# Skill Drill 6

The following exercises should help bring you up to speed doing basic algebraic manipulations. Questions in the first part of the Drill involve only abstract symbols and numbers, typical of problems you find in mathematics courses. But in the spirit of problem solving as you will find it in physics there are also a number of word problems and quantities with units.

1. This exercise is a tour of the main points covered in Review 6.

(a) Using cross multiplication find factor e in terms of the other quantities in the expression: $(a+b)/c = d/e$.

(b) Transpose terms so that d alone appears on the lhs of the equation: $a+b-c = -d+e$.

(c) Rearrange the parenthesis in each of the following expressions so as to put it in the form indicated to its right. In each case what is X in terms of a, b, c, and d?

$(a-b-c/2)+d \rightarrow a-b+(X)$

and

$(abc)/d \rightarrow ab/(X)$ .

(d) Using numerical values a=3, b=4, c=-5, and d=5 verify that

$$\frac{(a+b+c)d}{abc} = \frac{d}{bc} + \frac{d}{ac} + \frac{d}{ab}$$

(e) Using numerical values a=3, b=4, c=-5, and d=5 verify that

$(a+b)(c+d) = ac+bc+ad+bd$ .

(f) Using the distributive rule show that

$(a+b)(c+d) = ac + bc + ad + bd$

(g) Using the result of part (f) obtain the following useful "binomial expansions":

$(x-y)^2 = x^2 + y^2 - 2xy$

and

$(x+y)(x-y) = x^2 - y^2$ .

2. Find a factor which is common to every term in each of the following expressions. Rewrite each expression with the factor separated from other terms.

(a) $4ab + 2bc$

(b) $2b^2 - 6ab$

(c) $2x^2y + 4bxy + 2y^2x$

(d) $3x(a^2 + 2ay + y^2) + 6x^2(y + a)$
(NOTE: In this expression the common factor contains a binomial.)

3. Expand the following expressions into a polynomial in x by carrying out the indicated multiplications and grouping together like powers of x.

(a) $x(4x+3ax+2)$

(b) $(x+2)(x-5)$

(c) $x(x^2-3x+6)$

(d) $(2-x)(2x^2+4x-2)$

4. Solve the following equations. (Isolate x on the lhs, then calculate its numerical value.)

(a) $4x + 12 = x + 30$

(b) $4x - 3 = 5(x+3)$

(c) $\frac{5(x+4)}{3} = 8 + \frac{3x}{2}$

(d) $0.75x - 0.25x - 2.00 = 0.25x + 3.00$

5. Set up an equation to represent each of the following problems and solve for the unknown. Be sure to include units in the solution.

(a) Admission to the college play costs \$5.00 for the general public and \$3.00 for students and faculty. Half as many tickets were sold to students and faculty as to the general public, and a total of \$455 was collected. How many general admission tickets were sold?

(b) You buy two certificates of deposit, one of which yields 9.0% interest annually, while the other earns 8.5%. The amount invested in the second C.D. is twice the amount invested in the first. If the total amount earned in one year is \$780, how much is the value of the first C.D.?

(c) Two planes, one in New York and one in London, take off at the same time heading for each other's city, 3469 miles apart. The eastbound plane, travelling with the jet stream, flies at 700 miles/hour while the westbound plane travels at 600 mph. How long after taking off do they meet?

6. The following problems have a scientific context. You do not need to understand all the details, but you should be able to *rearrange each of the given equations algebraically* as directed.

(a) The location of the "center of mass" $X_{cm}$ of a molecule consisting of two atoms of masses $m_1$ and $m_2$ is given by

$$(m_1+m_2)X_{cm} = m_1x_1 + m_2x_2$$

where $X_{cm}$ and the position of the atoms $x_1$ and $x_2$ are measured from some common origin. Find $m_1$ in terms of the other quantities.

(b) When a marble of mass m having a speed $v_1$ rolls up an incline its speed decreases to $v_2$. If the height of the incline is h the two speeds are related very closely by the equation

$$\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_2^2 = mgh$$

where g is a constant (gravitational acceleration). Write an expression for $v_1$ in terms of the other quantities.

(c) The ability of an electrical component to permit a flow of electricity through it is described by a quantity called resistance R. When three components are connected in a certain standard way called a "parallel connection" the ability of the entire arrangement to conduct electricity $R_{eq}$ is related to the individual resistances $R_1$, $R_2$, and $R_3$ by the equation

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}.$$

($R_{eq}$ means "equivalent resistance.") Show that this expression can be rewritten as

$$R_{eq} = \frac{R_1R_2R_3}{R_2R_3 + R_3R_1 + R_1R_2}.$$