# Review 9 — Geometry II - Angles, Shape, and Size

Making a sketch which shows the geometry of a problem is usually not sufficient for solving a problem. Quantitative information has to be used, but in many cases this is not given directly in the statement of the problem. Dimensions or angles which are needed to find an answer may have to be deduced from other lengths and angles shown in the diagram. The geometrical facts which can be used to find such relationships, as well as important formulas for areas and volumes are reviewed in this section.

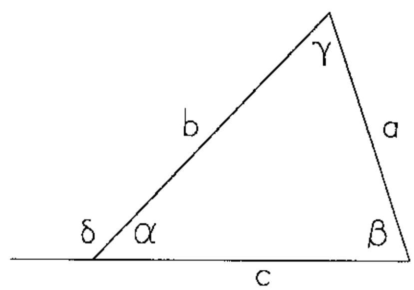
## ANGLES AND POLYGONS

**Angle measure.** Angles are most commonly measured in degrees. Radian measure, as an alternative for dealing with certain important types of problems, will be discussed in a later review.

Special names are given to angles which relate in a simple way to certain fractions of 360°, the angle swept out by a line making one complete rotation in a plane. Half a rotation results in a *straight angle* or 180°; a quarter rotation results in a *right angle* (90°). Any angle less than 90° is an *acute angle*. The "complement" of an acute angle is the difference between that angle and a straight angle. Any angle between 90° and 180° is called an *obtuse angle*.

**Angles of a triangle.** Angles in one part of a triangle can be related to other angles in that triangle using these facts:

*(a) The sum of the interior angles of a triangle is a straight angle.*




In terms of the triangle pictured at the right $\alpha + \beta + \gamma = 180°$.

*(b) An exterior angle equals the sum of the opposite interior angles.*

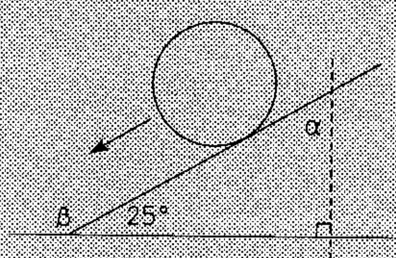
(An exterior angle is formed by extending one side of a triangle beyond the vertex.) In terms of the figure $\delta = \beta + \gamma$.

This problem applies these ideas to a drawing similar to one used in an example in the last Review:

A hoop is rolling down an inclined plane which makes an angle of 25° with respect to the horizontal.

(a) What angle does the incline make with respect to the vertical (α in this drawing)?

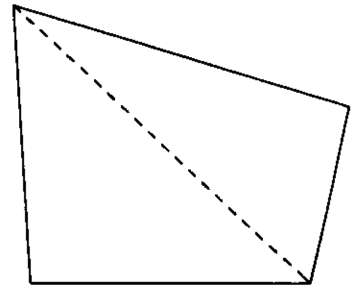
(b) When the hoop reaches the bottom of the incline, it runs out on a horizontal ramp. What angle does this ramp make with the incline (β in this drawing)?




DISCUSSION: (a) In the right triangle shown, the three interior angles add up to a straight angle, i.e., $180° = \alpha + 25° + 90°$. Hence $\alpha = 65°$.

(b) β is an exterior angle so that $\beta = \alpha + 90° = 155°$.

**Angles in a quadrilateral.** The following rule is easily understood when it is recognized that a diagonal line (dashed) drawn between opposite corners divides a four sided polygon into two triangles. Each of these triangles contributes 180° to the sum of the interior angles of the quadrilateral.

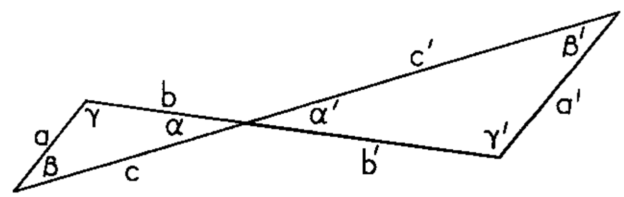


*The interior angles of a plane quadrilateral add up to two straight angles.*

## SIMILAR TRIANGLES

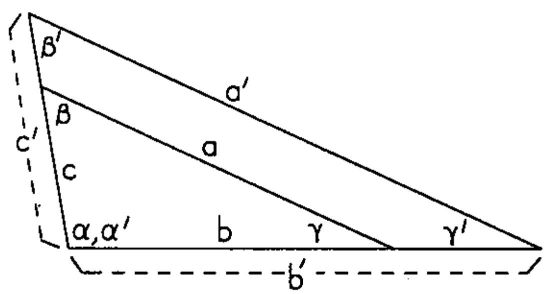
Triangles which are of the same shape but differ only in size and/or orientation are called *similar triangles.* They often may arise in a drawing when triangles have parallel sides or common vertices. In the two examples shown here sides a and a′ are parallel.

Three important properties of similar triangles are the following:




*(a) The three interior angles are the same in all similar triangles.*

In both pairs of triangles shown here $\alpha=\alpha'$, $\beta=\beta'$, and $\gamma=\gamma'$.



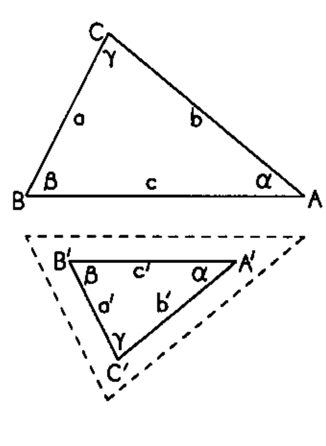

*(b) The three sides are in the same proportion in all similar triangles.*

In the examples of similar triangles shown here a:b:c = a′:b′:c′. A third property follows logically from this one:

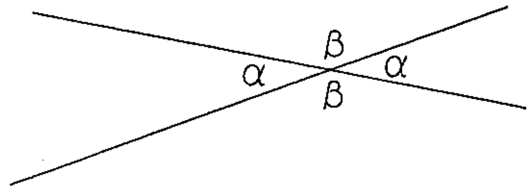
*(c) All the sides of a triangle are in the same proportion to the corresponding sides of any triangle similar to it.*

In the triangles shown above a:a′ = b:b′= c:c′.

**Mirror image triangles.** Some geometrical arrangements result in a pair of triangles whose shapes, while not identical, share all the characteristics of similar triangles just listed. This is the case when one of the triangles has the shape which the other would have were it reflected in a mirror. In the drawing the dashed lines show the mirror image of triangle ABC; triangle A′B′C′, which has the same shape, can be considered to be similar to ABC. Be careful, however, in setting up proportions among the side lengths to recognize which correspond to one another.



## ANGLES FORMED BY INTERSECTING STRAIGHT LINES

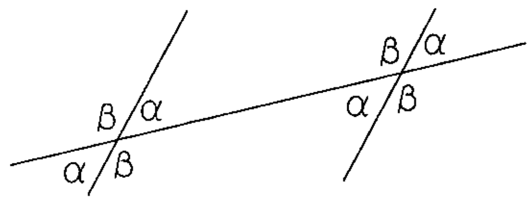



**Opposing angles.** Opposing angles are formed when two lines cross over each other. (In the drawing the two α's are opposing angles; so are the β's.)

> *Opposing angles formed by the intersection of two straight lines are equal.*

This explains why $\alpha = \alpha'$ in the first example of similar triangles in the last section.

**Angles formed by parallel lines.** As shown in the drawing:




> *A straight line intersects parallel straight lines at equal angles.*

This explains why sides a and a′ are parallel in the first two cases of similar triangles shown on the previous page.

**Angles formed by pairs of perpendiculars.** In physics diagrams lines drawn perpendicular or "*normal*" to other lines frequently occur; for example, horizontal and vertical lines. An extremely useful fact is the following:

> *If a pair of lines intersect at some angle, two other lines which are perpendicular to them intersect at the same angle.*

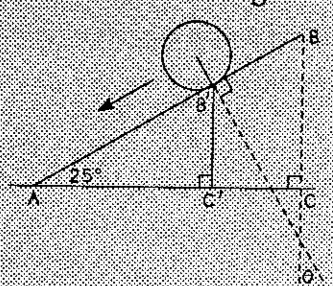
This example uses this idea to solve a problem involving similar triangles:

Referring to the last example of a hoop on an incline:

(a) What angle does the radius line which passes through the point of contact make with the vertical?

(b) If the hoop has rolled one-third of the way along the incline from the upper end, how much elevation above the lower horizontal ramp has it lost?

DISCUSSION: (a) Draw in a vertical (dashed line from point B) and extend the radius until it crosses it. (The acute angle of intersection is labelled θ.) Since the radius line is perpendicular to AB and the vertical line is perpendicular to AC, they intersect at the same angle which the incline makes with the horizontal. Thus $\theta = 25°$.

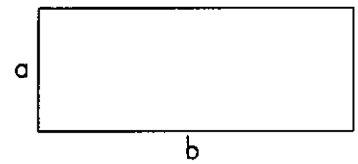



(b) Triangle AB′C′ is similar to triangle ABC. Hence B′C′ is in the same proportion to BC as AB′ is to AB; one third of the original elevation has been lost.

## AREAS AND VOLUMES

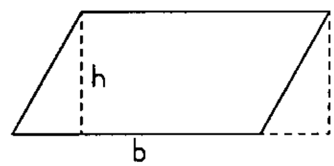
Areas and volumes of geometrical figures are needed often enough in physics problems so that it is worthwhile to remember the most important formulas. But, as elsewhere in physics, do not rely solely on rote memory; always be guided by your good sense. For instance all formulas for areas contain length measurements to the second power. Some formulas are simply extensions of other rules; several cases are noted below:

**Rectangle.** *Area* $= ab$ .



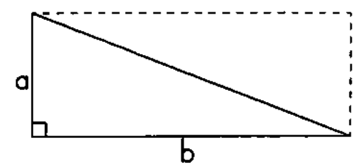

**Parallelogram.** A perpendicular of height h would cut off a triangle which would just fill in an area on the other side of the figure to form a rectangle; so

*Area* $= hb$.



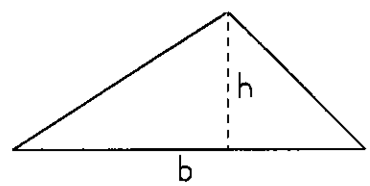

**Right triangle.** This is half a rectangle; so

*Area* $= \frac{1}{2}ab$ .



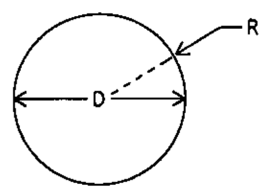

**Other triangles.** A perpendicular of height h (the "altitude") divides the triangle into two right triangles whose areas combine to give simply

*Area* $= \frac{1}{2}hb$.

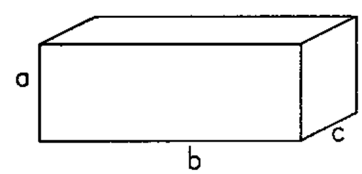



**Circles.** Know the formulas both in terms of radius R and diameter D:

*Area* $= \pi R^2 = \pi D^2/4$
*Circumference* $= 2\pi R = \pi D$.

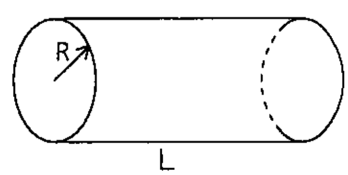



**Rectangular parallelepipeds (and cubes).** Surface area is the sum of the rectangular areas of all six faces. Also
*Volume* $= abc$ .




**Right circular cylinder**. The surface area includes two "end caps" and the cylindrical "wrapper"; thus

*Surface area* $= 2\pi R^2 + 2\pi RL$.




The volume may be thought of as being generated by sweeping the cross-sectional area along the axis a distance L; thus

*Volume* $= \pi R^2 L$ .

**Sphere.** A sphere has roughly half the volume and roughly half the surface area of a cube that would just hold it.
*Volume* $= (4/3)\pi R^3$ .
*Surface area* $= 4\pi R^2$ .