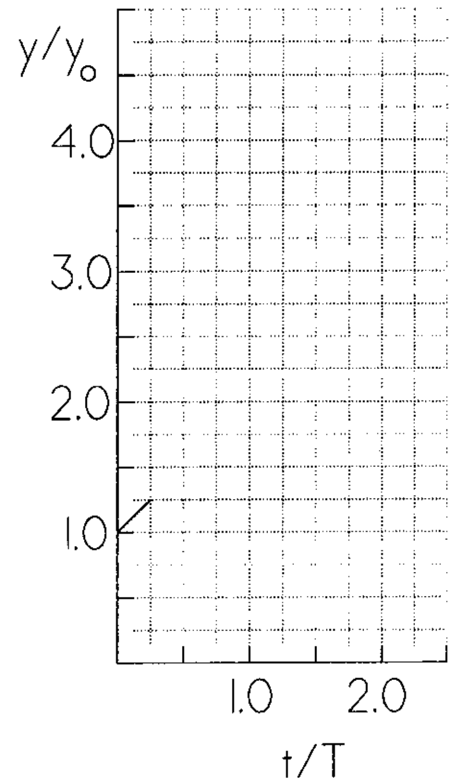
Skill Drill 16

Most of the exercises in this drill aim to reinforce your grasp of the fundamental character of the exponential function. Additional problems review the use of such functions to describe growth and decay in natural phenomena.

1. Construct a curve of y/y_o versus t/T using the following graphical approach:

(a) Starting on the vertical axis at $y/y_o = 1$, draw a line segment with slope = 1 out to $t/T = 0.25$. (This first step is done for you at the right.)

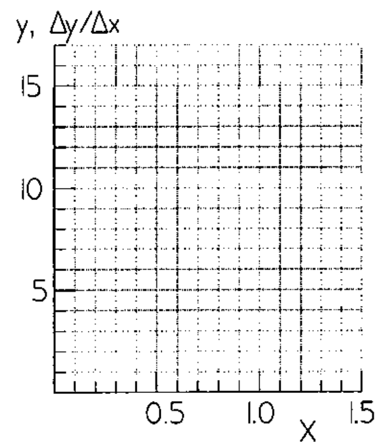


(b) Continue the curve by next drawing a line segment between $t/T = 0.25$ and 0.5 with a slope equal to y/y_o at $t/T = 0.25$.

(c) Add to the curve by successively drawing line segments for 4 more intervals of $t/T = 0.25$, each time with a slope equal to y/y_o at the start of the interval.

(d) On the same graph draw in several circled points calculated from the function $y/y_o = e^{t/T}$, and compare these with the approximation to an exponential curve just constructed.

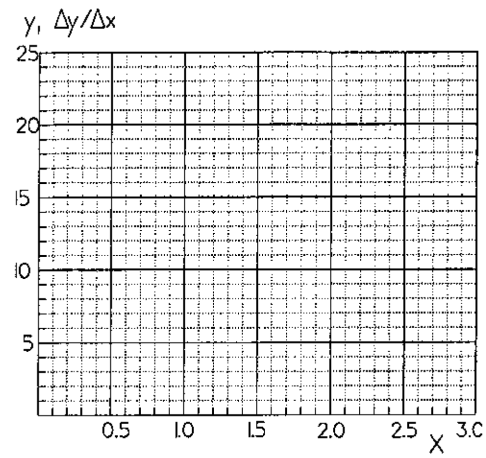
2. (a) Plot a curve of $y = 10^x$ by evaluating y at x = 0, 1/4, 1/2, 3/4, and 1, and connecting the points with a smooth curve. (If your calculator does not have a 10^x key, use the $\sqrt{}$ key to calculate the fourth and other roots.)



(b) Estimate the rates of increase of y at several points (x = 0.2, 0.4, 0.9) by drawing lines tangent to the curve at those points and calculating the slopes of the lines. Plot these rates of increase on the same graph.

(c) Compare the data you have plotted. What is the proportionality constant relating 10^x and its rate of growth?

3. This is essentially a repeat of question 2, except that it focuses on the function $y = e^x$.



(a) Use the e^x key on your calculator to plot the curve out to x = 3.

(b) Estimate rates of growth at x = 0.5, 1.5, and 2.5 and mark these values with circled points on your graph.

(c) From the plotted information determine how the rates of growth of e^x relate to e^x.

4. An example problem in Review 16 discussed exponential growth of a population given by $p = p_o e^{t/T}$, with $p_o = 1000$ animals and T = 0.50 years. The population growth may be also expressed using a power of 2 raised to (t/T_2), where T_2 is the doubling time.

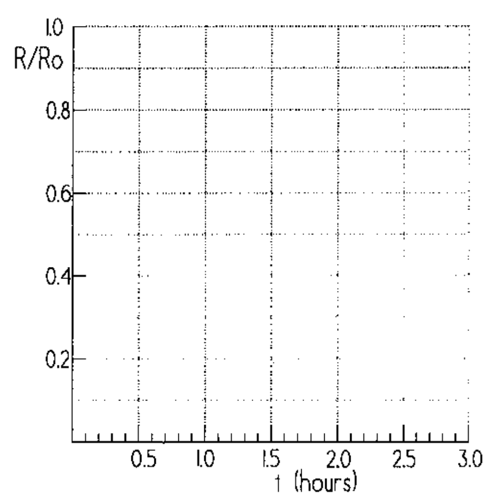
(a) Use the formula $2^x = e^{0.69x}$ to determine the doubling time. Compare with the value of 0.35 years estimated from the curve shown with the example.

(b) What is the population at $t = T_2$? – at $t = 2T_2$?

(c) The example gives the population at t = T as 2718 animals. What is it exactly one doubling time later, i.e., at $t = T + T_2$?

5. The intensity of radiation R given off by a radioactive substance decreases according to

$$R = R_o e^{-t/T}, \text{ where } T = 2.0 \text{ hours.}$$



(a) Plot R/R_o out to t = 3.0 hours.

(b) From the curve find the time it takes for the intensity of the radiation to decay to one-half its initial value. (This time is called the "half-life $T_{½}$.) Compare this with T.

(c) Without using a calculator determine the value of R/R_o at $t = 2T_{½}$?