Review 1 — Dealing with Numbers

If there is something which characterizes physics when compared with other fields of study, including its sister sciences, it is an extraordinary reliance on numbers to describe and verify conclusions. In this section we review important ways of writing numbers and how numbers can be efficiently combined to give answers with an appropriate degree of precision.

SCIENTIFIC NOTATION

Physics involves concepts which are described by numbers ranging from the unimaginably small to the astronomically large. The most convenient way to express numbers over a wide range is called "scientific notation."

> *In scientific notation numbers are represented by the product of a multiplying factor and a power of ten.*

(A "power of ten" is the number 10 raised to an integer exponent.) Negative as well as positive multiplying factors and exponents may be used. Some examples of numbers written in scientific notation and their ordinary decimal equivalents are the following:

$$35.00 \times 10^{6} = 35{,}000{,}000$$
$$-2.70 \times 10^{3} = -2700$$
$$4.3 \times 10^{-2} = 0.043$$

Besides being a compact way of writing quantities over a wide range of values, scientific notation also gives some idea of how precisely a quantity is known. All the digits in the multiplying factor in front of the power of ten are considered *significant* — they convey meaningful information and are to be "taken seriously." More will be said about "significant figures" later on.

Translating into scientific notation and back again. The above examples illustrate the meaning of the power of ten: 10^6 is simply a way of writing 1,000,000, 10^3 means 1000, and 10^{-2} means 0.01.

> *Given a number in decimal form, to find the power ten in its scientific notation equivalent: the exponent is the number of places from where you wish to place the decimal point in the multiplying factor to where it lies in the decimal form.*

The sign of the exponent depends on whether you count off places to the right (+) or left (–). How this procedure is used to work out one of the numerical examples above is discussed here:

Write 2700 in scientific notation, assuming 3 significant figures.

DISCUSSION: The multiplying factor is written with 3 digits to indicate three significant figures; a convenient value is 2.70 . (We could have also chosen 0.270, or 27.0, etc.; the power of ten simply would have to be different.) Counting off places to the *right* this way:

2 ↷(1) 7 ↷(2) 0 ↷(3) 0. → the exponent in the power of ten is +3.

Thus the scientific notation equivalent is 2.70×10^3.

Here is a second example in which the scientific notation equivalent contains a *negative* exponent in the power of ten:

> Write 0.043 in scientific notation, assuming 2 significant figures.
>
> DISCUSSION: Choose the multiplying factor to be 4.3. Then counting off to the *left* this way:
>
> $0\,.\,\overset{2}{\curvearrowleft}0\,\overset{1}{\curvearrowleft}4\;3 \rightarrow$ the exponent is -2.
>
> The equivalent is thus 4.3×10^{-2}.

To go from scientific notation to a decimal form the above procedure is reversed, as in this example:

> Write 35.00×10^{6} in decimal form.
>
> DISCUSSION: Move the decimal point in the multiplying factor 6 places to the *right*, as follows.(In other words multiply by 1,000,000.)
>
> $3\,5\,.\,\overset{1}{\curvearrowright}0\,\overset{2}{\curvearrowright}0\,\overset{3}{\curvearrowright}0\,\overset{4}{\curvearrowright}0\,\overset{5}{\curvearrowright}0\,\overset{6}{\curvearrowright}0 \rightarrow 35{,}000{,}000.$
>
> The final result is ambiguous about the number of significant figures.

CALCULATIONS AND ESTIMATES

Calculators. The use of electronic calculators is encouraged for finding numerical answers to physics problems, but this should be done only when appropriate and only with intelligence. Operating a calculator is not entirely error-free, but mistakes can often be caught if one has the habit of making quick estimates.

When your answer looks ridiculous it is wise to recheck your logic <u>and</u> your calculation!

Approximate answers. Scientific notation makes it simple to multiply and divide numbers to get an approximate answer without a calculator. As an example consider the following combination of three numbers:

$$\frac{(2700)(0.043)}{35{,}000{,}000} = ?$$

To get a quick estimate, the expression can be rewritten in scientific notation with each multiplying factor rounded off to a single digit. The calculation is then carried out using the usual rules for multiplying and dividing, as follows:

The multiplying factors are separately combined to find the multiplying factor in the answer. To find the exponent in the answer the individual exponents are added if there is a multiplication or subtracted if there is a division.

Thus for the combination of numbers given above

$$\frac{(2.70\times10^{3})(4.3\times10^{-2})}{3.500\times10^{7}} \simeq \frac{(3)(\not{4})}{\not{4}}\times10^{(3-2-7)} = 3\times10^{-6}$$

(The symbol $\simeq$ means "approximately equal." The "slash marks" / indicate that the 4's divide exactly into one another to give a value of 1, i.e., they "cancel.")

Order of magnitude estimates. With practice, getting a sense of the size of an answer by doing rough calculations can become routine and often can be done in one's head. This is particularly true when only an "order of magnitude" answer is wanted. An order of magnitude estimate is usually considered one in which the exact value is rounded off to the nearest factor of ten. Thus an order of magnitude result for the above calculation is

$$\frac{(2700)(0.043)}{35{,}000{,}000} \sim (10^{3})(10^{-2})(10^{-7}) = 10^{-6}$$

(The symbol $\sim$ stands for "is order of magnitude of-.")

SIGNIFICANT FIGURES

The number of digits used to write out a number can tell us how precise the number is meant to be. For instance something described by the number 2.54 can be supposed to have an actual value between 2.535 and 2.545. But simply writing 2.54 means we are uncertain about just where in that range the actual value falls. Only three figures are "significant".

Calculations and significant figures. When 2.54 is multiplied or divided by a number which is more precisely known, the result is only meaningful to three significant figures. For example, the most accurate value we can give for the product (2.54)(3.213) is 8.16. It is not 8.16102, the number which a calculator might display.

> *When multiplying or dividing, the answer cannot be more precise than the least precise factor in the calculation; this usually means that the answer has the same number of digits as the factor with the least number of significant figures.*

(For numbers somewhat greater than a multiple of ten, an extra digit does not imply greater precision. For instance, the three digit number 105, is just about as precise as the two digit number 95.)

In any event, do not confuse the number of decimal places with the number of significant figures. The number 0.0254 has the same number of significant figures as does 2.54; both should be regarded as being known to about 1 part in 254.

> *It is bad form as well as incorrect to write down all the digits displayed by your calculator. Your answer should be rounded off to the appropriate number of significant figures.*

Exact numbers. There is an exception to the above rule about significant figures of products and quotients. Some numbers have, by implication, an unlimited accuracy, and therefore cannot set a limit on the number of significant figures in the answer. For example, suppose we wish to calculate the diameter of a circle, given its radius. If the radius is 3.4 inches then the correct calculation is:

diameter = (2)(3.4 inches) = 6.8 inches.

Although the factor of 2 is written with only one digit, it might be thought of as having an unlimited number of significant figures (2.00000....). Another example is the irrational number $\pi = 3.14159...$; in any calculation, π can be written out, in principle, to as high degree of accuracy as is needed.

CALCULATING POWERS AND ROOTS

Squares and higher powers. These often appear in physics calculations. If a number is expressed in scientific notation it can be raised to a power n using the following rule:

> *First the multiplying factor is raised to the power n to give the multiplying factor of the answer; then the exponent in the power of ten is multiplied by n to give the exponent in the answer.*

That this rule follows from the rule for multiplying numbers expressed in scientific notation is illustrated by this example:

Take the square of 3×10^3, i.e., find $(3 \times 10^3)^2$.

DISCUSSION: The square can be written

$$(3\times10^3)(3\times10^3) = (3)(3) \times (10^3)(10^3) = 9 \times 10^{(3+3)}$$

or as summarized in the rule above

$$(3 \times 10^3)^2 = 3^2 \times 10^{2\times3} = 9 \times 10^6.$$

Fractional powers, or roots. Taking a root of a number expressed in scientific notation is analogous to forming an integer power. The rule can be stated this way:

> *The multiplying factor in the result is the root of the multiplying factor in the original number; the exponent is obtained by* <u>*dividing*</u> *the original exponent by the degree (square, cube, etc.) of the root.*

This rule is really the same as that given for taking integer powers; however in the case of a root the power is a fraction rather than an integer. This can be seen in the following example:

Take the square root of 3.6×10^3.

DISCUSSION: Taking a square root is the same as raising to a power of ½. Thus

$$\sqrt{3.6\times10^3} = (3.6\times10^3)^{1/2} = (36\times10^2)^{1/2} = \sqrt{36}\times10^{2/2} = 60.$$

Notice the trick in this example of making the power of ten an even power (10^2) so that it is easily raised to the ½-power.