Skill Drill 14

This drill covers the use of ratios or graphs in working problems involving various power functions. All the word problems are drawn from a science context.

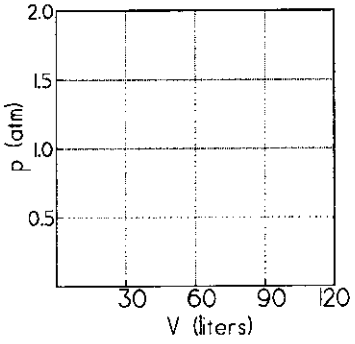
1. Practice with ratios: For the following power functions, what is y when x = 8, given that y = 2 when x = 2?

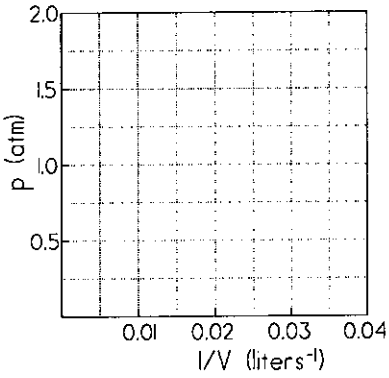
(a) $y = kx^3$ (b) $x = ky^3$ (c) $y = kx^{-2}$ (d) $y = A(kx)^{1/2}$

2. Boyle's Law (see example problem in Review 14) states that under most conditions the pressure p of a gas kept at constant temperature varies inversely with the volume V, i.e., p = k/V. Here is a table of data giving p and V for a sample of carbon dioxide gas at 295 K (approximately room temperature).

p(atmospheres)	V(liters)
0.5	112
0.8	68
1.0	54
1.5	37
1.8	30

(a) Plot these data on the p versus V graph at the right, and connect the points by a smooth curve. (b) Plot the data again on the p versus 1/V graph. (c) Draw a straight line which best fits the points plotted in the latter graph. From the slope of the line determine the constant k.



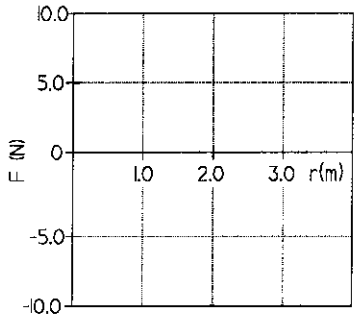


3. The attractive gravitational force exerted by the earth on an object located above the earth's surface depends as the inverse square on the distance of the object from the center of the earth. What is the pull of the earth on a satellite orbiting 26,000 mi above the surface, if the satellite weighs 1.0 ton when it is on the ground? The radius of the earth is 3960 mi.

4. Electrically charged bits of matter exert forces on one another which vary as the inverse square of their distance of separation r. Suppose, for a pair of electrical particles, the force F is written

$$F = \pm K/r^2$$

where the plus sign signifies a repulsive (outward directed) force which occurs when the charges on the two particles are of the same type ("positive" or "negative" charge), and the minus sign signifies an attractive (inward directed) force which occurs when the charges are of opposite type.



(a) On the graph at the right, plot F (in Newtons, N) vs. r for K = 9.0 $N \cdot m^2$, for both attractive and repulsive forces. (b) At what separation is the force one hundred times greater than that exerted at 1.0 meter? (c) At what separation is the force one-hundredth as large as that exerted at 1.0 meter?

5. Stefan's law of energy radiation from hot bodies can be written

$$W = kT^4$$

where W is the intensity of the energy radiated per unit time and T is the absolute temperature (in degree Kelvin, K) of the surface.

(a) The sun's surface temperature is about 6000 K. If the sun cooled by just 1000 K, by what factor would the energy radiated to the earth be reduced?
(b) The earth is presently 93 million miles from the sun. How much closer would it have to be to receive its present solar radiation from a 5000 K sun? (Intensity of the radiation varies as the inverse square of the distance, as in the example problem in Review 14 on light intensity.)

6. The frequency υ (hertz *or* vibrations per second) of a stretched string varies as the square root of the force F with which it is stretched, i.e.,

$$\upsilon = k\sqrt{F}\,.$$

The graph shows some data for a steel piano string. Make another plot of the frequency data versus $\sqrt{F}$. Draw a straight line through these points to verify the square root dependence. From this line determine the constant k.