Review 2 — Units to Go with the Numbers

In the examples discussed in Review 1 no mention is made of the nature of the things to which the numbers refer. But in fact, throughout physics all numbers are associated with physical entities which, in principle, can be measured. The unit of measurement must always appear with the number. For example:

- The length of the San Francisco Bay Bridge is 1378 *meters.*
- The time for the moon to circle the Earth is 27.3 *days.*

Students should always keep in mind that the numbers used in physics almost always are accompanied by some unit of measurement, the most common of which are so-called metric units.

SYSTEMS OF UNITS

Metric units. You will come across many different units in your study of physics. Some will be familiar to you, such as inches, seconds, and grams. Some will be unfamiliar such as newtons, pascals, and hertz. However the principle units used in much of physics are those based on the following three "metric" quantities:

- the meter (m), a unit of length,
- the second (s), a unit of time, and
- the kilogram (kg), a unit of mass or "amount of matter".

These three basic units (together with a few others related to electricity and some other branches of physics) are the foundation of a system of interrelated units known as the Système International, or S.I. units. The English system of units, including quantities such as the foot and the pound, because of its value in everyday life in the U.S. and in some parts of engineering, is also used to some extent in introductory physics.

Prefixes. Since the numbers which describe physical quantities range from the very large to the very small, a set of standard prefixes are used to designate convenient-sized units which differ by multiples of ten from the primary units. A familiar example is the centimeter (cm) which is a hundredth of a meter. Some other common prefixes are listed in the following table:

Multiple	Prefix	Symbol	Example
10^6	mega-	M	Power: 1 million watts (10^6 W) is a megawatt (1 MW).
10^3	kilo-	k	Mass: 1000 grams (10^3 g) is a kilogram (1 kg).
10^{-3}	milli-	m	Length:a thousandth of a meter (10^{-3} m) is a millimeter (1 mm).
10^{-6}	micro-	µ	Time: a millionth of a second (10^{-6} s) is a microsecond (1 µs).
10^{-9}	nano-	n	Length: a millionth of a millimeter (10^{-9} m) is a nanometer (1 nm).

SIZING UP AN ANSWER

Sizes of things. How big or small are some of these units? Just as numerical estimates are useful before doing a detailed calculation, it is worthwhile to have a sense for the physical size of the quantities involved in problems. Your ability to estimate the approximate size of things will develop with experience. But as a beginning, consider the following list of comparisons among some important units or familiar objects:

Length:
- 2½ cm is about an inch
- a meter is about 3 feet (length of a yardstick)
- a km is almost 2/3 mile
- a typical adult's arm span or height is almost 2 m

Mass and volume:
- a liter (1000 cm^3) is about a quart
- a liter of water has a kg of mass
- a kg of stuff weighs about 2½ pounds
- an average adult's mass is about 70 kg

Time:
- 10^5 seconds is somewhat more than a day
- a human heart beats about once per second
- it takes about 2½ seconds for an object to fall from the top of a 10 story building to the ground, or about ½ second from a table top to the floor.

Making estimates. Finding the exact equivalent of a measurement made in one set of units in terms of another set of units (conversion of units) will be discussed in a later section of this review. But it isn't necessary to go through a formal conversion process to estimate the size of something in familiar terms, as in the following example:

A runner covers 10^6 cm during a race . Is it likely that the runner is a sprinter?

DISCUSSION: The answer is NO. There are 100 cm in a meter so that 1,000,000 cm (10^6 cm) contains 10,000 meters or 10 km. The race is 7 to 8 miles — definitely not a sprint.

COMBINATIONS OF UNITS

An example: speed. Calculations in physics often require that we combine quantities having different units. The result is a quantity whose unit is a combination of the units which went into the calculation. As an example consider a determination of speed, the rate at which an object moves from one position to another:

What is the speed of an automobile which travels 300 meters in 60 seconds?

DISCUSSION: Speed, the rate of change of position, is the number of units of length moved divided by the time it takes. Using the symbol Δ, which can be read "change in" (or something similar), the required calculation is

$$Speed = \frac{change\ in\ distance}{change\ in\ time} = \frac{\Delta\, d}{\Delta\, t} = \frac{300\ m}{60\ s} = 5.0\ m/s$$

The unit m/s, which also may be written $m{\cdot}s^{-1}$, stands for "meters per second". It can be thought of as the number of meters moved per unit time.

Dimensionless ratios. In physics there is no meaning to an answer which is unaccompanied by units, *except* when the answer refers to the comparative size of two quantities having the same units. For example the number of times the diameter of a circle goes into its circumference is $\pi = 3.1416..$, a so-called "dimensionless" or unit-less number.

Getting the units right. To be sure of the correct units in the results of a calculation, it is strongly recommended that units are included with all the numerical quantities you write down. Then

- *the several units which appear in an expression multiply, divide, or cancel one another to form a resultant unit.*

As an example, consider the following problem:

An electron travels from the back end of a television picture tube to the screen, a distance of 50 cm, in 2.5 μs. What is its average speed?

DISCUSSION: It may prove convenient to change to other units during the calculation so that the final units will be more conventional. For instance

$$Speed = \frac{50\ cm}{2.5\ \mu s} = \frac{50\times 10^{-2} m}{2.5\times 10^{-6} s} = 20.\times 10^{4}\ m/s\ .$$

Combinations of units with special names. Some combinations of units are used so often in physics or in technical work that they are given special names. For instance, the term "knot" stands for nautical miles per hour. This unit is used in the following example, which shows how specially named composite units can be manipulated.

An aircraft flies in a straight line at 400 knots for 1.5 hours. How far does it travel?

DISCUSSION: By writing out the composite unit "knots" in terms of its component units, a cancellation of units occurs. The result is then purely in units of length, as follows:

Distance= (400 naut.mi/~~hr~~)(1.5 ~~hr~~) = 600 naut.miles .

Many specially named units will crop up in your study of physics. It is worthwhile to know precisely how each of the more common ones are defined.

CONVERSION OF UNITS

Conversion factors. Knowing the value of a physical quantity in terms of some particular unit, you may frequently want to know its equivalent in terms of another unit. Change (or "conversion") of units can be made by multiplying (or dividing) the original unit by an appropriate "conversion factor" whose value can be found, if necessary, from information listed in a table.

A conversion factor is simply the ratio of a quantity stated in one unit to the same quantity stated in another unit. Thus both units will appear when the conversion factor is written out. For instance, suppose an answer stated in feet is to be converted to the answer stated in meters. A table might tell you "1 m = 3.28 ft"; appropriate conversion factors are either (3.28 ft/meter) or (1 m/3.28 ft).

Multiply or divide by the appropriate conversion factor so that the unwanted unit cancels, leaving the desired unit in the result.

This illustrated for a meters-to-feet conversion as follows:

How many feet does a runner go in a 200-meter dash?

DISCUSSION: Multiplication by the first of the above conversion factors results in a cancellation of "meters". Thus

$$\text{Distance in feet} = (200\ \cancel{m})(3.28\ \text{ft}/\cancel{m}) = 656\ \text{ft}.$$

Chains of conversions. Sometimes it is necessary to convert between two units for which conversion information cannot be found directly from a table. In such a case several known conversion factors can be used so that all the necessary cancellations of units take place. Here is an example, again involving length units:

How many meters are in exactly 1 mile? Use the following conversion information: 5280 feet are in a mile, 12 inches are in a foot, 2.54 cm are in an inch, and 100 cm are in a meter.
DISCUSSION: Chain together the appropriate conversion factors as follows.

$$\text{Distance in meters} = (1\ \cancel{mi})\left(\frac{5280\ \cancel{ft}}{1\ \cancel{mi}}\right)\left(\frac{12\ \cancel{in}}{1\ \cancel{ft}}\right)\left(\frac{2.54\ \cancel{cm}}{1\ \cancel{in}}\right)\left(\frac{1\ \text{m}}{100\ \cancel{cm}}\right)$$

$$= 1609\ \text{m}.$$

All the numerical conversion factors in this problem happen to be exact, as they are defined to have these values by international agreement. Therefore the result of the unit conversion is good to as many significant figures as is needed. (We chose to round off to four figures in this example.)

Unlike those in the above example, some conversion factors are determined from laboratory measurements. For these, the number of significant figures after conversion are limited by the number of significant figures in the conversion factors.