The Simple Pendulum

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Objectives

The purposes of this experiment are: (1) to study the motion of a simple pendulum, (2) to study simple harmonic motion, (3) to learn the definitions of period, frequency, and amplitude, (4) to learn the relationships between the period, frequency, amplitude and length of a simple pendulum and (5) to determine the acceleration due to gravity using the theory, results, and analysis of this experiment.

Theory

A simple pendulum may be described ideally as a point mass suspended by a massless string from some point about which it is allowed to swing back and forth in a place. A simple pendulum can be approximated by a small metal sphere which has a small radius and a large mass when compared relatively to the length and mass of the light string from which it is suspended. If a pendulum is set in motion so that is swings back and forth, its motion will be periodic. The time that it takes to make one complete oscillation is defined as the period T. Another useful quantity used to describe periodic motion is the frequency of oscillation. The frequency f of the oscillations is the number of oscillations that occur per unit time and is the inverse of the period, $f = 1/T$. Similarly, the period is the inverse of the frequency, $T = l/f$. The maximum distance that the mass is displaced from its equilibrium position is defined as the amplitude of the oscillation.

When a simple pendulum is displaced from its equilibrium position, there will be a restoring force that moves the pendulum back towards its equilibrium position. As the motion of the pendulum carries it past the equilibrium position, the restoring force changes its direction so that it is still directed towards the equilibrium position. If the restoring force $\vec{F}$ is opposite and directly proportional to the displacement x from the equilibrium position, so that it satisfies the relationship

$$\vec{F} = -k\,\vec{x} \tag{1}$$

then the motion of the pendulum will be simple harmonic motion and its period can be calculated using the equation for the period of simple harmonic motion

$$T = 2\pi\sqrt{\frac{m}{k}}\,. \tag{2}$$

It can be shown that if the amplitude of the motion is kept small, Equation (2) will be satisfied and the motion of a simple pendulum will be simple harmonic motion, and Equation (2) can be used.

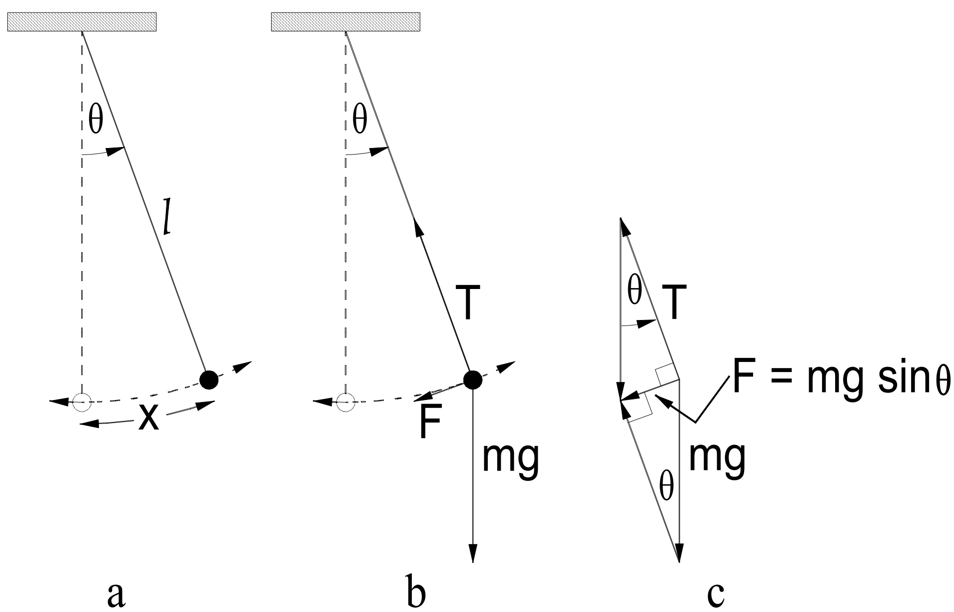

Figure 1. Diagram illustrating the restoring force for a simple pendulum.

The restoring force for a simple pendulum is supplied by the vector sum of the gravitational force on the mass. mg, and the tension in the string, T. The magnitude of the restoring force depends on the gravitational force and the displacement of the mass from the equilibrium position. Consider Figure 1 where a mass m is suspended by a string of length l and is displaced from its equilibrium position by an angle θ and a distance x along the arc through which the mass moves. The gravitational force can be resolved into two components, one along the radial direction, away from the point of suspension, and one along the arc in the direction that the mass moves. The component of the gravitational force along the arc provides the restoring force F and is given by

$$F = -\,mg\sin\theta \tag{3}$$

where g is the acceleration of gravity, θ is the angle the pendulum is displaced, and the minus sign indicates that the force is opposite to the displacement. For small amplitudes where θ is small, $\sin\theta$ can be approximated by θ measured in radians so that Equation (3) can be written as

$$F = -\,mg\,\theta\,. \tag{4}$$

The angle θ in radians is $\frac{x}{l}$, the arc length divided by the length of the pendulum or the radius of the circle in which the mass moves. The restoring force is then given by

$$F = -\,mg\,\frac{x}{l} \tag{5}$$

and is directly proportional to the displacement x and is in the form of Equation (1) where $k = \frac{mg}{l}$. Substituting this value of k into Equation (2), the period of a simple pendulum can be found by

$$T = 2\pi\sqrt{\frac{m}{\left(mg/l\right)}} \tag{6}$$

and

$$T = 2\pi\sqrt{\frac{l}{g}}\,. \tag{7}$$

Therefore, for small amplitudes the period of a simple pendulum depends only on its length and the value of the acceleration due to gravity.

Apparatus

The apparatus for this experiment consists of a support stand with a string clamp, a small spherical ball with a 125 cm length of light string, a meter stick, a vernier caliper, and a timer. The apparatus is shown in Figure 2.

Procedure

1. The simple pendulum is composed of a small spherical ball suspended by a long, light string which is attached to a support stand by a string clamp. The string should be approximately 125 cm long and should be clamped by the string clamp between the two flat pieces of metal so that the string always pivots about the same point.

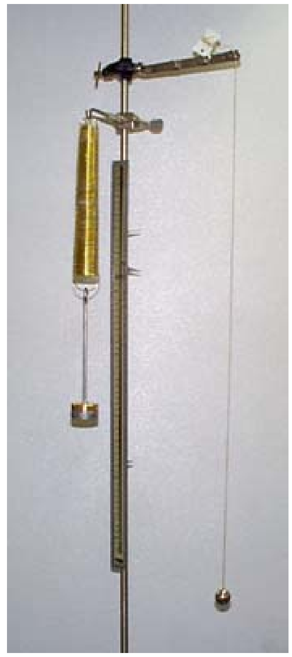

Figure 2. Apparatus for simple pendulum.

2. Use a vernier caliper to measure the diameter d of the spherical ball and from this calculate its radius r. Record the values of the diameter and radius in meters.

3. Prepare an Excel spreadsheet like the example shown in Figure 3. Adjust the length of the pendulum to about .10 m. The length of the simple pendulum is the distance from the point of suspension to the center of the ball. Measure the length of the string l_s from the point of suspension to the top of the ball using a meter stick. Make the following table and record this value for the length of the string. Add the radius of the ball to the string length l_s and record that value as the length of the pendulum $l = l_s + r$.

4. Displace the pendulum about 5º from its equilibrium position and let it swing back and forth. Measure the total time that it takes to make 50 complete oscillations. Record that time in your spreadsheet.

5. Increase the length of the pendulum by about 0.10 m and repeat the measurements made in the previous steps until the length increases to approximately 1.0 m.

6. Calculate the period of the oscillations for each length by dividing the total time by the number of oscillations, 50. Record the values in the appropriate column of your data table.

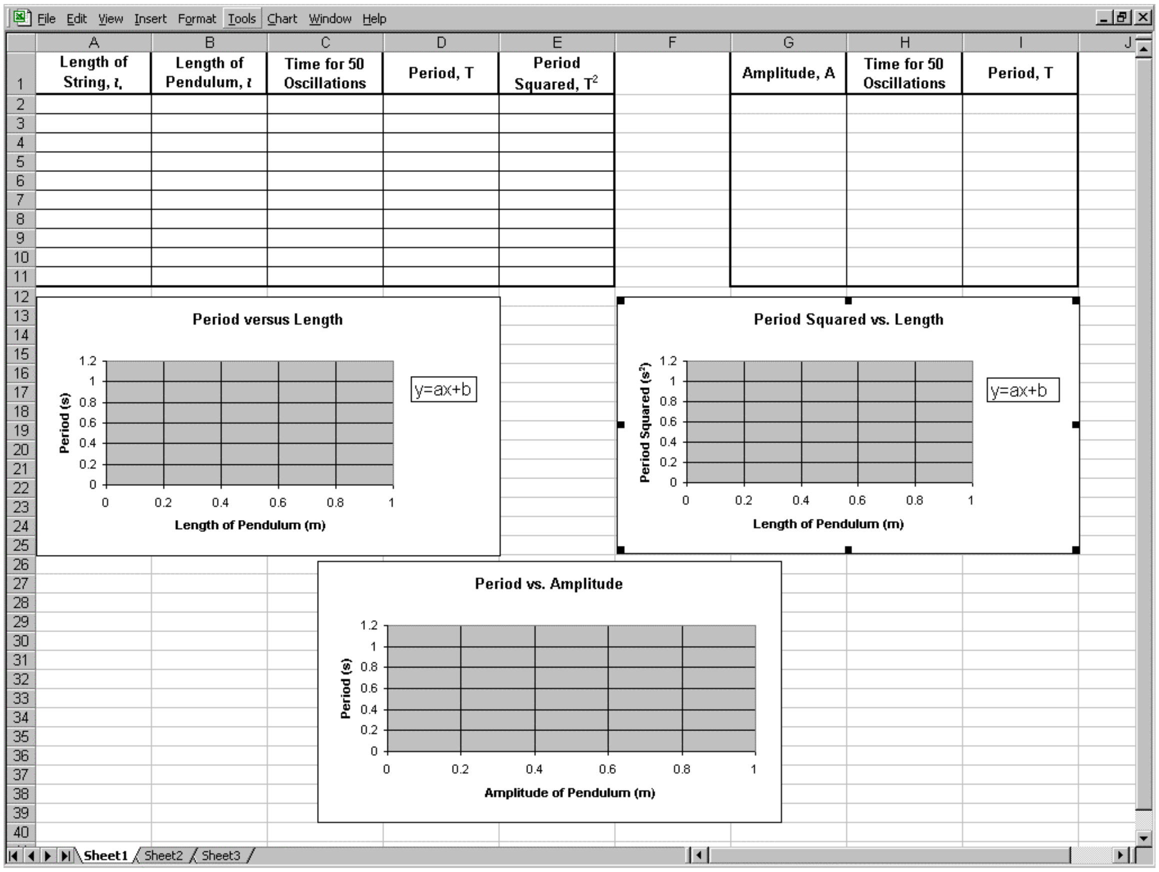

Figure 3. Example of Excel spreadsheet for recording and analyzing data.

7. Graph the period of the pendulum as a function of its length using the chart feature of Excel. The length of the pendulum is the independent variable and should be plotted on the horizontal axis or abscissa (x axis). The period is the dependent variable and should be plotted on the vertical axis or ordinate (y axis).

8. Use the trendline feature to draw a smooth curve that best fits your data. To do this, from the main menu, choose *Chart* and then *Add Trendline . . .* from the dropdown menu. This will bring up a *Add Trendline* dialog window. From the *Trend* tab, choose *Power* from the *Trend/Regression* type selections. Then click on the *Options* tab and select *Display equations on chart* option.

9. Examine the power function equation that is associated with the trendline. Does it suggest the relationship between period and length given by Equation (7)?

10. Examine your graph and notice that the change in the period per unit length, the slope of the curve, decreases as the length increases. This indicates that the period increases with the length at a rate less than a linear rate. The theory and Equation (7) predict that the period depends on the square root of the length. If both sides of Equation 7 are squared then

$$T^2 = \frac{4\pi^2}{g} l . \tag{8}$$

If the theory is correct, a graph of T^2 versus l should result in a straight line.

11. Square the values of the period measured for each length of the pendulum and record your results in the spreadsheet.

12. Use the chart feature again to graph the period squared, T^2, as a function of the length of the pendulum l. The period squared is the dependent variable and should be plotted on the y axis. The length is the independent variable and should be plotted on the x axis.

13. Examine your graph of T^2 versus l and check to see if there is a linear relationship between T^2 and l so that the data points lie along a line.

14. Use the trendline feature to perform a linear regression to find a straight line that best fits your data points. This time from the *Add Trendline* dialog window. choose *Linear* from the *Trend/Regression* type selections. Click on the *Options* tab and once again select the *Display equations on chart* option. This should draw a straight line that best fits the data and should display the equation for this straight line.

15. Equation (8), $T^2 = \frac{4\pi^2}{g} l$, is of the form y=ax+b where $y=T^2$, $a = \frac{4\pi^2}{g}$, $x=l$, and b=0. A graph of T^2 versus l should therefore result in a straight line whose slope, a, is equal to $\frac{4\pi^2}{g}$. From the equation for the trendline, record the value for the slope, a, and from the equation $a=\frac{4\pi^2}{g}$ find g, the acceleration due to gravity.

16. Compare your result with the accepted value of the acceleration due to gravity 9.8 m/s^2. Calculate the percent difference in your result and the accepted result.

    % Difference = [(your result - accepted value)/accepted value] x 100%

17. Using the accepted value of the acceleration due to gravity and Equation 7 calculate the period of a simple pendulum whose length is equal to the longest length measured in Table 1. Compare this theoretical result with the measured experimental result and calculate the percent difference.

% Difference = [(Experimental Result - Theoretical Result)/Theoretical Result] x 100%.

18. The equation for the period of a simple period, Equation (7), was developed by assuming that the amplitude is small. The range of amplitudes over which Equation

(7) is valid is to be determined by measuring the period of a simple pendulum with different amplitudes.

19. Adjust the length of the pendulum to about 0.6 m. Measure the period of the pendulum when it is displaced 5°, 10°, 15°, 20°, 25°, 30°, 40°, 50°, and 60° from its equilibrium position. Make a table to record the period T as a function of the amplitude A.

20. Using your data, make a graph of the period versus the amplitude.

21. Measure the length of the pendulum and use Equation (7) to calculate the period of the pendulum. Add this theoretical point to your graph for the period with zero amplitude.

22. Examine your graph for the behavior of the period with amplitude. What conclusions can you draw from your data regarding the range of amplitudes over which Equation (7) is valid?

Questions

1. How would the period of a simple pendulum be affected if it were located on the moon instead of the earth?

2. What effect would the temperature have on the time kept by a pendulum clock if the pendulum rod increases in length with an increase in temperature?

3. What kind of graph would result if the period T were graphed as a function of the square root of the length, $\sqrt{l}$.

4. What effect does the mass of the ball have on the period of a simple pendulum? What would be the effect of replacing the steel ball with a wooden ball, a lead ball, and a ping pong ball of the same size?