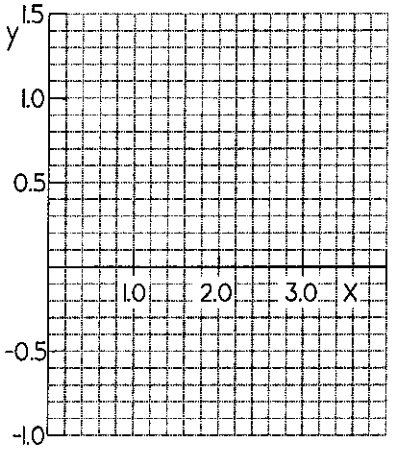
Skill Drill 17

It is fortuitous that this drill on the use of logarithms, as the last drill in the book, bears upon many ideas which were reviewed previously. Besides exponentials (which are closely related to logarithms), you will have to consider proportional expressions, linear equations and graphs, and power functions.

1. Without using a calculator, find $y = \log_2 x$ for $x = 2^{3/2}$, 2, $2^{1/2}$, 1, and $2^{-1/2}$. Plot y versus x on the graph at the right, and connect the points with a smooth curve.



2. Plot the function $y = \ln x$ on the graph at the right, using your calculator to find enough points to draw a smooth curve. Draw a line tangent to the curve at $x = 1$ and determine its slope. Compare with the slope at $x = 1$ for the $\log_2$ curve.

3. Without a calculator find numerical values for the following:

$\log_2 2$	$\log_2 4$	$\ln 1$
$\log_{10} 1$	$\log_{10} 0.01$	$\ln (1/e)$

4. Sound level in decibels is related to sound intensity I (W/m^2) by

$$dB = 10 \log_{10}(I/I_o),$$

where $I_o = 10^{-12}\ W/m^2$. Find I for these sounds: (a) Jet engine at 160 ft (150 dB); (b) Average rock band (120 dB); (c) Car interior at highway speed (90 dB); (d) Busy street (70 dB).

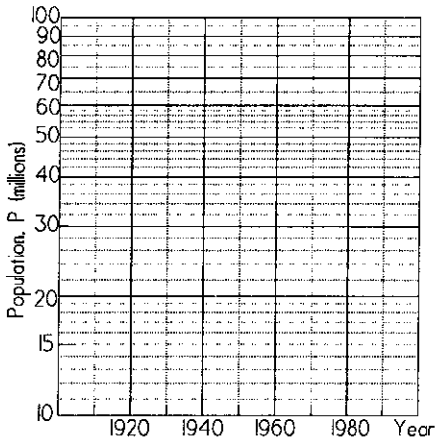
5. The exponentials to the natural base e and to base 2 are related by $e^x = 2^{kx}$. What is k? (You may use the **lnx** key on your calculator.)

6. The natural log and log to the base 2 are related by $k \ln x = \log_2 x$. What is k? (You may use the **lnx** key on your calculator.)

7. Suppose a certain country's population growth during the present century grew according to the following table.

Year	Population (millions)
1900	10
1920	15
1940	22
1960	33
1980	49

These data can be expressed as an exponential function $P = P_o\, e^{t/T}$ (a) On the right make a semi-log plot of the data and draw a straight line through the points. (b) From the slope of the line determine the characteristic time T. (c) Assuming the same population trend in previous centuries, approximately how many years ago was the population 1000 persons?



8. The time τ it takes for a pendulum to make a complete swing back and forth depends upon the length L of the pendulum according to a power law: $\tau = A\, L^n$. Use a log-log plot of the data listed in the table below to find n, as follows:

(a) Plot the data on the graph at the right and draw a straight line through the points. (b) Evaluate the slope of the logarithmic plot and set it equal to n, i.e., $n = (\Delta \log \tau / \Delta \log L)$. Does this agree with the power law given in Review 13? (c) What slope do you get evaluating $(\Delta \ln \tau / \Delta \ln L)$?

L(m)	τ(s)
0.10	0.6
0.25	1.0
0.55	1.5
1.0	2.0
4.0	4.0