicated. It should be understood that the choice of coordinate directions and origin are largely arbitrary. Certainly the final answer to a problem should not depend on an arbitrary choice, although the numbers which must be used later in the solution may reflect that decision.

**(2) Choosing the equation(s).** With an understanding of the problem well in hand you are ready to ask yourself some crucial questions:

- *What is the given information and what information is to be determined?*

- *Can I think of an equation which relates these? Or, do I know a relationship which gives an intermediate result which can be used in another equation to get an answer?*

- *Is there any useful information, not explicitly stated in the problem, which can be tacitly assumed?*

Although simple rote memorization of formulas is discouraged, with experience the most important equations inevitably will be remembered. Often the problem, while not identical to any of them, will remind you of problems you have done in the past. In any event in choosing a relationship

- *one should always be guided by an understanding of the underlying physics and of the limits of applicability of the equation.*

For instance, the equations which apply to the above problem must have to do with *uniformly-accelerated* motion since this describes freely falling objects; such equations cannot be used if acceleration is non-uniform. The solution of the problem continues as follows:

> The given quantities are the initial position $y_o$ and initial speed $v_o$, as well as the position y at the end of the falling motion. The acceleration a is also a known quantity. An equation which relates all of these to the unknown time t at the end of the fall is the quadratic expression
>
> $$y = \tfrac{1}{2}at^2 + v_o t + y_o \, .$$

**(3) Checking on the equation.** Have you remembered the equation correctly?

*The equation should appear to make sense, or at least it should not appear to be nonsense.*

For instance, in the example, you would be amiss to forget the term involving $v_o$; the distance an object has moved after some time has elapsed surely depends on how fast it starts off. Moreover

- *the equation should be dimensionally correct; every term should have the same units.*

In the example the term $\frac{1}{2}at^2$ has the dimensions of distance, as expected: (meter/second$^2$)×(second)$^2$ = meters. (Checking dimensions does not, of course, allow us to obtain the dimensionless factor ½.)

**(4) Calculating the answer.**

*In most cases it is more efficient to do the necessary algebra before substituting in numbers.*

The reason for this hint is that less writing is required, cancellation of terms is easily recognized, and simple arithmetic can be performed while the expression is least cumbersome. Finally

- *numerical quantities should be substituted with units included.*

The numbers which are substituted must take into account the coordinate axes which you have elected to use. In the drawing the +y axis points upwards whereas $v_o$ points downwards. Consequently $v_o$ has a negative numerical value. Likewise the acceleration *a* has a negative numerical value, since a weight rising in the +y direction would be slowed down by gravity. The choice of origin sets y = 0. (Practice using other choices of axes is given in the Skill Drill.) In keeping with these pointers, the remaining step in the solution of the example problem is as follows:

In "standard form" the quadratic expression given on the previous page is

$$0 = \tfrac{1}{2}at_2 + v_o t + (y_o - y)$$

from which we can identify the coefficients $A = \frac{1}{2}a$, $B = v_o$, and $C = y_o - y$. The numerical values which are consistent with the choice of coordinates are $v_o = -5$ m/s, $y_o = 30$ m, and y = 0. The acceleration $a = -9.8$ m/s$^2$. Hence, using the quadratic formula for the roots of the equation, we have

$$t = \frac{-v_o \pm\sqrt{v_o^2 - 2a(y_o - y)}}{a}$$

$$= \frac{5.0\text{m/s} \pm\sqrt{(-5.0\text{m/s})^2 - 2(-9.8\text{m/s}^2)(30\text{m} - 0)}}{-9.8\text{m/s}^2}$$

$$= 2.0 \text{ s, } \underline{\text{and}} \ -3.0 \text{ s} .$$

The first of these two roots corresponds to the desired answer; the ball reaches the ground in 2.0 seconds.

Notice that using units along with numbers in the final calculation shown above provides a way of checking both the dimensions of the terms and the units in the answer, which should always be written down. Finally

- *one should always ask: Does the answer seem reasonable?*

For instance, in this example it is likely that your experience with similar problems will lead you to realize that *seconds* is the right order of magnitude of the time for a body to fall the height of a building.

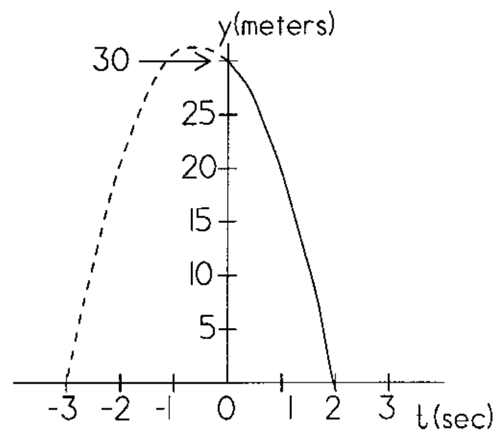
**(5) Asking further questions.** Try to engage physics problems with an inquisitive spirit, particularly in homework assignments. For instance, you might wonder whether there is some meaning to the negative root (-3.0 s) obtained in the example above. Although most of the time a graph is not needed, in this case a quickly drawn graph can be a guide to understanding, as discussed below.

## GRAPHS AND INTERPRETATION

**Graph of the falling body problem.** The quadratic equation representing uniformly accelerated motion

$$y = \frac{1}{2}at^2 + v_o t + y_o$$

plots as a parabola on a graph of position y versus time t. The curve (taking the choice of axes as in the last section) is shown here.




Review 12 gave some useful tips which could be used for making an approximate sketch of this graph: the coefficient ½a is negative, so the parabola is an upside-down bowl; also the constant term $y_o$ is the intercept. Since the body continues to fall after leaving the top of the building the top of the "bowl" must be to left of the t=0 position.

**Interpretation.** The original problem asked the question: when does the ball strike the ground? The answer, t = 2.0 s, corresponds to the point on the graph where the parabola passes through the horizontal (y = 0) axis.

The part of the parabola in the -t region of the graph is plotted as a dashed curve; the "mysterious" answer, t = -3.0 s, corresponds to the point where the dashed parabola passes through the horizontal axis. Hence we are led to the following interpretation of the negative root: a ball rising from the ground with an appropriate speed 3 seconds *before* we start our stopwatch (t = -3.0 s) would reach a maximum height, and then at t = 0 be falling with a speed $v_o$. Of course, it would again reach the ground at t = 2.0 s, as the curve illustrates.