Chapter 12

You're Getting Warm: Thermodynamics

In This Chapter

- Converting between temperature scales
- Working with linear expansion
- Calculating volume expansion
- Using heat capacities
- Understanding latent heat

Thermodynamics is the study of heat. It's what comes into play when you drop an ice cube into a cup of hot tea and wait to see what happens — if the ice cube or the tea wins out.

In physics, you often run across questions that involve thermodynamics in all sorts of situations. This chapter refreshes your understanding of the topic and lets you put it to use with practice problems that address thermodynamics from all angles.

Converting Between Temperature Scales

REMEMBER

You start working with questions of heat by establishing a scale for measuring temperature. The temperature scales that you work with in physics are Fahrenheit, Celsius (formerly centigrade), and Kelvin.

Fahrenheit temperatures range from 32° for freezing water to 212° for boiling water. Celsius goes from 0° for freezing water to 100° for boiling water. Following are the equations you use to convert from Fahrenheit (F) temperatures to Celsius (C) and back again:

$$C = \frac{5}{9}(F - 32)$$

$$F = \frac{9}{5}C + 32$$

The Kelvin (K) scale is a little different: Its 0° corresponds to *absolute zero,* the temperature at which all molecular motion stops. Absolute zero is at a temperature of –273.15° Celsius, which means that you can convert between Celsius and Kelvin this way:

$$K = C + 273.15$$

$$C = K - 273.15$$

To convert from Kelvin to Fahrenheit degrees, use this formula:

$$F = \frac{9}{5}(K - 273.15) + 32 = \frac{9}{5}K - 459.67$$

Technically, you don't say "degrees Kelvin" but rather "Kelvins," as in 53 Kelvins. However, people persist in using "degrees Kelvin," so you may see that usage in this book as well.

EXAMPLE

Q. What is 54° Fahrenheit in Celsius?

A. The correct answer is 12° C.

1. Use this equation:

$$C = \frac{5}{9}(F - 32)$$

2. Plug in the numbers:

$$C = \frac{5}{9}(F - 32) = (0.55) \cdot (54 - 32) = 12^\circ C$$

1. What is 23° Fahrenheit in Celsius?

Solve It

2. What is 89° Fahrenheit in Celsius?

Solve It

3. What is 18° Celsius in Fahrenheit?

Solve It

4. What is 18° Celsius in Kelvin?

Solve It

5. What is 18° Kelvin in Celsius?

Solve It

6. What is 57° Kelvin in Fahrenheit?

Solve It

Getting Bigger: Linear Expansion

Ever try to open a screw-top jar by running hot water over it? That hot water makes the lid of the jar expand, making it easier to turn. This simple solution is physics on the job — it's all about *thermal expansion.*

You can see an example of thermal expansion in Figure 12-1, where a bar is undergoing expansion in one direction, called *linear expansion.*

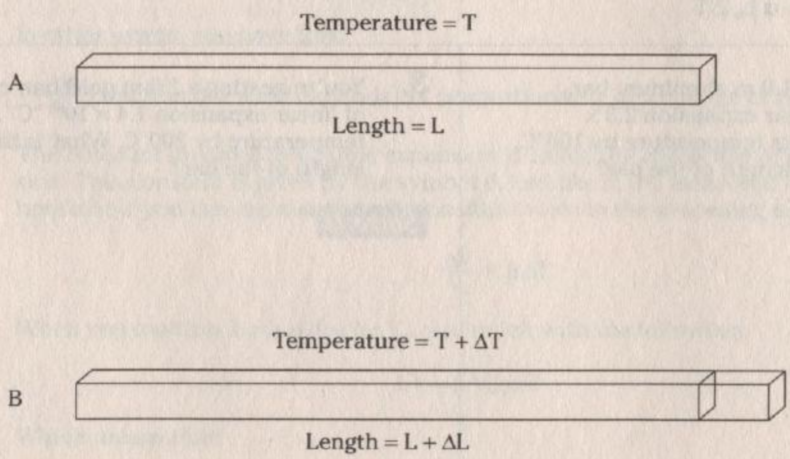

Figure 12-1: Linear expansion.

When you talk about the expansion of a solid in any one dimension under the influence of heat, you're talking about *linear expansion.* When you raise the temperature a small amount, this equation applies:

$$T_f = T_o + \Delta T$$

Linear expansion results in an expansion in any linear direction of the following:

$$L_f = L_o + \Delta L$$

If the temperature goes down a small amount, this equation applies:

$$T_f = T_o - \Delta T$$

You get a contraction instead of an expansion:

$$L_f = L_o - \Delta L$$

Like the coefficient of friction, a coefficient is in play here — the *coefficient of linear expansion,* which is given the symbol α. So you can write this:

$$\frac{\Delta L}{L_o} = \alpha \Delta T$$

This equation is usually written in this form:

$$\Delta L = \alpha L_o \Delta T$$

Here, α is usually measured in 1/°C, or $°C^{-1}$.

EXAMPLE

Q. You're heating a 1.0 m steel bar, coefficient of linear expansion 1.2×10^{-5} $°C^{-1}$, raising its temperature by 5°C. What is the final length of the bar?

A. The correct answer is $1.0 + 6.0 \times 10^{-5}$ m.

1. Use this equation:

$$\Delta L = \alpha L_o \Delta T$$

2. Plug in the numbers:

$$\Delta L = \alpha L_o \Delta T = (1.2 \times 10^{-5}) \cdot (1.0) \cdot (5) = 6.0 \times 10^{-5} \text{ m}$$

3. The final length is

$$L_f = L_o + \Delta L = 1.0 + 6.0 \times 10^{-5} \text{ m}$$

7. You're heating a 1.0 m aluminum bar, coefficient of linear expansion 2.3×10^{-5} $°C^{-1}$, raising its temperature by 100°C. What is the final length of the bar?

Solve It

8. You're heating a 2.0 m gold bar, coefficient of linear expansion 1.4×10^{-5} $°C^{-1}$, raising its temperature by 200°C. What is the final length of the bar?

Solve It

9. You're heating a 1.5 m copper bar, coefficient of linear expansion $1.7 \times 10^{-5}\ °C^{-1}$, raising its temperature by 300°C. What is the final length of the bar?

Solve It

10. You're heating a 2.5 m lead bar, coefficient of linear expansion $2.9 \times 10^{-5}\ °C^{-1}$, raising its temperature by 40°C. What is the final length of the bar?

Solve It

Plumping It Up: Volume Expansion

In addition to linear expansion, physics problems can ask you to find volume expansion. Linear expansion takes place in only one dimension, but volume expansion happens in all three dimensions.

In other words, you have this:

$$\frac{\Delta V}{V_o} \text{ (fraction the solid expands) is proportional to } \Delta T \text{ (change in temperature)}$$

The constant involved in volume expansion is called the *coefficient of volume expansion.* This constant is given by the symbol β, and like α, it's measured in $°C^{-1}$. Using β, here's how you can express the relationship shown in the preceding equation:

$$\frac{\Delta V}{V_o} = \beta \Delta T$$

When you multiply both sides by V_o, you're left with the following:

$$\Delta V = \beta V_o \Delta T$$

Which means that:

$$V_f = V_o + \beta V_o \Delta T$$

Q. You're heating a 1.0 m^3 steel block, coefficient of volume expansion 3.6×10^{-5} °C^{-1}, raising its temperature by 45°C. What is the final volume of the bar?

A. The correct answer is $1.0 + 1.6 \times 10^{-3}$ m^3.

1. Use this equation:

$$\Delta V = \beta V_o \Delta T$$

2. Plug in the numbers:

$$\Delta V = \beta V_o \Delta T = (3.6 \times 10^{-5})\cdot(1.0)\cdot(45) = 1.6 \times 10^{-3} \text{ m}^3$$

3. The final volume is

$$V_f = V_o + \Delta V = 1.0 + 1.6 \times 10^{-3} \text{ m}^3$$

11. You're heating a 2.0 m^3 aluminum block, coefficient of volume expansion 6.9×10^{-5} °C^{-1}, raising its temperature by 30°C. What is the final volume of the block?

Solve It

12. You're heating a 2.0 m^3 copper block, coefficient of volume expansion 5.1×10^{-5} °C^{-1}, raising its temperature by 20°C. What is the final volume of the block?

Solve It

13. You're heating a 1.0 m^3 glass block, coefficient of volume expansion $1.0 \times 10^{-5}\ °C^{-1}$, raising its temperature by 27°C. What is the final volume of the block?

Solve It

14. You're heating a 3.0 m^3 gold block, coefficient of volume expansion $4.2 \times 10^{-5}\ °C^{-1}$, raising its temperature by 18°C. What is the final volume of the block?

Solve It

Getting Specific with Heat Capacity

It's a fact of physics that it takes 4186 J to raise the temperature of 1.0 kg of water by 1° C. But it takes only 840 J to raise the temperature of 1.0 kg of glass by 1° C.

You can relate the amount of heat, Q, it takes to raise the temperature of an object to the change in temperature and the amount of mass involved. Use this equation:

$$Q = m \cdot c\ \Delta T$$

In this equation, Q is the amount of heat energy involved (measured in Joules if you're using the MKS system), m is the amount of mass, ΔT is the change in temperature, and c is a constant called the *specific heat capacity,* which is measured in J/(kg-°C) in the MKS system.

REMEMBER

So it takes 4186 J of heat energy to warm up 1.0 kg of water 1.0°C. One calorie is defined as the amount of heat needed to heat 1.0 g of water 1.0°C, so 1 calorie equals 4.186 J. Nutritionists use the food energy term *Calorie* (capital C) to stand for 1000 calories, 1.0 kcal, so 1.0 Calorie equals 4186 J. And when you're speaking in terms of heat, you have another unit of measurement to deal with: the British Thermal Unit (Btu). 1.0 Btu is the amount of heat needed to raise one pound of water 1.0°F. To convert Btus to Joules, use the relation that 1 Btu equals 1055 J.

If you add heat to an object, raising its temperature from T_o to T_f, the amount of heat you need is expressed as:

$$\Delta Q = m \cdot c \cdot (T_f - T_o)$$

EXAMPLE

Q. You're heating a 1.0 kg copper block, specific heat capacity of 387 J/(kg-°C), raising its temperature by 45°C. What amount of heat do you have to apply?

A. The correct answer is 17,400 J.

1. Use this equation:

$$Q = m \cdot c \cdot \Delta T$$

2. Plug in the numbers:

$$Q = m \cdot c \cdot \Delta T = (387) \cdot (1.0) \cdot (45) = 17{,}400 \text{ J}$$

15. You're heating a 15.0 kg copper block, specific heat capacity of 387 J/(kg-°C), raising its temperature by 100°C. What heat do you have to apply?

Solve It

16. You're heating a 10.0 kg steel block, specific heat capacity of 562 J/(kg-°C), raising its temperature by 170°C. What heat do you have to apply?

Solve It

17. You're heating a 3.0 kg glass block, specific heat capacity of 840 J/(kg-°C), raising its temperature by 60°C. What heat do you have to apply?

Solve It

18. You're heating a 5.0 kg lead block, specific heat capacity of 128 J/(kg-°C), raising its temperature by 19°C. What heat do you have to apply?

Solve It

19. You're cooling a 10.0 kg lead block, specific heat capacity of 128 J/(kg-°C), lowering its temperature by 60°C. What heat do you have to extract?

Solve It

20. You're cooling a 80.0 kg glass block, specific heat capacity of 840 J/(kg-°C), lowering its temperature by 16°C. What heat do you have to extract?

Solve It

21. You put 7600 J into a 14 kg block of silver, specific heat capacity of 235 J/(kg-°C). How much have you raised its temperature?

Solve It

22. You add 10,000 J into a 8.0 kg block of copper, specific heat capacity of 387 J/(kg-°C). How much have you raised its temperature?

Solve It

Changes of Phase: Latent Heat

Heating blocks of lead is fine, but if you heat that lead enough, sooner or later it's going to melt. When it melts, its temperature stays the same until it liquefies, and then the temperature of the lead increases again as you add heat. So why does its temperature stay constant as it melts? Because the heat you applied went into melting the lead. There's a latent heat of melting that means that so many Joules must be applied per kilogram to make lead change phase from solid to liquid.

REMEMBER

The units of latent heat are J/kg.

There are three phase changes that matter can go through — solid, liquid, and gas — and each transition has a latent heat:

- **Solid to liquid:** The latent heat of melting (or heat of fusion), L_f, is the heat per kilogram needed to make the change between the solid and liquid phases (such as when water turns to ice).
- **Liquid to gas:** The latent heat of vaporization, L_v, is the heat per kilogram needed to make the change between the liquid and gas stages (such as when water boils).
- **Solid to gas:** The latent heat of sublimation, L_s, is the heat per kilogram needed to make the change between the solid and gas phases (such as the direct sublimation of dry ice (CO^2) to the vapor state).

The latent heat of fusion of water is about 3.35×10^5 J/kg. That means it takes 3.35×10^5 J of energy to melt 1 kg of ice.

EXAMPLE

Q. You have a glass of 50.0 g of water at room temperature, 25°C, but you'd prefer ice water at 0°C. How much ice at 0.0°C do you need to add?

A. The correct answer is 15.6 g.

1. The heat absorbed by the melting ice must equal the heat lost by the water you want to cool. Here's the heat lost by the water you're cooling:

$$\Delta Q_{water} = m \cdot c\ \Delta T = m \cdot c \cdot (T_f - T_o)$$

2. Plug in the numbers:

$$\Delta Q_{water} = m \cdot c \cdot \Delta T = m \cdot c \cdot (T_f - T_o) = (0.050)\ (4186)\ (0 - 25) = -5.23 \times 10^3\ J$$

3. So the water needs to lose 5.23×10^3 J. How much ice would that melt? That looks like this, where L_m is the latent heat of melting:

$$\Delta Q_{ice} = m_{ice} \cdot L_m$$

4. You know that for water, L_m is 3.35×10^5 J/kg, so you get this:

$$\Delta Q = m_{ice} \cdot L_m = m_{ice} \cdot 3.35 \triangle 10^5$$

5. You know that equation has to be equal to the heat lost by the water, so you can set it to:

$$\Delta Q_{ice} = \Delta Q_{water}$$

In other words:

$$m_{ice} = \frac{\Delta Q_{water}}{L_m} = \frac{5.23 \times 10^3\ J}{3.35 \times 10^5\ J/kg}$$

6. You know that the latent heat of melting for water is $L = 3.35 \times 10^5$ J/kg, which means that:

$$m_{ice} = \frac{5.23 \times 10^3}{3.35 \times 10^5} = 1.56 \times 10^{-2}\ kg$$

So you need 1.56×10^{-2} kg, or 15.6 g of ice.

23. You have 100.0 g of coffee in your mug at 80°C. How much ice at 0.0°C would it take to cool 100.0 g of coffee at 80°C to 65°C?

Solve It

24. You have 200.0 g of cocoa at 90°C. How much ice at 0.0°C do you have to add to the cocoa (assuming that it has the specific heat capacity of water) to cool it down to 60°C?

Solve It

Chapter 13

Under Pressure: From Solid to Liquid to Gas

In This Chapter

- Handling heat convection and heat conduction
- Dealing with heat radiation
- Using Avogadro's Number
- Working with molecular motion

This chapter is concerned with all kinds of things heat related — heat transfer, for example, which is about how fast heat travels along iron bars, aluminum pot handles, and more. Want to know how long you can hold that pot over an open flame? This chapter is for you.

You also find here the famous ideal gas law, which connects the pressure, volume, and temperature of a gas. Want to know the pressure of a volume of gas at a particular temperature? Just turn to the ideal gas law, and it'll give you the answer.

Plenty of thermodynamics comes up in this chapter, from how fast heat travels to what pressure gas has at a certain temperature.

In this chapter, you discover the three primary means by which heat moves: convection, conduction, and radiation.

How Heat Travels: Convection

Take a look at Figure 13-1, where you see a pot of water being heated. The water in that pot moves in the pattern you see in the figure; as it circulates, the hot water moves from the bottom to the top.

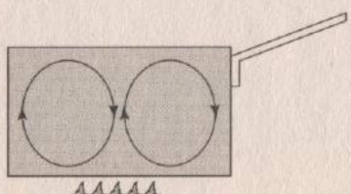

Figure 13-1: A convection example.

That's the first method of heat transfer: convection. *Convection* allows heat to move by the motion of heated matter.

EXAMPLE

Q. How does heat move from the bottom of the pot to the top in Figure 13-1?

A. The correct answer is convection.

The heat moves through convection. The water heats up near the flame and then rises in the pot.

1. How does heat move from the bottom inside a stove to the top?

Solve It

2. How does heat move from the Earth's surface to 2000 ft, where an eagle is coasting?

Solve It

How Heat Travels: Conduction

Take a look at Figure 13-2, where you see a metal pot being heated — and the handle's getting warm. The mechanism that transfers heat between points on the pot's handle is called *conduction.*

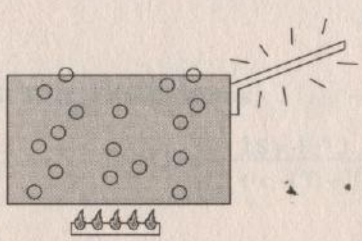

Figure 13-2: An example of heat conduction.

How much heat travels between two points via conduction? The amount of heat is proportional to the difference in temperature, so for heat conduction between the ends of a bar of metal, you'd have this:

$$Q \propto \Delta T$$

As you may expect, a bar twice as wide conducts twice the amount of heat. In general, the amount of heat conducted is proportional to the cross-sectional area:

$$Q \propto A$$

Also, the longer the bar, the less heat makes it all the way through. In fact, the conducted heat turns out to be inversely proportional to the length of the bar:

$$Q \propto \frac{1}{L}$$

Finally, the amount of heat transferred depends on the amount of time that passes — twice the time, twice the heat. That makes sense too. Here's how you express that mathematically:

$$Q \propto t$$

So here's what you get when you put it all together (k is some constant yet to be determined):

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

What is the constant k? This constant is called the thermal conductivity of a material, and its units are J/(s-m-°C).

Q. The thermal conductivity of copper is 390 J/(s-m-°C). How much heat is conducted per second between two points on a copper rod, cross section 0.1 m^2, 1.0 m apart, with a temperature difference of 69°C?

A. The correct answer is 2690 J/sec.

1. Use this equation:

$$Q = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

2. Solve for Q/t:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T \cdot t}{L}$$

3. Plug in the numbers:

$$\frac{Q}{t} = \frac{k \cdot A \cdot \Delta T \cdot t}{L} = \frac{(390) \cdot (0.1) \cdot (69)}{1.0} =$$

2690 J/sec

3. The thermal conductivity of steel is 79 J/(s-m-°C). How much heat is conducted per second between two points on a steel rod, cross section 0.3 m^2, 2.0 m apart, with a temperature difference of 34°C?

Solve It

4. The thermal conductivity of silver is 420 J/(s-m-°C). How much heat is conducted per second between two points on a silver rod, cross section 0.40 m^2, 1.5 m apart, with a temperature difference of 91°C?

Solve It

5. If you have a steel rod, thermal conductivity 79 J/(s-m-°C), with a cross section 0.30 m^2 and a length of 50 cm, how long do you have to wait for 1000.0 J to be transferred from one end to the other if the temperature difference is 90°C?

Solve It

6. If you have a brass rod, thermal conductivity 110 J/(s-m-°C), with a cross section 0.50 m^2 and a length of 73 cm, how long do you have to wait for 3000.0 J to be transferred from one end to the other if the temperature difference is 180°C?

Solve It

7. You have a mystery substance, length 2.0 m, cross section 0.40 m^2, and apply a temperature difference of 69°C across its length. If the substance conducts 1000.0 J of heat in 0.50 sec, what is its thermal conductivity?

Solve It

8. You have an unknown material, length 1.0 m, cross section 0.30 m^2, and apply a temperature difference of 97°C across its length. If the substance conducts 1400.0 J of heat in 0.60 sec, what is its thermal conductivity?

Solve It

How Heat Travels: Radiation

The third way that heat travels, after convection and conduction, is *radiation*. Anything hot radiates heat in the form of electromagnetic radiation, which is the way the sun warms the Earth. The sun can't warm the Earth due to convection or conduction, because a vacuum exists between here and there. Instead, the sun beams light to the Earth in a wide range of frequencies, and that energy warms the Earth's surface.

The amount of heat transferred this way is proportional to the amount of time that the radiant object beams energy:

$$Q \propto t$$

As you may also expect, the amount of heat radiated is proportional to the total area doing the radiating — twice as much area doing the radiating, twice as much heat radiated. So you can write this equation, where A is the area doing the radiating:

$$Q \propto A \cdot t$$

You'd expect temperature, T, to be in here somewhere — the hotter an object, the more heat radiated. Hold on to your hat: It turns out that the amount of heat radiated is proportional to T in Kelvins to the fourth power, T^4. So now you have

$$Q \propto A \cdot t \cdot T^4$$

To make this relationship an equation, all you need to add is a constant, which is measured experimentally. This constant, called the Stefan-Boltzmann constant, σ, goes in like this:

$$Q = \sigma \cdot A \cdot t \cdot T^4$$

The value of σ is 5.67×10^{-8} J/(s-m^2-K^4).

This equation works only for objects that are so-called perfect emitters, however. Most objects are not perfect emitters, so you have to add another constant that depends on the substance you're working with. This constant is called its *emissivity,* e. So this equation, the Stefan-Boltzmann law of radiation, becomes

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

An object's emissivity depends on what it's made of, so you can expect emissivity to vary from material to material.

EXAMPLE

Q. A person has a surface area of 1.8 m^2, an emissivity of 0.65, and a temperature of 37°C. How much power does this person radiate?

A. The correct answer is 610 W.

1. Use this equation:

$$Q = e \cdot \sigma \cdot A \cdot t \cdot T^4$$

2. Solve for power, Q/t:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4$$

3. Plug in the numbers, remembering to convert from Celsius to Kelvin:

$$\frac{Q}{t} = e \cdot \sigma \cdot A \cdot T^4 =$$

$$(0.65) \cdot (5.67 \times 10^{-8}) \cdot (1.8) \cdot (37 + 273)^4 =$$

610 W

9. A coffeepot has a temperature of 90°C and a surface area of 0.20 m^2. If its emissivity is 0.95, what power is it radiating?

Solve It

10. A toaster has a temperature of 120°C and a surface area of 0.15 m^2. If its emissivity is 0.90, what power is it radiating?

Solve It

11. If a 200 g coffeepot holds 300 g of coffee, its temperature is 80°C, and its surface area is 0.18 m^2, how long will it take for the coffee to cool to 75°C, given that the pot's emissivity is 0.90 (omitting the fact that the pot is heated by radiation coming from its surroundings)?

Solve It

12. If a 100 g teacup holds 200 g of tea, its temperature is 82°C, and its surface area is 0.07 m^2, how long will it take for the tea to cool to 76°C, given that the cup's emissivity is 0.60 (omitting the fact that the cup is heated by radiation coming from its surroundings)?

Solve It

A Biggie: Avogadro's Number

A certain amount of mass contains a specific number of atoms, and you can figure out just how many atoms a certain amount of mass has. In particular, a *mole* is defined as the number of atoms in 12.0 g of carbon isotope 12. (Carbon isotope 12, also called carbon-12 or just carbon 12, is the most common version of carbon, although some carbon atoms have a few more neutrons in them — carbon 13, actually — so the average works out to about 12.011.)

That number of atoms has been measured as 6.022×10^{23}, which is called Avogadro's Number, N_A. That number represents many atoms. Now you know how many atoms are in 12.0 g of carbon 12.

Does 12.0 g of sulfur have the same number of atoms? No. Just check a periodic table of the elements. You find that the atomic mass of sulfur is 32.06. But 32.06 what? It turns out to mean 32.06 atomic units, u, where each atomic unit is $\frac{1}{12}$ of the mass of a carbon 12 atom. So if a mole of carbon 12 has a mass of 12.0 g, and the mass of an average sulfur atom is bigger than the mass of a carbon 12 atom by this ratio

$$\frac{\text{Sulfur mass}}{\text{Carbon 12 mass}} = \frac{32.06}{12 \text{ u}}$$

a mole of sulfur atoms must have this mass:

$$\frac{32.06}{12 \text{ u}}(12.0 \text{ g}) = 32.06 \text{ g}$$

Note that sulfur and carbon are composed of simple atoms; if you're dealing with a composite substance such as water, you have to think in terms of molecules instead. So instead of the atomic mass in cases like these, you look for the molecular mass (when atoms combine, you have molecules), which is also measured in atomic mass units. The molecular mass of water is 18.0153 u, so 1 mole of water molecules has a mass of 18.0153 g.

EXAMPLE

Q. How many molecules are in 10.0 g of water?

A. The correct answer is 3.3×10^{23} molecules.

1. You know that 1 mole of water has this mass:

   1 mole of water = 18.0 g

2. A mole has 6.022×10^{23} molecules, so 10.0 g has this many molecules:

$$6.022 \times 10^{23} \frac{(10.0)}{(18.0)} = 3.3 \times 10^{23}$$

13. You have 10.0 g of calcium, atomic mass 40.08 u. How many atoms do you have?

Solve It

14. You have 16.0 g of silicon, atomic mass 28.09 u. How many atoms do you have?

Solve It

15. You have 29.0 g of zinc, atomic mass 65.41 u. How many atoms do you have?

Solve It

16. You have 3.0 g of copper, atomic mass 63.546 u. How many atoms do you have?

Solve It

Ideally Speaking: The Ideal Gas Law

You can relate the pressure, volume, and temperature of an ideal gas with the ideal gas equation. An *ideal gas* is one whose molecules act like points; no interaction occurs among molecules except *elastic collisions* (that is, where kinetic energy is conserved). In practice, all gases act like ideal gases to some extent, so the ideal gas law holds fairly well. Here's that law:

$$P \cdot V = n \cdot R \cdot T$$

Here, P is pressure; n is the number of moles of gas you have; R is the *universal gas constant,* which has a value of 8.31 J(/mole-K); and T is measured in Kelvins. The volume, V, is measured in cubic meters, m^3. Sometimes, you'll see volume given in liters, where $1.0\ L = 10^{-3}\ m^3$. Using this law, you can predict the pressure of an ideal gas, given how much you have of it, its temperature, and the volume you've enclosed it in.

One mole of ideal gas takes up 22.4 L of volume at 0°C and one atmosphere pressure, which is $1.013 \times 10^5\ N/m^2$, where N/m^2 (Newtons per square meter) is given its own units, Pascals, abbreviated Pa.

You can also write the ideal gas law a little differently by using Avogadro's Number, N_A, and the total number of molecules, N:

$$P \cdot V = n \cdot R \cdot T = (N/N_A)\, R \cdot T$$

The constant R/N_A is also called Boltzmann's constant, k, and it has a value of 1.38×10^{-23} J/K. Using this constant, the ideal gas law becomes

$$P \cdot V = N \cdot k \cdot T$$

Q. You have 1.0 moles of air in your tire at 0°C, volume 10.0 L. What is the gauge pressure within the tire?

A. The correct answer is 1.3×10^5 Pa.

1. Use this equation:

   $$PV = nRT$$

2. Solve for P:

   $$P = nRT / V$$

3. **Plug in the numbers to get the pressure pushing out:**

   $$P = nRT / V = (1.0)(8.31)(273) / (10 \times 10^{-3}) = 2.3 \times 10^5 \text{ Pa}$$

4. **Subtract the pressure from the surrounding air, which pushes in, assuming that the air is at 0°C too:**

   $$P = 2.3 \times 10^5 - 1.013 \times 10^5 = 1.3 \times 10^5 \text{ Pa}$$

17. You have 2.3 moles of air in your tire at 0°C, volume 12.0 L. What pressure is the tire inflated to?

Solve It

18. You have a bottle of 2.0 moles of gas, volume 1.0 L, temperature 100°C. What is the pressure inside the bottle?

Solve It

Molecules in Motion

The average kinetic energy of molecules in a gas is

$$KE_{avg} = \frac{3}{2} \cdot k \cdot T$$

Here, k is Boltzmann's constant, 1.38×10^{-23} J/K. Now you can determine the average kinetic energies of the molecules in a gas. And because you can determine the mass of each molecule if you know what gas you're dealing with, you can figure out their average speeds at various temperatures.

EXAMPLE

Q. What is the speed of air molecules at 28°C?

A. The correct answer is 517 m/s.

1. Use this equation:

$$KE_{avg} = \frac{3}{2} \cdot k \cdot T$$

2. Plug in the numbers:

$$KE_{avg} = \frac{3}{2} kT = \frac{3}{2}(1.38 \times 10^{-23})(301) = 6.23 \times 10^{-21} \text{ J}$$

3. You know that:

$$KE = \frac{1}{2} m \cdot v^2$$

4. Solve for v:

$$v = \sqrt{\frac{2KE}{m}}$$

5. Plug in the numbers. Air is mostly nitrogen, and each nitrogen atom has a mass of about 4.65×10^{-26} kg:

$$v = \sqrt{\frac{2KE}{m}} = 517 \text{ m/s} = 1150 \text{ mph}$$

19. What is the average speed of air molecules at 43°C?

Solve It

20. What is the average speed of air molecules at 900°C?

Solve It

Chapter 14

All about Heat and Work

In This Chapter

- Understanding the laws of thermodynamics
- Working with isobaric processes
- Handling isochoric processes
- Calculating isothermal processes

This chapter is all about the laws of thermodynamics. Those laws describe all kinds of heat processes. You also discover how to handle heat processes where pressure is constant, or volume is constant, or temperature is constant.

The First Law of Thermodynamics

The first law of thermodynamics says that energy is conserved. The internal energy in a system, U_o, changes to a final value U_f when heat Q is absorbed or released by the system and the system does work W on its surroundings, or the surroundings do work W on the system, such that:

$$U_f - U_o = \Delta U = Q - W$$

The value Q is positive when the system absorbs heat and negative when the system releases heat. The quantity W is positive when the system does work on its surroundings and negative when the surroundings do work on the system.

This law is useful because it says that total energy — work plus heat — is conserved.

Q. Say that a motor does 1000 J of work on its surroundings while releasing 3000 J of heat. By how much does its internal energy change?

A. The correct answer is –4000 J.

1. You know that the motor does 1000 J of work on its surroundings, so you know that its internal energy decreases by 1000 J.
2. The motor also releases 3000 J of heat while doing its work, so the internal energy of the system decreases by an additional 3000 J. Think of negative values of work and heat as flowing out of the system as negative, making the total change of internal energy this:

$$\Delta U = -1000 - 3000 = -4000 \text{ J}$$

1. You have a motor that absorbs 3000 J of heat while doing 2000 J of work. What is the change in the motor's internal energy?

Solve It

2. You have a motor that absorbs 2500 J of heat while doing 1700 J of work. What is the change in the motor's internal energy?

Solve It

Constant Pressure: Isobaric Processes

Take a look at Figure 14-1, in which a lid with a weight on it keeps constant pressure on a gas as that gas expands. Work that's done at constant pressure is called *isobaric* work.

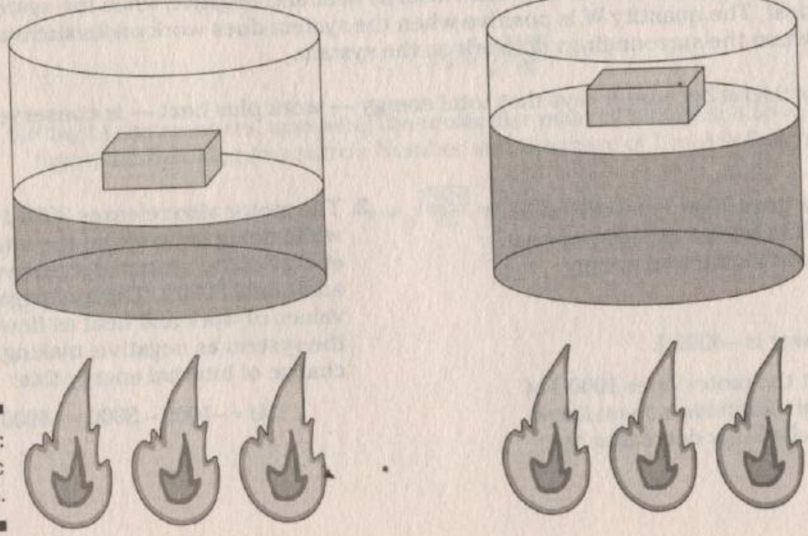

Figure 14-1: An isobaric system.

So the question is: What work is the system doing as it expands? Work = F·s, and F = P·A, where P is the pressure and A is the area. That means that:

$$W = F \cdot \Delta s = P \cdot A \cdot \Delta s$$

On the other hand, $A \cdot \Delta s = \Delta V$, the change in volume, so you have

$$W = P \cdot \Delta V$$

You can see what this looks like graphically for an isobaric process in Figure 14-2, in which the volume is changing while the pressure stays constant. Because $W = F\Delta V$, the work is the area beneath the graph as shown in the figure.

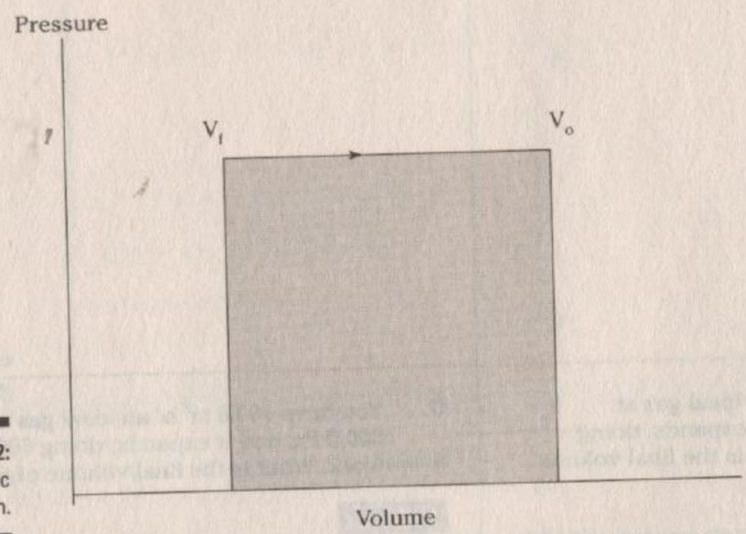

Figure 14-2: An isobaric graph.

All you've got to do is to plug in the numbers:

$$W = P \cdot \Delta V = (200) \cdot (120 - 60) = 12{,}000 \text{ J}$$

You get your answer: The gas did 12,000 J of work as it expanded under constant pressure.

Q. You have 60.0 m^3 of an ideal gas at 200.0 Pa and heat the gas until it expands to a volume of 120 m^3. How much work did the gas do?

A. The correct answer is 12,000 J.

1. Use this equation:

$$W = P \cdot \Delta V$$

2. Plug in the numbers:

$$W = P \cdot \Delta V = (200)\,(120 - 60) = 12{,}000 \text{ J}$$

3. You have 50.0 m^3 of an ideal gas at 1000.0 Pa and heat the gas until it expands to a volume of 300.0 m^3. How much work did the gas do?

Solve It

4. You have 300.0 m^3 of an ideal gas at 1500.0 Pa and heat the gas until it expands to a volume of 900.0 m^3. How much work did the gas do?

Solve It

5. You have 50.0 m^3 of an ideal gas at 1000.0 Pa, and the gas expands, doing 3000.0 J of work. What is the final volume of the gas?

Solve It

6. You have 100.0 m^3 of an ideal gas at 300.0 Pa, and it expands, doing 6000.0 J of work. What is the final volume of the gas?

Solve It

Constant Volume: Isochoric Processes

When an ideal gas's pressure increases at constant volume, how much work is done? Because

$$W = P \cdot \Delta V$$

the answer is simple: No work is being done. This is called an *isochoric* process, and you can see a graph of what's happening in Figure 14-3.

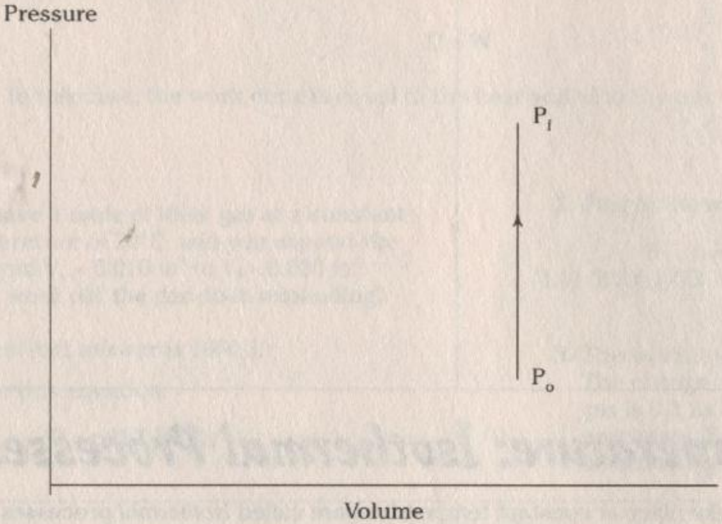

Figure 14-3: An isochoric graph.

As you see in the figure, the pressure rises while the volume stays the same, so no work is actually being done.

EXAMPLE

Q. You have 60.0 m^3 of an ideal gas at 200.0 Pa and heat the gas until its pressure is 300.0 Pa at the same volume. How much work did the gas do?

A. The correct answer is none.

1. Use this equation:

$$W = P \cdot \Delta V$$

2. Because $\Delta V = 0$, no work was done. This process is an isochoric process.

7. You have 50.0 m^3 of an ideal gas at 1000.0 Pa and heat the gas until it has a pressure of 3000.0 Pa, still at the same volume. How much work did the gas do?

Solve It

8. You have 300.0 m^3 of an ideal gas at 1500.0 Pa and heat the gas until it has a pressure of 6000.0 Pa. How much work did the gas do?

Solve It

Constant Temperature: Isothermal Processes

Processes that take place at constant temperature are called *isothermal* processes. What's the work look like in this case as the volume changes? Because PV = nRT, the relation between P and V is

$$P = \frac{n \cdot R \cdot T}{V}$$

That relationship looks like the graph you see in Figure 14-4.

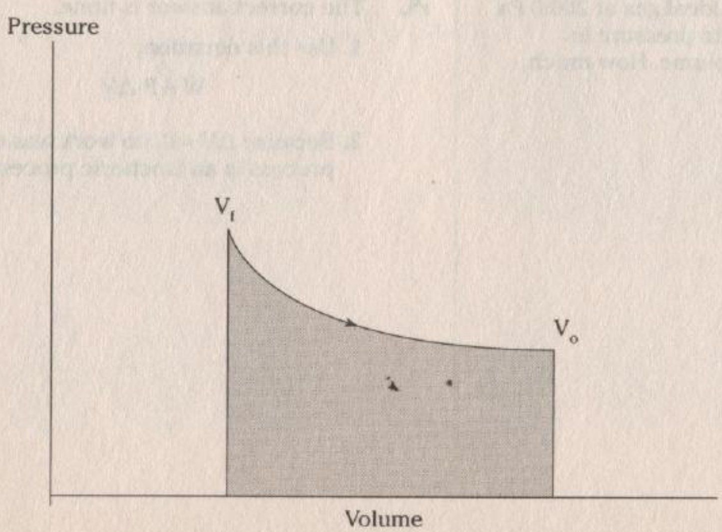

Figure 14-4: An isothermal graph.

As before, the work done is the area underneath the graph. What is that area? The work done is given by this equation in this case, where ln is the natural log:

$$W = n \cdot R \cdot T \ln(V_f / V_o)$$

Note that because in an isothermal process the temperature stays constant, and because for an ideal gas the internal energy = (3/2)·n·R·T, the internal energy doesn't change. So you have

$$\Delta U = 0 = Q - W$$

In other words:

$$Q = W$$

In this case, the work done is equal to the heat added to the gas.

EXAMPLE

Q. You have 1 mole of ideal gas at a constant temperature of 20°C, and you expand the gas from $V_o = 0.010\ m^3$ to $V_f = 0.020\ m^3$. What work did the gas do in expanding?

A. The correct answer is 1690 J.

1. Use this equation:

$$W = n \cdot R \cdot T \cdot \ln(V_f / V_o)$$

2. Plug in the numbers:

$$W = n \cdot R \cdot T \cdot \ln(V_f / V_o) = (1.0)\ (8.31)\ (273.15 + 20)\ \ln(0.020 / 0.010) = 1690\ J$$

3. The work done by the gas was 1690 J. The change in the internal energy of the gas is 0 J, as it must be in isothermal processes. Because Q = W, the heat added to the gas is equal to 1690 J.

9. You have 1 mole of ideal gas at a constant temperature of 30.0°C, and you expand the gas from $V_o = 2.0\ m^3$ to $V_f = 3.0\ m^3$. What work did the gas do in expanding?

Solve It

10. You have 0.60 mole of ideal gas at a constant temperature of 25°C, and you expand the gas from $V_o = 1.0\ m^3$ to $V_f = 3.0\ m^3$. What heat was supplied to the gas in expanding?

Solve It

At Constant Heat: Adiabatic

The last type of thermodynamic process is the *adiabatic* process, in which the total heat in the system is held constant. Take a look at the system in Figure 14-5; everything is surrounded by an insulating substance, so the heat from the system isn't going anywhere. When a change occurs, then, it's going to be an adiabatic change.

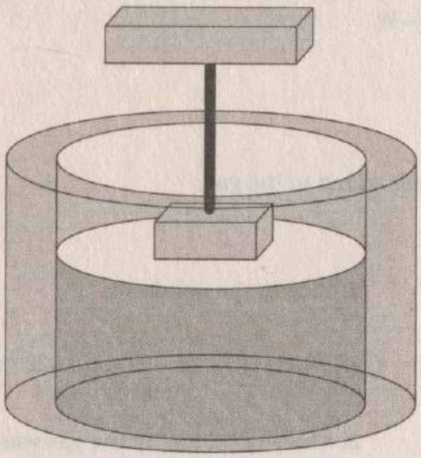

Figure 14-5: An adiabatic system.

Okay, what's the work done during an adiabatic process? Here, $\Delta Q = 0$, so $\Delta U = -W$. Because the internal energy of an ideal gas is $U = (3/2) \cdot n \cdot R \cdot T$, the work done is

$$W = \frac{3}{2} n \cdot R \cdot (T_o - T_f)$$

In other words, if the gas does work in an adiabatic process, the work comes from a change in temperature. If the final temperature is lower, the system does work on its surroundings.

You can see what P versus V looks like for an adiabatic process in Figure 14-6. The adiabatic curve here, called an *adiabat,* is not the same as the isothermal curves, called *isotherms.* The work done when the total heat in the system is constant is the area under the adiabat, as shown in Figure 14-6.

You can relate the initial pressure and volume to the final pressure and volume in an adiabatic process this way:

$$P_o \cdot V_o \gamma = P_f \cdot V_f \gamma$$

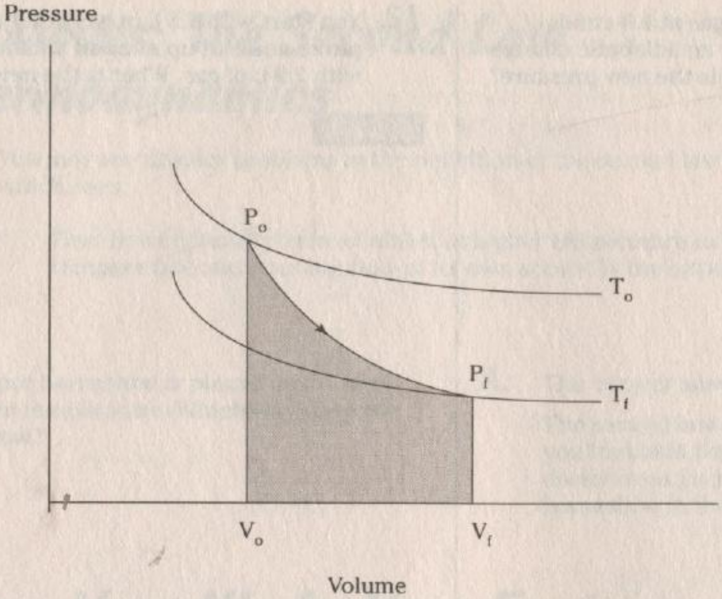

Figure 14-6: An adiabatic graph.

What's γ? It's the ratio of the specific heat capacity of an ideal gas at constant pressure divided by the specific heat capacity of an ideal gas at constant volume, c_p/c_v. What are c_p and c_v?

$$C_v = \frac{3}{2} \cdot R$$

$$C_p = \frac{5}{2} \cdot R$$

That makes the ratio γ this:

$$\gamma = \frac{C_p}{C_v} = \frac{5}{3}$$

So you can connect pressure and volume at any two points along an adiabat this way:

$$P_o \cdot V_o^{5/3} = P_f \cdot V_f^{5/3}$$

These processes are adiabatic; no heat is gained or lost.

EXAMPLE

Q. You start with 3.0 L of gas at 1.0 atmosphere and end up, after an adiabatic change, with 6.0 L of gas. What is the new pressure?

A. The correct answer is 0.31 atmosphere.

1. Use this equation:

$$P_f = \frac{P_o \cdot V_o^{5/3}}{V_f^{5/3}}$$

2. Plug in the numbers:

$$P_f = \frac{P_o \cdot V_o^{5/3}}{V_f^{5/3}} = \frac{(1.0)(3.0)^{5/3}}{(6.0)^{5/3}} = 0.31 \text{ atmosphere}$$

11. You start with 1.0 L of gas at 1.0 atmosphere and end up after an adiabatic change with 3.0 L of gas. What is the new pressure?

Solve It

12. You start with 1.5 L of gas at 1.7 atmosphere and end up after an adiabatic change with 2.9 L of gas. What is the new pressure?

Solve It

13. You have 1.0 mole of ideal gas that undergoes an adiabatic change, going from 30.0°C to 10.0°C. What work did the gas do?

Solve It

14. You have 3.0 moles of ideal gas that undergo an adiabatic change, going from 23°C to 69°C. What work was done on the gas?

Solve It

Heat Moves: The Second Law of Thermodynamics

You may see physics problems in the definition of the second law of thermodynamics, which says,

> Heat flows naturally from an object at higher temperature to an object at lower temperature, and does not flow of its own accord in the opposite direction.

EXAMPLE

Q. A red-hot horseshoe is placed on an anvil at room temperature. Which way does the heat flow?

A. The correct answer is into the anvil.

The second law of thermodynamics tells you that heat flows from hotter objects to cooler ones, so heat flows from the red-hot horseshoe to the room-temperature anvil.

Making Heat Work: Heat Engines

You know how a steam engine works: Heated steam does the work. Physics makes a study of this process, and you can see what's going on diagrammatically in Figure 14-7. Heat is supplied to a heat engine, which does work and sends its exhaust to a lower-temperature heat sink. (Often, the heat sink is just the surroundings of the heat engine, as is the case with steam engines.)

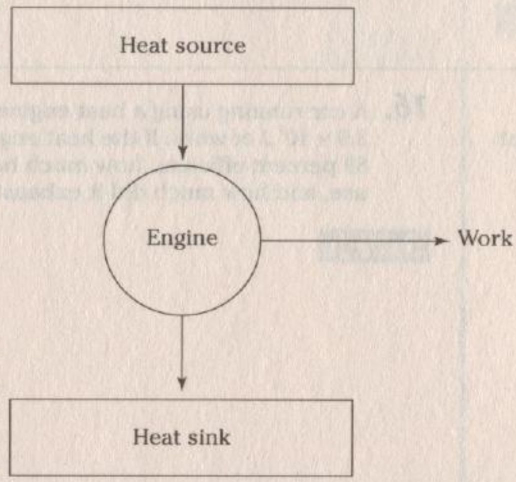

Figure 14-7: A heat engine.

Say that the heat supplied to the engine is Q_h, and the heat sent to the heat sink is Q_c (h and c stand for the hot and cold reservoirs of heat). In that case, you could say that the efficiency of the work engine in terms of turning heat into work is

$$\text{Efficiency} = \frac{\text{Work}}{\text{Heat input}} = \frac{W}{Q_h}$$

So if all the input heat is converted to work, the efficiency is 1.0. If none of the input heat is converted to work, the efficiency is 0.0.

Note that because total energy is conserved, the heat into the engine must equal the work done plus the heat sent to the heat sink, which means that:

$$Q_h = W + Q_c$$

That in turn means you can rewrite the efficiency in terms of just Q_h and Q_c this way:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

EXAMPLE

Q. Your car is powered by a heat engine and does 3.0×10^7 J of work getting you up a small hill. If the heat engine is 80 percent efficient, how much heat did it use, and how much did it exhaust?

A. The correct answer is 3.75×10^7 J, 0.75×10^7 J.

1. Use this equation:

$$\text{Efficiency} = \frac{W}{Q_h} = \frac{3.0 \times 10^7 \text{ J}}{Q_h} = .80$$

2. Solve for Q_h:

$$\frac{3.0 \times 10^7 \text{ J}}{.80} = Q_h = 3.75 \times 10^7 \text{ J}$$

So the input heat was 3.75×10^7 J.

3. Use this equation:

$$Q_h = W + Q_c$$

4. Solve for Q_c:

$$Q_h - W = Q_c$$

5. Plug in the numbers:

$$Q_h - W = 3.75 \times 10^7 - 3.0 \times 10^7 = 0.75 \times 10^7 = Q_c$$

So the output heat was 0.75×10^7 J.

15. A car running on a heat engine does 7.0×10^7 J of work. If the heat engine is 76 percent efficient, how much heat did it use, and how much did it exhaust?

Solve It

16. A car running using a heat engine does 3.9×10^7 J of work. If the heat engine is 89 percent efficient, how much heat did it use, and how much did it exhaust?

Solve It

17. A 63 percent efficient heat engine does 3.8×10^{10}J of work. How much heat did it use, and how much did it exhaust?

Solve It

18. An 87 percent efficient heat engine does 4.5×10^{10}J of work. How much heat did it use, and how much did it exhaust?

Solve It

19. A heat engine does 4.6×10^{7} J of work when supplied 8.9×10^{7} J. What is its efficiency?

Solve It

20. A heat engine does 8.1×10^{7} J of work when supplied 10.9×10^{7} J. What is its efficiency?

Solve It

Maximum Efficiency: Carnot Heat Engines

An engineer named Sadi Carnot figured out that the maximum possible efficiency of a heat engine is this:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

The temperatures T_h and T_c are the temperatures of the heat source and heat sink, respectively, measured in Kelvin.

That efficiency is the best a heat engine can get, assuming that no irreversible losses of energy occur due to friction or other causes. When you have a heat engine that does the best a heat engine can do, you have a Carnot engine, and the preceding equation is the expression for its efficiency.

EXAMPLE

Q. The heat source for a Carnot engine is at 100°C, and the heat sink is at 20°C. What is the engine's efficiency?

A. The correct answer is 21percent.

1. Use this equation:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h}$$

2. Plug in the numbers:

$$\text{Efficiency} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273.15 + 20)}{(273.15 + 100)} = 21\%$$

21. The heat source for a Carnot engine is at 87°C, and the heat sink is at 49°C. What is the engine's efficiency?

Solve It

22. The heat source for a Carnot engine is at 67°C, and the heat sink is at 29°C. What is the engine's efficiency?

Solve It

The Third Law of Thermodynamics

This chapter finishes with the third law of thermodynamics, which says

> You cannot reach absolute zero through any process which uses a finite number of steps.

In other words, you can get closer and closer to absolute zero step by step, but you can't actually reach it. This fact has been demonstrated in practice. Physicists have been able to approach absolute zero until they're just a fraction of a degree away, but no one has been able to reach it.