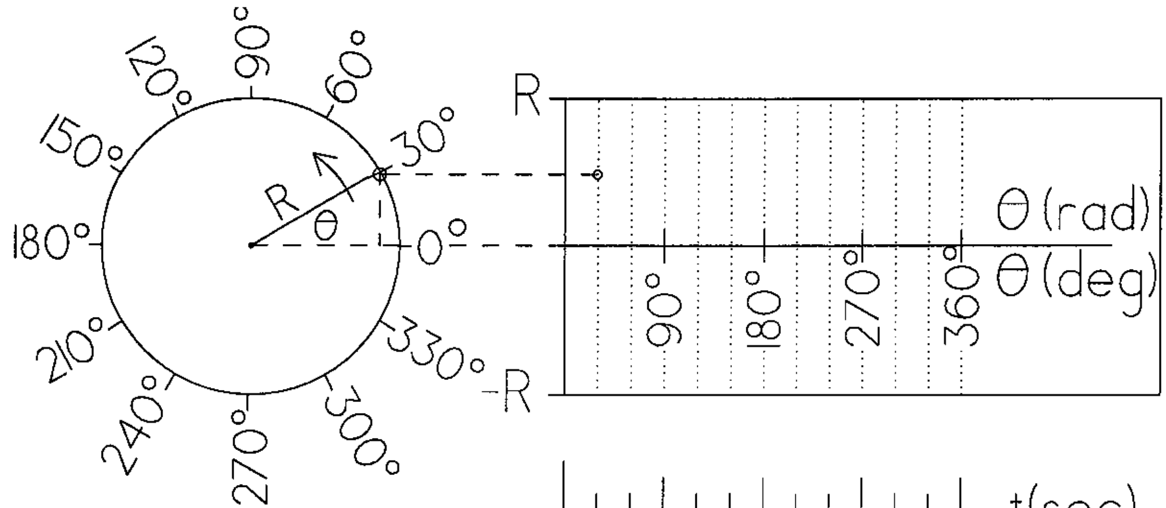
Skill Drill 15

Two exercises in this drill should give you some insight into certain aspects of sinusoidal functions not explored in the Review: their connection with rotational motion and with wave phenomena. Other questions should help your understanding of the interrelationships among sinusoidal functions and the use of radian measure.

1. This is an exercise to help you visualize the shape of a sinusoidal curve as well as to demonstrate its relationship to rotational motion.

Consider the rotating wheel of radius R pictured below. A spot is painted on the rim at the end of a radial line which at any moment makes an angle θ with respect to the horizontal. The rotation is uniform and time t = 0 when the radial line is horizontal, so that $\theta = kt$.



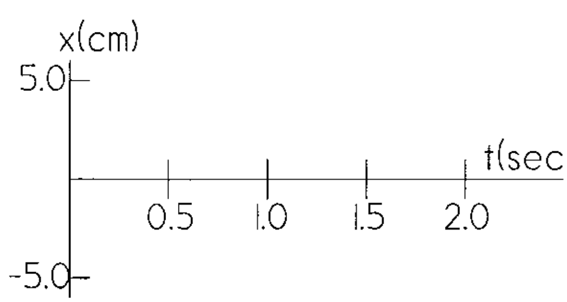
(a) Draw the radial line and spot at each of the marked angular positions from 0° to 360°. (The 30° position is shown in the drawing.) For each position plot the height of the spot above the axis versus angle by drawing with a ruler a horizontal line from the spot to the appropriate point on the graph. (b) Connect the plotted points by a smooth curve; you will recognize this as the curve of sin kt. (c) Label the horizontal axis with equivalent radian values at each of the positions labeled above in degrees. (d) Taking $k = 2\pi$, mark each of these positions on the t(sec) axis with the equivalent values of time.

2. A spring-mass oscillator moves back and forth between positions x = ±5.0 cm in a manner described by a sinusoidal function. A complete oscillation requires 1.0 second.

(a) Taking t = 0 when x = 5.0 cm, sketch a curve representing the motion on the accompanying graph.

(b) On the same graph sketch the curve of the motion if x = 0 when t = 0.

(c) There is an alternative curve which also satisfies the conditions stated in (b). Draw this curve as well.



3. Beneath the end of a pier the level of the water is observed to move up and down as waves move by according to the equation

$$y = (10.\text{cm}) \cos (\pi/2)t \ ,$$

where y is the height of the water surface with respect to the average height. t is the time in seconds and the argument is in radians.

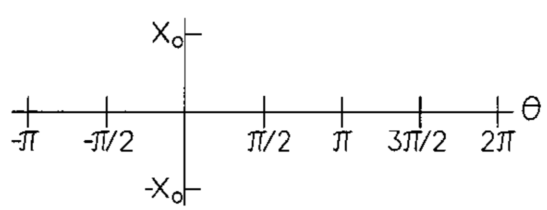
(a) How long is does one oscillation take?

(b) What is the total distance the surface moves (highest to lowest point) during an oscillation?

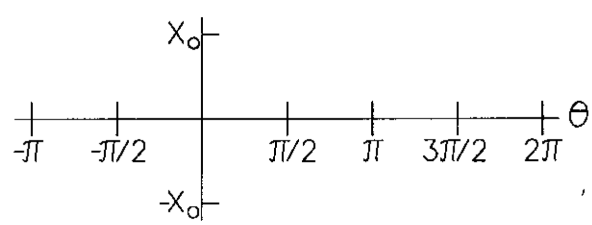
(c) What is y at t = 2 sec?

4. On each of the graphs sketch curves of the indicated functions (arguments in radians):

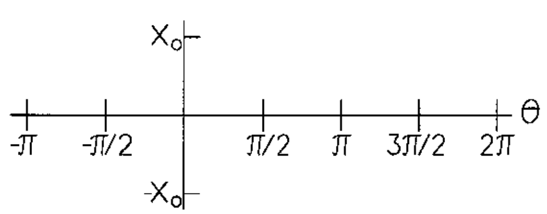
(a) $x = x_o \sin (\theta + \pi)$.



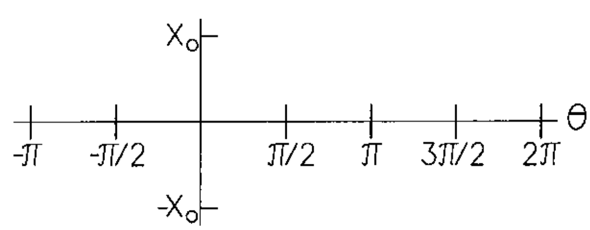
(b) $x = x_o \sin (\theta - \pi/4)$.



(c) $x = x_o \sin (\theta + \pi/4)$.



(d) $x = x_o \cos (\theta - \pi/2)$.



5. From an inspection of the curves drawn in the preceding question, determine the phase angle θ_o in each of the following identities:

(a) $\sin(\theta + \theta_o) = - \sin \theta$

(b) $\cos(\theta + \theta_o) = \sin \theta$

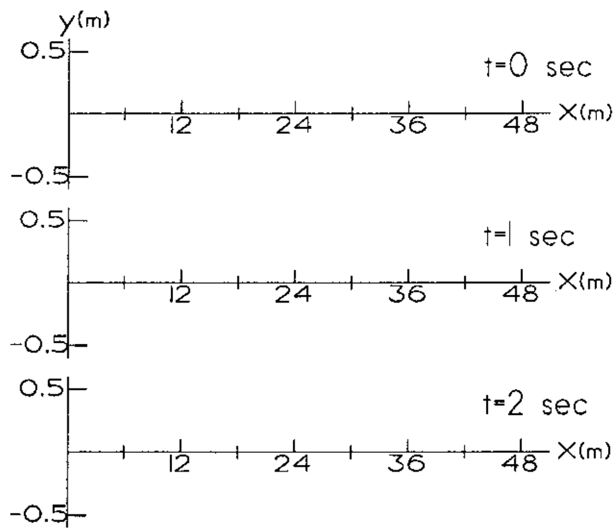
6. Verify each of the equations in the preceding problem by substituting into one of the following general trig identities:

$$\sin(\theta \pm \phi) = \sin \theta \cdot \cos \phi \pm \cos \theta \cdot \sin \phi$$
$$\cos(\theta \pm \phi) = \cos \theta \cdot \cos \phi \mp \sin \theta \cdot \sin \phi$$

7. Travelling waves use mathematics similar to that used to treat oscillations: the wave profile along a direction x (such as the undulating surface of water) can be sketched out using a sinusoidal function with argument (k'x); a phase angle kt determines how the undulation is shifted along x with time. Consider, for instance, the height y of the surface in a water wave given by

$$y = (0.50 \text{ m}) \cos (k'x - kt)$$

where $k' = \pi/12$ rad/m and $k = \pi/2$ rad/sec. Sketch curves giving a profile of the water surface at t = 0, 1 sec, and 2 sec. (HINT: After sketching the first curve, evaluate kt and shift the curve accordingly.)



8. The diameters D of the sun and moon and their distances R from the earth are as follows: sun, $D = 1.4 \times 10^6$ km, $R = 150 \times 10^6$ km; moon, $D = 3.5 \times 10^3$ km, $R = 380 \times 10^3$ km. What angle does each of these astronomical bodies subtend when viewed from the earth - in radians? - in degrees?

9. An aircraft search light beam spreads out at an angle of 2.0°, illuminating a patch of clouds directly above at 10,000 ft elevation. What is angular spread of the beam in radians? (Do not use a DEG/RAD conversion key on your calculator.) From this, estimate the width in feet of the beam at cloud level.