## Lab. 6

## Torque and Static Equilibrium

Goal: In a previous lab, you investigated the conditions for static equilibrium of a particle. In this lab, we extend this earlier work by considering the equilibrium of an extended object.

Theory: Since an object in static equilibrium is not translating, the sum of the forces along any axis must be zero,

$$
\begin{aligned}
& \sum F_{x}=0 \\
& \sum F_{y}=0
\end{aligned}
$$

In addition, since the object is not rotating, the sum of all torques about any axis is also zero. The torque, $\tau$, generated by a force, $F$, acting at a distance, $x$, from the axis of rotation is given by

$$
\tau=F x \sin \theta
$$

where $\theta$ is the angle between the force and the position vector joining the axis and the point where the force, $F$, is acting. See Fig. 1.


Fig. 1. The torque on the object due to the force, $F$, is $F x \sin \theta$.

The sign convention for torques is as follows. If the force acts to rotate the object about the chosen axis in the counter-clockwise direction, the torque is positive. Similarly, if the force acts to rotate the object in the clockwise direction, the negative sign is used. In Fig. 1, the torque is positive.

The brass beam used in this lab may be considered to have a uniform mass distribution in spite of the small number of holes drilled into it. For the purposes of computing the torque on a uniform object due to its weight, the mass of the object may be considered to be concentrated at the center. In other words, when computing the torque, the weight of a uniform object acts at the center of mass.

Procedure for Part 1: Configure your apparatus as shown in Fig. 2. Choose a mass M of about 250 g . The tendency of the brass beam to rotate about the pivot is balanced by the spring gauge. Attach the spring guage to the beam with a loop of string so that the point of attachment will slide along the beam. For this part of the lab, always keep the beam horizontal and the spring gauge and its attached string vertical, that is, the angle between the string and the beam is always $90^{\circ}$. Starting at the end of the beam near the mass M, measure the force, F , on the spring gauge every few centimeters along the beam until you
reach the maximum range of the gauge; be sure to use the largest spring gauge available. Plot a graph of the force, F, versus the inverse of the distance between the pivot and the point at which the force of the spring scale was applied. Can you fit a straight line through these points?


Fig. 2. Configuration of the apparatus for Part 1. Be sure the beam is horizontal and the spring gauge and attached string are vertical.

The slope of the line on your graph is equal to the torque generated by the force in the spring about the pivot in the counter-clockwise direction; see Fig. 2. Because the beam is stationary, that is, in static equilibrium, the net torque about the pivot (or any other point, for that matter) is zero. The magnitude of the counter-clockwise (positive, as shown in Fig. 2) torque due to the force of the spring gauge exactly equals that of the clockwise (negative) torque caused by gravity acting on the beam and the mass, M. Compare the slope of the line on your graph with the clockwise torques due to gravity.

Finally, remove your mass, M, and place a different one at another position on the beam. Apply the force of the spring gauge to the beam at a point about two-thirds along the beam away from the pivot, still keeping the spring gauge and string perpendicular to the beam. Recall the conditions of static equilibrium. Calculate the force applied to the beam by the pivot; since there are no forces on the beam in the $x$-direction, you need only consider the $y$-direction. Calculate the torques acting on the beam about any two points and show that in each case, the net torque is always zero to within a small error due to uncertainties in the spring gauge, etc.

Procedure for Part 2: Configure your apparatus as shown in Fig. 3. In this configuration, there is no pivot and the beam is freely suspended. Avoid angles of $90^{\circ}$ and do not exceed the maximum range of the spring gauges. To be certain that the configuration does not change while your are making your measurements, you may wish to tape the strings supporting the spring gauges to the upper support. In a detailed sketch, record the magnitude of all the forces, the angle each force makes with the beam and the point at which each force is applied to the beam. Also record the angle the beam makes with the vertical. Calculate the $x$ and $y$ components of each force and show that the sum of the $x$-components and the sum of the $y$-components are each zero; you may find it convenient to orient the $x$-axis along the beam. In addition, sum the torques about two different points on the beam and show that in each case, the sum of the torques also vanishes.


Fig. 3. Configuration of the apparatus for Part 2.

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